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GENERATION OF FAR INFRARED AS A DIFFERENCE FREQUENCY*

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This Letter is to report the observation of far-infrared radiation generated as the difference frequency between two near-infrared frequencies. Quartz was used as the nonlinear mixing element. The near-infrared frequencies were emitted by one neodymium glass laser. Input and output beams were collinear and phase matched. The output had a frequency of about 100 cm^{-1} and was detected by a gallium-doped germanium photoconductor operated at liquid-helium temperature.

Whereas sum and difference frequencies in the visible and the near infrared, and difference frequencies in the microwave range, have been generated, we believe this to be the first time that a difference frequency in the far infrared has been observed.

In quartz, the phase-matching condition can be satisfied if ω_1 is an extraordinary ray and ω_2 an ordinary ray, where $\omega_1-\omega_2=\omega_3$.

Since the output frequency is lower than the infrared absorption frequencies, the nonlinear susceptibility coefficients to be used in this experiment are approximately equal to the ones used in the dc case.[1]

From the nonlinear susceptibility tensor of quartz, the amplitude P_0 of the polarization wave generating an ordinary ray at ω_3 is found to be

$$P_0=X_{11}E^2\cos^2\theta\cos3\alpha, \tag{1}$$

while for the extraordinary way

$$P_e=X_{11}E^2\cos^2\theta\sin3\alpha+(X_{14}E^2\sin\theta\cos\theta)/2, \tag{2}$$

where θ is the angle between the incident beam and the optical axis of the crystal and α is the angle between the plane of incidence and the positive x axis of the crystal. Neglecting absorption at ω_3, the radiation pattern of a polarization wave at the difference frequency can be shown to have an angular dependence

$$D(d,a,\varphi)=\left[\frac{\sin[k_3d(1-\cos\varphi)/2]}{k_3d(1-\cos\varphi)/2}\right]^2\left[\frac{2J_1(k_3a\sin\varphi)}{k_3a\sin\varphi}\right]^2, \tag{3}$$

providing all frequencies are phase matched.[2] Here d is the length of the crystal in the direction of the beam, $2a$ is the diameter of the beam, φ is the angle between the axis of the input beam and the direction of the output radiation, and k_3 is the magnitude of the propagation vector at ω_3. Unlike the case of second-harmonic generation, where all beams have approximately the same apex angle at the focus, Eq. (3) shows that since the output in the far-infrared case has a much longer wavelength than the input, its apex angle can be made to be much larger. This allows the crystal to be placed inside the resonator of the laser which generates the input beams. The output beam can then be brought out of the resonator with only very small losses by a mirror which passes the input beams through a hole in its center, as in Fig. 1.

At the high input energy used (9000 J into six flash lamps) the laser emits a wavelength

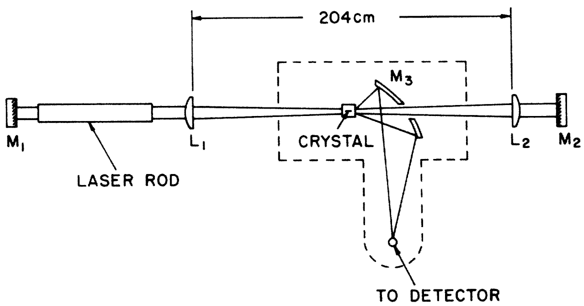

FIG. 1. Experimental arrangement showing the quartz crystal inside the resonator. M_1 and M_2 are ultrahigh-reflectivity mirrors. Laser rod dimensions $20 \times 1.2 \times 1.2$ cm. The dotted line indicates an enclosure for flushing with dry nitrogen.

band from 1.059 to 1.073 μ. From available data on the refractive indices of crystal quartz,[3,4] the matching angle, θ, for generation of the difference frequency, 100 cm^{-1}, between 10 600 and 10 715 Å was calculated to be 53° 36′ for ω_3 as an ordinary ray and 56° for ω_3 as an extraordinary ray.

Three 18-mm-long by 15-mm-square crystals were cut. For crystal A, $\alpha = 30°$ and $\theta = 53° \, 36'$ while for crystal B, $\alpha = 60°$ and $\theta = 56°$. Crystal C is a Z cut.

Typical curves are shown in Fig. 2. Figure 2(a) is the signal produced by a high-pressure mercury lamp (GE H400-A4) such as is conventionally used as a far-infrared source, chopped at 1000 cps, filling the aperture of

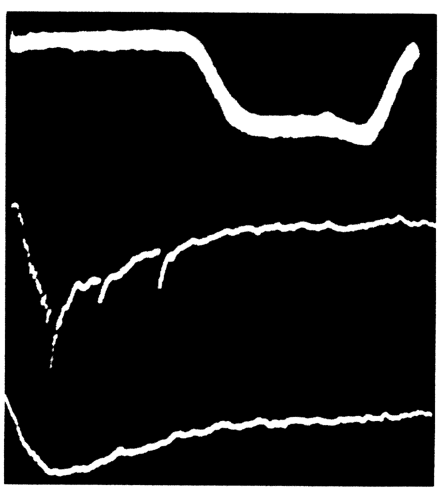

FIG. 2. Top: signal produced by a high-pressure mercury lamp, passed through the same filters and polarizer as the difference signal. Center: output from crystal B, with the polarizer transmitting e rays. Bottom: output from crystal B, with the polarizer rotated through 90° relative to the center trace.

