absorption in crossed fields[16] shows that the Franz-Keldysh exponential edge is not dramatically affected by low magnetic fields, which is in agreement with the two-band model. In a magnetic field of 100 kG and electric field 5×10^4 V/cm, the predicted decrease of cyclotron frequency in germanium amounts to 6-7%. Another optical crossed-field investigation in this material[15] seems to confirm this prediction. The approach presented is also valid for more general formulation of the Franz-Keldysh effect than that given by Tharmalingam[17] and for the description of tunneling phenomena in diodes in the presence of a magnetic field (Haering and Adams[18]).

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COHERENT ELASTIC SCATTERING AND INCOHERENT INELASTIC SCATTERING IN PROTON-NUCLEUS COLLISIONS AT HIGH ENERGY*

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Recent data on proton-nucleus scattering at 19.3 GeV/c show typical diffraction patterns.[1] For heavy nuclei they are similar to those observed from nuclear scattering at low energy, while somewhat different characteristics are shown by light nuclei. The difference may be attributed to incoherent scattering from nucleons in the nucleus which is more important for light nuclei where the transparency is larger due to the shorter paths of small-impact-parameter projectiles within the constant-density nuclear matter.

The present note uses a simple scattering model in order to extract geometrical and dynamical information from the experimental data about the target nuclei. The diffraction patterns are interpreted with the strong-absorption model (SAM)[2] which takes into account both the shadow effects of elementary plus nuclear inelastic open channels and the effect of Coulomb interaction. The large-angle cross section for light nuclei is interpreted as a free proton-nucleus scattering operating between the projectile and a small fraction of the target nucleons. This incoherent scattering is mainly an inelastic process by which the target nucleus may be transformed either into an excited state or directly into a daughter nucleus by the recoil or knock-out, respectively, of a few of its nucleons. Such events were not separated experimentally from the true elastic-scattering events.

Spin effects are believed to be small and are neglected in our analysis. We therefore use for the coherent elastic amplitude the spin-nonflip amplitude

$$f(\theta)=\frac{i}{2k}\sum_{l=0}^{\infty}(2l+1)[1-\eta_l\exp(2i\sigma_l)]P_l(\cos\theta), \quad (1)$$

where η_l is the nuclear reflection coefficient and σ_l is the Coulomb phase shift for the lth partial wave. k is the c.m. momentum.

The Coulomb interaction in our case contributes significantly only at relatively small angles. The Coulomb scattering at such small angles is due to the long-range part of the interaction and may be well approximated by the Rutherford formula[3]:

$$f_c(\theta)=-\frac{n}{2k\sin^2(\theta/2)}[1-v^2\sin^2(\theta/2)]^{1/2}, \quad (2)$$

where v is the c.m. velocity and n is the Sommerfeld parameter.

The SAM approximates η_l by

$$\mathrm{Re}\eta_l=(1-\epsilon)g(l)+\epsilon, \quad (3a)$$

$$\mathrm{Im}\eta_l=\mu dg/dl, \quad (3b)$$

where g is a continuous function of l which approximates unit step function at $L_0+\frac{1}{2}\approx kR$. R is the sum of the radii of the colliding particles. The real phase shifts enter via the parameter μ which is responsible for the po-

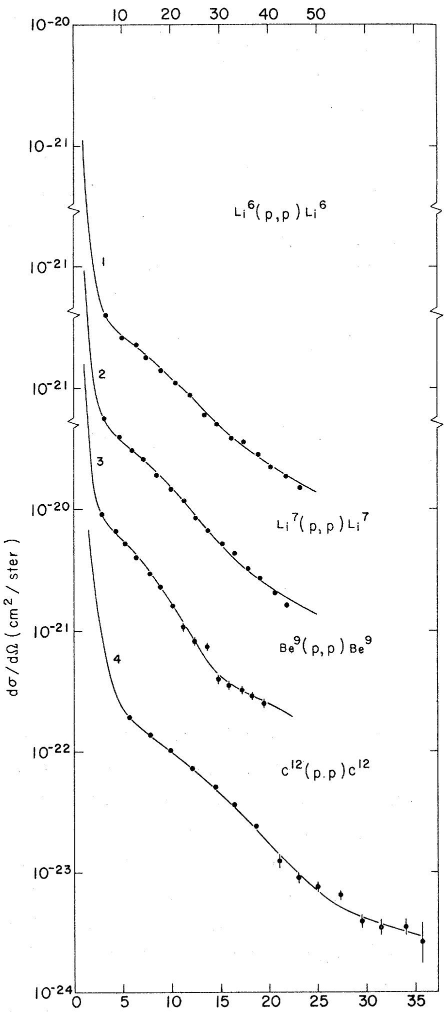

FIG. 1. Differential cross section for elastic scattering of 19.3-GeV/c protons by Li^6, Li^7, Be^9, and C^{12}. For C^{12}, the lab momentum of the proton is 21.5 GeV/c. The first three plots have the upper abscissa and the last one, the lower.

