where V_M is the operator describing the action of the M-vertex function and $|M\bar{M}-\bar{M}M\rangle$ is the antisymmetric combination of the initial and final mesons which contributes to the Johnson-Treiman relations in the t channel. From the requirement that the exchanged meson state $|X\rangle$ have vanishing components for all the $C=+1$ states described above, it follows that the expansion of $|X\rangle$ in the eigenstates of $SU(6)_W$ contains only the antisymmetric multiplets $\underline{35}_F$, $\underline{280}$, and $\underline{280}^*$.

[10]This transformation has been used by T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956); A. Bohr, Nucl. Phys. 10, 486 (1959). For detailed discussion of the application of this transformation to W spin see reference 11.

[11]H. J. Lipkin and S. Meshkov, to be published.

[12]H. Harari, reference 3.

CONSEQUENCES OF CURRENT COMMUTATION RELATIONS IN THE NONLEPTONIC HYPERON DECAYS*

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It was emphasized by Gell-Mann[1] that the space integrals of the time components of vector and axial-vector unitary spin currents give rise to the algebra $SU(3)\otimes SU(3)$. The weak interactions are beautifully formulated in the quark model of $SU(3)\otimes SU(3)$.[2]

In this Letter we discuss consequences of the current commutation relations and of the partially conserved axial-vector current hypothesis[3] (PCAC) in the nonleptonic hyperon decays. Our basic idea consists in relating the matrix elements of the nonleptonic hyperon decays to those between octet baryon states of bilinear products of vector and axial-vector unitary spin currents by means of equal-time commutation relations of current components.

The nonleptonic weak Hamiltonian is written in the form of current×current,[4]

$$H^{(\mathrm{w})}=(G/\sqrt{2})[(F_\mu{}^1+iF_\mu{}^2)\cos\theta+(F_\mu{}^4+iF_\mu{}^5)\sin\theta][(F_\mu{}^1-iF_\mu{}^2)\cos\theta+(F_\mu{}^4-iF_\mu{}^5)\sin\theta], \tag{1}$$

$$F_\mu{}^i=j_\mu{}^i+j_{5\mu}{}^i, \tag{2}$$

where the superscripts refer to unitary spins. In terms of the quark fields, $j_\mu{}^i$ and $j_{5\mu}{}^i$ have the transformation properties of $\bar{q}\gamma_\mu\lambda_i q$ and $\bar{q}\gamma_\mu\gamma_5\lambda_i q$, respectively.

First we shall show that the commutators of $H^{(\mathrm{w})}$ with the space integrals of $j_{50}{}^i$ $(i=1,2,3)$ are simply related to the nonleptonic decay matrices on the assumption of PCAC. The decay amplitude of $\beta\to\alpha+\pi$ is written by reduction as

$$\langle\alpha\pi^i|\int d^3x\,H^{(\mathrm{w})}(\vec{x},0)|\beta\rangle=-i\int d^4y\langle\alpha|[j_\pi{}^i(y),\int d^3x\,H^{(\mathrm{w})}(\vec{x},0)]_-\theta(y)|\beta\rangle\exp(-ip_\pi y), \tag{3}$$

where

$$(\Box-\mu^2)\varphi_\pi{}^i(y)=j_\pi{}^i(y), \tag{4}$$

and the state vectors are covariantly normalized such that $\langle\alpha|\beta\rangle=2E_\alpha\delta(\vec{p}_\alpha-\vec{p}_\beta)$. We insert a complete set of the intermediate states and take account of $E_\beta=E_\alpha+E_\pi$ where the physical decay process $\beta\to\alpha+\pi$ occurs. Then we get

$$(2\pi)^3\delta(\vec{p}_\pi+\vec{p}_\alpha-\vec{p}_\beta)\sum_n\left\{\langle\alpha|H^{(\mathrm{w})}(0)|n\rangle\langle n|j_\pi{}^i(0)|\beta\rangle\frac{\delta(\vec{p}_n-\vec{p}_\alpha)}{2E_n(E_\alpha-E_n)}\right.$$
$$\left.-\langle\alpha|j_\pi{}^i(0)|n\rangle\langle n|H^{(\mathrm{w})}(0)|\beta\rangle\frac{\delta(\vec{p}_n-\vec{p}_\beta)}{2E_n(E_n-E_\beta)}\right\}. \tag{5}$$