M_3 and of the detector and passing through the same filters and polarizer as the difference signal. The filters, a "hot" black polyethylene sheet and a 1-mm-thick quartz window at 4.2°K give the detector a short-wavelength cutoff at about 80 μ. Its long-wavelength cutoff is about 125 μ.

Figure 2(b) shows the output from crystal B with the polarizer transmitting e rays. Spikes due to the difference frequency are clearly observable.

Figure 2(c) shows the output from crystal B with the polarizer rotated through 90°. No spikes appear but a smooth signal with a long time constant is still present.

Both the spikes and the smooth signal were shown to be caused by far-infrared radiation: They were transmitted by black polyethylene and blocked by a NaCl crystal. The same signal as in Fig. 2(c) was obtained from crystal C and from a piece of fused silica. It is not polarized and increases in magnitude when the quartz filter is taken out of the detector assembly, giving the detector a cutoff much shorter than 80 μ. This indicates that the smooth signal is of thermal origin.

The spikes always occur in the extraordinary polarization from crystal B and in both polarizations from crystal A. They occur at different moments during the laser pulse and they vary in amplitude. To explain this, we note that the two input frequencies occur simultaneously only at random moments during the laser pulse. Moreover, in deriving Eq. (3) we assumed a polarization wave of uniform phase across the beam. In practice this is not so, both because of the multimode nature of the laser[5] and because another pair of input frequencies with the same difference frequency may give a different phase.

The predictable polarization shows that the spikes are indeed caused by the difference frequency.

The occurrence of spikes in both polarizations from crystal A shows that $X_{14} \neq 0$, indicating that Kleinman's symmetry does not apply in this case.[6]

Because of the variations from shot to shot, no precise measurements of wavelength or output power have been made as yet.

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ELLIPTIC KRUSKAL-SCHWARZSCHILD SPACE*

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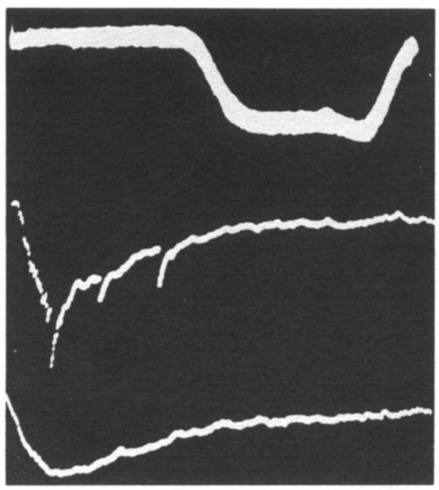
Recent interest in gravitational collapse has focused fresh attention on Kruskal-Schwarzschild space. The usual interpretation of Kruskal's[1] maximal extension of the general relativistic Schwarzschild space of an isolated mass point is as follows[2]: For an infinite past and future in Kruskal time v, three-space consists of two disconnected quasi-Euclidean branches, each possessing a spherically symmetric cuspidal singularity (analogous to that obtained by sticking a sharp pencil into a stretched rubber sheet); at time $v=-1$ these two branches spring a connection at their cusps, which develops into a smooth bridge or "wormhole" reaching its maximum radius $2m$ at $v=0$; thereafter the bridge shrinks and finally breaks off again at $v=1$. (We adopt Kruskal's notation throughout, except for writing m and t for his m^* and T.)

It seems, however, hard to believe that every mass point P should have the effect of splitting the spatial universe in two, thus necessitating a second copy of ours which, though quite unknowable to us, is nevertheless visible at the same time as ours to some (albeit doomed) observers sufficiently close to P. Thus, for example, there could exist a set of observers in our universe, densely and permanently orbiting P along a sphere of given radius, and yet distinct from, and without possible knowledge of, another identical set also circling P but in the other universe.

Some light can be shed on these difficulties by considering the limit K_0 of Kruskal space K as $m \to 0$. For consistency, this limit should be equivalent to Minkowski space M. If an identification of points is required to make K_0 into M, a similar identification would be indicated for K. Unfortunately, Kruskal's coordinates are unsuitable for carrying out this limiting procedure directly. But one may simply observe that each of the two Kruskal regions $u+v \gtrless 0$ represents all of Schwarzschild space, which in the limit becomes all of Minkowski space. One possible identification, which, however, it is difficult to specify quantitatively, consists in regarding each mass point as a widely separated cusp pair in the same quasi-Minkowskian space.[1,2] But we here wish to suggest the identification of event pairs (u, v, θ, φ) and $(-u, -v, \theta, \varphi)$[3] in Kruskal's scheme, since they correspond to events with the same Schwarzschild (and thus ultimately Minkowskian) coordinates (t, r, θ, φ).

Such identification is closely analogous to the possible "elliptic" identification of space-time antipodes in De Sitter's universe, which was very fully discussed by Schrödinger[4] —though it is not analogous to that discussed earlier by Eddington[5] for both the Einstein and the De Sitter universes, since that involved the identification of spatial antipodes only. In our case, the identification has an obvious blemish: We have introduced a singularity where there was none before—on the 2-sphere $u=0=v$. (It is easily seen that the identification at all these points destroys the triplet "past-future-elsewhere," a topological feature associated with each regular point.) Nevertheless, this singularity may not be considered fatal, since (i) it lasts but a single v instant, and (ii) the curvature does not become unbounded near it. As in Schrödinger's case, however, another price must be paid for the elliptic interpretation of Kruskal space, namely the essential ambiguity of the arrow of time in the



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