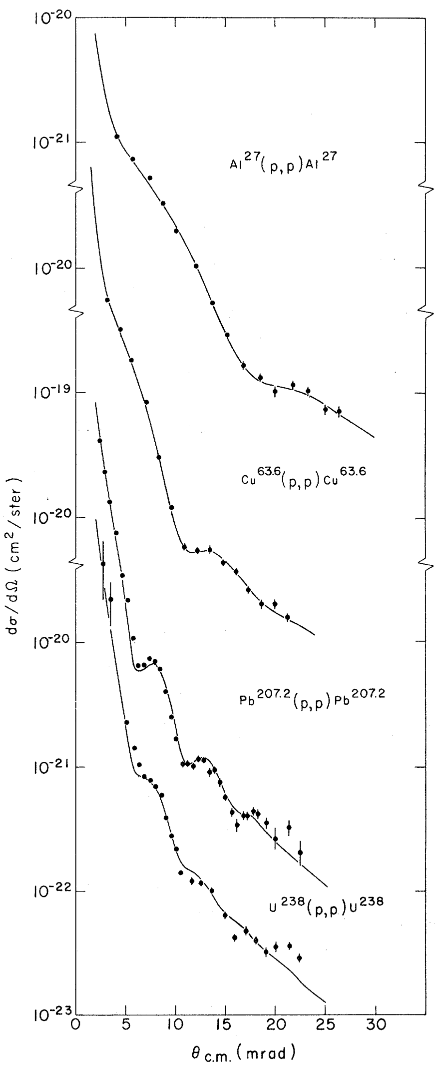

FIG. 2. Differential cross section for elastic scattering of 19.3-GeV/c protons by Al^{27}, $Cu^{63.6}$, $Pb^{207.2}$, and U^{238}.

tential scattering, and ϵ allows for small transparency for partial waves of low l.

The analytical treatment of the SAM is independent of the specific functional form of $g(l)$. The only practical requirement is that dg/dl possess a simple Fourier transform:

$$F(\Delta\theta)=\int_{-\infty}^{\infty}\frac{dg}{dl}\exp[-i(l-L_0)\theta]dl, \quad (4)$$

where Δ is the "rounding" parameter of g.

The differential cross section for elastic scattering, up to second order in Δ/L_0, is given by

$$\frac{d\sigma}{d\Omega}=R^2\left(\frac{\theta}{\sin\theta}\right)[F(\Delta\theta)]^2\left\{(1-\epsilon)^2\left[\frac{J_1(kR\theta)}{\theta}\right]^2+\left[\mu+\left(\frac{\sin\theta}{R^2\theta}\right)^{1/2}f_C(\theta)\right]^2[J_0(kR\theta)]^2\right\}. \quad (5)$$

In the following analysis we have chosen the Woods-Saxon shape for the absorption amplitude,

$$g=\{1+\exp[(L_0-l)/\Delta]\}^{-1}. \quad (6)$$

For this choice

$$F(\Delta\theta)=\pi\Delta\theta/\sinh\pi\Delta\theta. \quad (7)$$

The total nuclear cross section is

$$\sigma_{\text{tot}}=2\pi R^2(1-\epsilon)[1+\tfrac{1}{3}\pi^2(d/R)^2], \quad (8)$$

where d is the spatial diffuseness of the interaction region,

$$\Delta=kd. \quad (9)$$

The total reaction cross section is

$$\sigma_{\text{abs}}=\pi R^2\{(1-\epsilon)^2[1+\tfrac{1}{3}\pi^2(d/R)^2]+[(1-\epsilon)^2-\tfrac{1}{6}(\mu/\Delta)^2](2d/R)\}. \quad (10)$$

Following Ref. 1, a correction term

$$\frac{d\sigma}{d\Omega}=N(A)\left[\frac{k\sigma_{\text{tot}}(p,p)}{4\pi}\right]^2 e^{-10|t|} \quad (11)$$

is added to account for the incoherent inelastic scattering. $N(A)$ is the number of nucleons in the target that act as independent scattering centers. $\sigma_{\text{tot}}(p,p)$ is taken from proton-proton elastic scattering at 19.33 GeV/c.[4] $|t|$ is the four-momentum transfer squared. The fit obtained is illustrated in Figs. 1 and 2. The parameters are listed in Table I where they are compared with those measured from

Table I. Parameters obtained from a best fit.