On the other hand, the matrix element of the equal-time commutator of $\int d^3x\, H^{(\mathrm{w})}(\vec{x},0)$ with $J_5{}^i = -i\int j_{50}{}^i(\vec{x},0)d^3x$ is

$$\langle\alpha|[\int d^3x\, H^{(\mathrm{w})}(\vec{x},0), J_5{}^i]|\beta\rangle$$
$$=(2\pi)^3\frac{m\mu^2 g_A}{g_r K^{NN\pi}(0)}\delta(\vec{p}_\alpha-\vec{p}_\beta)\sum_n\left\{\langle\alpha|H^{(\mathrm{w})}(0)|n\rangle\langle n|j_\pi{}^i(0)|\beta\rangle\frac{\delta(\vec{p}_n-\vec{p}_\alpha)}{2E_n(E_\beta-E_n)[\mu^2-(E_n-E_\beta)^2]}\right.$$
$$\left.-\langle\alpha|j_\pi{}^i(0)|n\rangle\langle n|H^{(\mathrm{w})}(0)|\beta\rangle\frac{\delta(\vec{p}_n-\vec{p}_\beta)}{2E_n(E_n-E_\alpha)[\mu^2-(E_n-E_\alpha)^2]}\right\}. \tag{6}$$

Here we have used the consequence of PCAC,[3]

$$\partial_\mu j_{5\mu}{}^i=[-im\mu^2 g_A/g_r K^{NN\pi}(0)]\varphi^i, \tag{7}$$

where m is the nucleon mass, g_A is the axial-vector β-decay coupling constant, $g_r{}^2/4\pi=14.6$, φ^i is the renormalized pion field, and $K^{NN\pi}(0)$ is the πN vertex at zero momentum transfer $[K^{NN\pi}(-\mu^2)=1]$. In the limit of large $p_\alpha(=p_\beta)$, we have $E_\alpha\to E_\beta$ in the right-hand side. We assume that the summation over the intermediate states converges sufficiently rapidly that the order of the summation over n and limiting process $p_\alpha\to\infty$ can be interchanged. Then $\mu^2-(E_\alpha-E_n)^2$ tends to μ^2 as $p_\alpha\to\infty$, since $E_n-E_\alpha=(M_n{}^2-m_\alpha{}^2)/(E_n+E_\alpha)$ because of the momentum conservation. Thus we are led to

$$\lim_{p_\alpha\to\infty}\langle\alpha|[J_5{}^i,\int d^3x\, H^{(\mathrm{w})}(\vec{x},0)]_-|\beta\rangle/\delta(\vec{p}_\alpha-\vec{p}_\beta)$$
$$\approx\frac{mg_A}{g_r K^{NN\pi}(0)}\lim_{p_\alpha\to\infty}\langle\alpha\pi^i|\int d^3x\, H^{(\mathrm{w})}(\vec{x},0)|\beta\rangle/\delta(\vec{p}_\alpha+\vec{p}_\pi-\vec{p}_\beta). \tag{8}$$

If m_α were equal to m_β, and the pion mass were zero, one thinks of Eq. (8) as holding exactly. Since we apply it to the decays of the octet baryon into the octet baryon + the pion, we can expect the equality to be sufficiently accurate.[5] In this way, the covariant matrix elements are related to the equal-time commutators.

The explicit structure of $H^{(\mathrm{w})}$ enables us to write down the commutators at equal time[6]:

$$[H^C, J_5{}^3]=-\tfrac{1}{2}H^v, \tag{9a}$$

$$[H^C,J_5{}^1]=(G/\sqrt{2})\cos\theta\sin\theta[j_{5\mu}{}^3(j_\mu{}^4-ij_\mu{}^5)+\tfrac{1}{2}(j_\mu{}^1+ij_\mu{}^2)(j_{5\mu}{}^6-ij_{5\mu}{}^7)+(j_\mu\to j_{5\mu})]+\text{H.c.}, \tag{9b}$$

$$[H^C,-iJ_5{}^2]=(G/\sqrt{2})\cos\theta\sin\theta[j_{5\mu}{}^3(j_\mu{}^4-ij_\mu{}^5)-\tfrac{1}{2}(j_\mu{}^1+ij_\mu{}^2)(j_{5\mu}{}^6-ij_{5\mu}{}^7)+(j_\mu\to j_{5\mu})]-\text{H.c.}, \tag{9c}$$

$$[H^v, J_5{}^3]=-\tfrac{1}{2}H^C, \tag{9d}$$

$$[H^v,J_5{}^1]=(G/\sqrt{2})\cos\theta\sin\theta[j_\mu{}^3(j_\mu{}^4-ij_\mu{}^5)+\tfrac{1}{2}(j_\mu{}^1+ij_\mu{}^2)(j_\mu{}^6-ij_\mu{}^7)+(j_\mu\to j_{5\mu})]+\text{H.c.}, \tag{9e}$$