	R (F) SAM	R (F) Ref. 5	r_0 (F) SAM	r_0 (F) Ref. 5	d (F) SAM	d (F) Ref. 5	$\mu/4\Delta$	ϵ	$N(A)$
Li^6	2.19		1.20		0.60	~0.57	−0.05	0.38	3.9
Li^7	2.17		1.14		0.56	~0.57	−0.07	0.29	3.8
Be^9	2.67		1.29		0.46	~0.45	−0.05	0.43	4.1
C^{12}	2.51		1.10		0.48	~0.50	−0.17	0.23	4.3
Al^{27}	3.33		1.11		0.57	~0.59	−0.16	0.10	4.1
$Cu^{63.6}$	4.49		1.12		0.63	~0.57	−0.17	0.0	7.0
$Pb^{207.2}$	7.09	~7.0	1.20	~1.18	0.50	~0.52	−0.22	0.0	8.9
U^{238}	7.21		1.16		0.56	~0.64	−0.27	0.0	10.2

Table II. Comparison between experimental and theoretical cross sections.

	σ_{tot} (F^2) Theor.	σ_{tot} (F^2) Exptl.	σ_{abs} (F^2) Theor.	σ_{abs} (F^2) Exptl.	σ_{el} (F^2) Theor.	σ_{el} (F^2) Exptl.
Li^6	23.4	23.2	19.2	19.4	4.2	3.8
Li^7	25.6	25.0	20.2	20.8	5.4	4.2
Be^9	28.0	27.8	22.5	22.7	5.5	5.1
C^{12}	34.2	33.5	24.9	25.4	9.3	8.1
Al^{27}	69.2	68.7	46.8	47.2	22.4	21.5
$Cu^{63.6}$	135.1	136.0	84.0	85.0	51.1	51.0
$Pb^{207.2}$	321.3	329.0	180.1	175.0	141.2	154.0
U^{238}	332.9	···	187.1	···	145.8	···

electron scattering.[5] Comparison between experiment and theory for the total absorption and total nuclear scattering is carried out in Table II.

The following conclusions may be drawn from our analysis: (1) The strong interaction form factor is similar to the electromagnetic form factor. (2) The real part of the nucleus-proton nuclear potential is repulsive at the surface of the nucleus (the real phase shifts are negative at the surface). (3) The cross section for low-energy nuclear channels is small for heavy nuclei and increases with decreasing mass number. (4) The mean free path of 20-GeV/c protons in nuclear matter is small in comparison with the radius of medium-weight nuclei.

While this work was being completed, the authors received a report from W. E. Frahn and G. Wiechers describing a similar analysis[6] for heavy nuclei.

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PARITY MIXING IN Hf^{180}†

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Scharff-Goldhaber and McKeown[1] have recently measured the L internal-conversion coefficients for the 57.6-keV $8^- \to 8^+$ transition in Hf^{180} (Fig. 1) and found that no mixture of exclusively odd-parity multipolarities (i.e., $E1$, $M2$, etc.) would fit the data. Scharff-Goldhaber and McKeown show that their result can be interpreted as indicating that the transition has mixed parity, 90.5% $E1$ + 9.5% $M1$, although (as they point out and Hager and Seltzer[2] discuss in more detail) penetration effects offer another possible explanation. The purpose of this note is, first, to show that the parity mixing that must be invoked in order to fit the conversion-coefficient data is inconsistent with the properties of the 501-keV crossover to the 6^+ state, and second, to point out that the selection rule that inhibits the emission of $E1$ radiation in the 57.6-keV transition must also affect the decay of the state by $M1$ radiation.

To illustrate the first point, we note that a positive-parity component in the 1143-keV state would permit an $E2$ transition to the 6^+ state at 642 keV. Angular-correlation studies[3] show that the 501-keV gamma ray is 3.6% quadrupole.[4] Since the correlation cannot distinguish between parities, this 3.6% may be taken as an upper limit on the $E2$ component. In combination with the lifetime and branching ratio, this upper limit leads to $\tau_{E2} \geq 8.4 \times 10^6$ sec for the 501-keV gamma ray. Interpreting the Scharff-Goldhaber and McKeown result as arising from a parity violation leads to $\tau_{M1} = 5.2 \times 10^5$ sec for the 57.6-keV transition.

The relevant part of the wave function for the 1143-keV state can be written

$$\Psi = \psi_{K'=8^-}^{8-} + i\mathcal{F}\varphi_K^{8+},$$

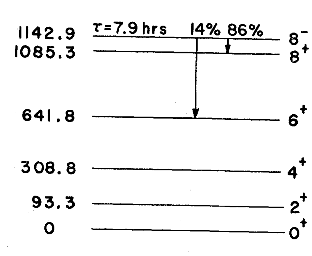

FIG. 1. The low-lying states of Hf^{180}. The 57.6-keV $E1$ transition between the 8^- and 8^+ levels is inhibited by about 10^{16}.