$$[H^v,-iJ_5{}^2]=(G/\sqrt{2})\cos\theta\sin\theta[j_\mu{}^3(j_\mu{}^4-ij_\mu{}^5)-\tfrac{1}{2}(j_\mu{}^1+ij_\mu{}^2)(j_\mu{}^6-ij_\mu{}^7)+(j_\mu\to j_{5\mu})]-\text{H.c.}, \tag{9f}$$

where H^C and H^v denote the parity-conserving (pc) and parity-nonconserving (pv) parts of $H^{(\mathrm{w})}$, respectively, and H.c. denotes a Hermitian conjugate. By use of these commutation relations, we have reduced the number of independent amplitudes of $\beta\to\alpha+\pi$; we have related the $\beta\to\alpha+\pi$ decay to the transition matrix element of $\beta\to\alpha$ caused by the "interactions" which are explicitly given by the right-hand sides.

The right-hand sides give rise to $\underline{8}_s$ and $\underline{27}$. The C quantum number, the charge parity of the $Y=I=0$ member, is $-$ for $\underline{8}_s$ and $\underline{27}$ arising from the right-hand side of Eqs. (9a), (9b), and (9c). Since $\underline{8}_a$, $\underline{8}_s$, and $\underline{27}$ composed from the octet baryons and antibaryons have $C=+$, we are led to

$$\langle B|[H^C, J_5^{\,i}]_-|B'\rangle = 0 \quad (i=1,2,3). \tag{10}$$

This is a slight generalization of the fact that a pv pole transition is forbidden between the octet baryons in the case of octet spurion.[7]

We enumerate consequences in the pv amplitudes. Without octet dominance or the like,

$$\begin{aligned}
\Xi_0^{\,0} &= -(\sqrt{3}/20)a(27) + (\sqrt{3}/20)a(8_{ss}) + (3/80)^{1/2}a(8_{as}),\\
\Xi_-^{\,-} &= [(\sqrt{6})/20]a(27) - [(\sqrt{6})/20]a(8_{ss}) - (3/40)^{1/2}a(8_{as}),\\
\Lambda_0^{\,0} &= -(\sqrt{3}/20)a(27) + (\sqrt{3}/20)a(8_{ss}) - (3/80)^{1/2}a(8_{as}),\\
\Lambda_-^{\,0} &= [(\sqrt{6})/20]a(27) - [(\sqrt{6})/20]a(8_{ss}) + (3/40)^{1/2}a(8_{as}),\\
\Sigma_0^{\,+} &= -(\sqrt{2}/10)a(27) - (3\sqrt{2}/20)a(8_{ss}) - (1/40)^{1/2}a(8_{as}),\\
\Sigma_-^{\,-} &= \tfrac{3}{10}a(27) - \tfrac{3}{10}a(8_{ss}) - (1/20)^{1/2}a(8_{as}),\\
\Sigma_+^{\,+} &= \tfrac{1}{2}a(27).
\end{aligned} \tag{11}$$

We easily see

$$\sqrt{2}\,\Xi_0^{\,0} + \Xi_-^{\,-} = 0, \tag{12a}$$

$$\sqrt{2}\,\Lambda_0^{\,0} + \Lambda_-^{\,0} = 0, \tag{12b}$$

$$\Sigma_-^{\,-} - \sqrt{2}\,\Sigma_0^{\,+} - \Sigma_+^{\,+} = 0. \tag{12c}$$

Equations (12a) and (12b) are the $\Delta I=\frac{1}{2}$ rules for the Ξ and the Λ decays, respectively. Equation (12c) is not the $\Delta I=\frac{1}{2}$ rule for the Σ decays because the sign before $\Sigma_+^{\,+}$ is wrong. However, because $\Sigma_+^{\,+}$ is experimentally either purely s wave or purely p wave, this sign cannot be experimentally determined in any way. Therefore, the consequence of Eq. (12c) cannot be distinguished from the $\Delta I=\frac{1}{2}$ rule.[8] Next,

$$2\Xi_-^{\,-} + \Lambda_-^{\,0} = (\sqrt{\tfrac{3}{2}})\Sigma_+^{\,+} + \sqrt{3}\,\Sigma_0^{\,+}. \tag{13}$$

If $\Sigma_+^{\,+}$ is dominantly p wave, this holds good within experimental error.

Let us assume octet dominance in the bilinear products of the vector and axial-vector unitary spin currents. We get an additional relation

$$\Sigma_+^{\,+} = 0, \tag{14}$$

and therefore

$$2\Xi_-^{\,-} + \Lambda_-^{\,0} - \sqrt{3}\,\Sigma_0^{\,+} = 0. \tag{15}$$

This is the relation originally suggested by Lee, Sugawara, and others[9] by use of specific assumptions. A rigorous proof was given for the pv amplitudes by Gell-Mann[7] only on the assumption of the current×current interactions, CP invariance, and octet dominance.

The present discussions give the pv amplitudes in terms of the three independent matrix elements between the octet baryons in the general case, and in terms of the two in the case of octet dominance. All of the predicted relations are satisfied very well.[10,11] One of the most important results is that the $\Delta I=\frac{1}{2}$ rule is derived for the Λ and the Ξ decays without octet dominance and that the sum rule experimentally equivalent to the $\Delta I = \frac{1}{2}$ rule is derived for the Σ decay.

In contrast, we encounter a serious difficulty in the pc amplitudes. According to the present discussions [see Eq. (10)], all of the pc amplitudes are forbidden by SU(3) and PCAC. There is a proper reason why the pc amplitudes are forbidden. Suppose that the system is transformed into the rest system of α and β in Eq. (6). The pion has no mass or energy-momentum, and therefore the process which Eq. (6) describes is β (at rest) $\rightarrow \alpha$ (at rest) $+\pi$ (without energy momentum). The p-wave decay (the pc process) is evidently forbidden in this situation.[12]

We shall show a way of overcoming the difficulty in the pc amplitudes, and give further applications elsewhere.

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[1]M. Gell-Mann and Y. Ne'eman, The Eightfold Way (W. A. Benjamin, Inc., New York, 1964), p. 11; Phys. Rev. 125, 1067 (1962).

[2]M. Gell-Mann, Physics 1, 63 (1964).

[3]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[4]We may add a numerical factor α to $j_{5\mu}^{i}$; the following discussions are unaltered by this factor.

[5]The left-hand side is defined off the energy shell where $E_\alpha^2 - E_\beta^2 = m_\alpha^2 - m_\beta^2$ and $p_\pi = 0$, while the right-hand side is on the energy shell where $(E_\alpha - E_\beta)^2 - (p_\alpha - p_\beta)^2 = \mu^2$. In the limit $p_\alpha \rightarrow \infty$, $(E_\alpha - E_\beta)^2 - (p_\alpha - p_\beta)^2 = (m_\alpha^2 - m_\beta^2)^2 O(p^{-2})$ in the left-hand side. In the limit $p_\pi \rightarrow 0$ and $p_\alpha \rightarrow \infty$ where Eq. (8) is exact, the equal-time commutator terms neglected in the right-hand side of Eq. (3) can be shown to be of higher order in $1/p_\alpha$ as compared with the term retained there. In the approximation of neglecting the analytic continuation from $p_\pi = 0$ to $p_\pi^2 = \mu^2$, therefore, those terms can be justifiably neglected.

[6]We assume that commutators between a space and a time component do not involve a singularity higher than a δ function. This is not always the case; see, for example, J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

[7]M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964); B. W. Lee and A. R. Swift, Phys. Rev. 136, B228 (1964).

[8]I am grateful to Professor R. F. Dashen for clarifying this point.

[9]B. W. Lee, Phys. Rev. Letters 12, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) 31, 213 (1964); S. Okubo, Phys. Letters 8, 362 (1964); B. Sakita, Phys. Rev. Letters 12, 379 (1964); S. P. Rosen, Phys. Rev. Letters 12, 408 (1964).

[10]For the present status of the experimental data, see F. S. Crawford, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 825; M. L. Stevenson, J. P. Berge, J. R. Hubbard, G. R. Kalbfleisch, J. B. Shafer, F. T. Solmitz, S. G. Wojcicki, and P. G. Wohlmut, Phys. Letters 9, 349 (1964); R. H. Dalitz, lecture notes given at the Proceedings of the International School of Physics of Physics "Enrico Fermi," 1964 (Academic Press, Inc., New York, to be published).

[11]In the tadpole model of octet scalar mesons, the coupling of the scalar octet with the octet baryons is dominantly of F type, as is deduced from the mass splitting [see S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964); M. Suzuki, Progr. Theoret. Phys. (Kyoto) 32, 166 (1964)]. If we assume this mechanism in addition to octet dominance, we get from Eq. (11), $\Xi_-^-:\Lambda_-^0:\sqrt{3}\Sigma_0^+ = 1:-1:1$. This ratio is qualitatively in good agreement with experiment.

[12]This was pointed out by Professor R. F. Dashen.