PROJECTED HARTREE-FOCK SPECTRA IN LIGHT NUCLEI

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The nonradial Hartree-Fock (HF) theory has been successfully applied to s-d shell nuclei with the assumption of an inert O^{16} core.[1] It has also been extended to the description of states arising from excitations of nucleons from the p shell.[2] It is shown here that the HF theory, together with angular-momentum projection, is a useful approximation to the configuration-mixing calculations[3] which become rapidly too complicated as the number of particles and of shells increases. The method consists of finding deformed single-particle orbitals μ, and hole orbitals λ, such that the determinental wave function

$$|\varphi\rangle = \sum_{i=s}^{M} \sum_{j=s}^{N} b_{\mu_i}^{\dagger} b_{\lambda_j} |0\rangle \tag{1}$$

will satisfy $\delta\langle\varphi|H|\varphi\rangle = 0$. Here $|0\rangle$ is the spherical ground state of O^{16}. Equivalently, λ and μ must be eigenstates of the HF Hamiltonian h given as follows in the $jm\tau$ representation:

$$\langle jm\tau|h|j'm\tau\rangle = \epsilon_j \delta_{jj'} + \sum_{i=s}^{M} \langle jm\tau, \mu_i|v|j'm\tau, \mu_i\rangle - \sum_{i=s}^{N} \langle jm\tau, \lambda_i|v|j'm\tau, \lambda_i\rangle. \tag{2}$$

M is the number of particles in the s-d shell and N is the number of holes in the p shell. The ϵ_j, determined from the relative binding energies of O^{15}, O^{16}, and O^{17}, are given in reference 2. The interaction v is taken to be a Gaussian with the Rosenfeld mixture. The HF state $|\varphi\rangle$ is a deformed intrinsic state with axial symmetry. It is composed of a superposition of states with various angular momenta J. The energies E_J of these states are extracted from $|\varphi\rangle$ by angular-momentum projection[8]:

$$E_J = \langle\varphi|HP_K^J|\varphi\rangle / \langle\varphi|P_K^J|\varphi\rangle. \tag{3}$$

When $|\varphi\rangle$ has axial symmetry it follows that

$$\langle\varphi|P_K^J|\varphi\rangle = (J+\tfrac{1}{2}) \int_0^{\pi} \sin\theta d\theta\, d_{KK}^J(\theta) \times \langle\varphi|\exp(-i\theta J_y)|\varphi\rangle, \tag{4}$$

$$\langle\varphi|HP_K^J|\varphi\rangle = (J+\tfrac{1}{2}) \int_0^{\pi} \sin\theta d\theta\, d_{KK}^J(\theta) \times \langle\varphi|H\exp(-i\theta J_y)|\varphi\rangle. \tag{5}$$

$d_{KK}^J(\theta)$ is the reduced rotation matrix. The integrals (4) and (5) were evaluated numerically. A typical overlap function $\langle\varphi|\exp(-i\theta J_y)|\varphi\rangle$ computed with the Ne^{20} deformed state is shown on Fig. 1. The energy E_J thus calculated is to be compared to the observed binding energy of the state J relative to the O^{16} binding energy.

Consider first F^{19}. Positive-parity $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{9}{2}^+$ and negative-parity $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{3}{2}^-$ states have been identified. The former are three-particle ($3p$) states which can be obtained by exact diagonalization.[4] The latter are states

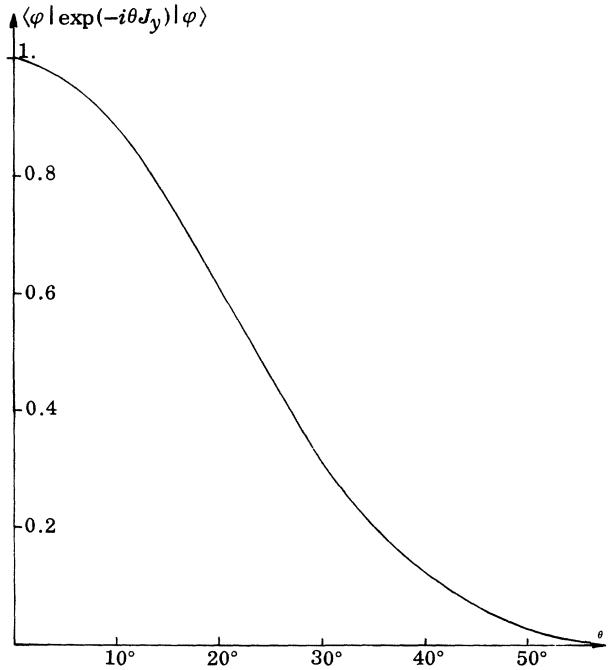

FIG. 1. The calculated overlap function in Ne^{20}, $\langle\varphi|\exp(-i\theta J_y)|\varphi\rangle$.

arising from a $k=\frac{1}{2}^-$ band produced by four-particle one-hole ($4p$-$1h$) configurations.[7] In Fig. 2 the states obtained by projecting angular momentum from the $3p$ and $4p$-$1h$ deformed intrinsic states are compared to the identified experimental levels. In addition the projected $3p$ states are compared to the $3p$ states obtained by exact diagonalization and they agree quite well.

In O^{18} the $2p$ states can be calculated by exact diagonalization.[4] It is known, however, that not all positive-parity states of O^{18} can be accounted for by $2p$ configurations.[5] For example, there is a 0^+ state observed at 5.33 MeV and a 2^+ state at 5.25 MeV. But the diagonalization of $2p$ states yields no 0^+ state between 2.7 and 13 MeV and no 2^+ state between 3.3. and 8.7 MeV. Since Federman and Talmi[6] have shown that $2p$ and $4p$-$2h$ states do not mix appreciably, the energy of the above-mentioned 0^+ and 2^+ states should be obtained by projecting angular momentum from an intrinsic $4p$-$2h$ state. The results are shown in Table I.

Table I. Calculated and experimental energies of levels arising from various configurations in $A=18$, 19, and 20 nuclei.

Nucleus	J	Experimental	Calculated	Configuration
Ne^{20}	0^+	−33.02	−33.7	$4p$
	2^+	−31.39	−32.5	$4p$
	4^+	−28.77	−29.9	$4p$
	6^+	−25.42	−26.3	$4p$
	8^+		−22.5	$4p$
F^{19}	$\frac{1}{2}^+$	−20.18	−20.5	$3p$
	$\frac{3}{2}^+$	−18.63	−18.5	$3p$
	$\frac{5}{2}^+$	−19.99	−19.9	$3p$
	$7/2^+$		−15.2	$3p$
	$9/2^+$	−17.39	−18.0	$3p$
	$11/2^+$		−11.9	$3p$
	$13/2^+$		−15.8	$3p$
F^{19}	$\frac{1}{2}^-$	−20.07	−19.1	$4p$-$1h$
	$\frac{3}{2}^-$	−18.73	−17.5	$4p$-$1h$
	$\frac{5}{2}^-$	−18.84	−18.0	$4p$-$1h$
	$7/2^-$		−14.6	$4p$-$1h$
	$9/2^-$		−15.5	$4p$-$1h$
	$11/2^-$		−10.7	$4p$-$1h$
O^{18}	0^+	−12.19	−11.5	$2p$
	0^+	−8.56	−8.7	$2p$
	0^+	−6.86	−6.0	$4p$-$2p$
	0^+		1.5	$2p$
	2^+	−10.21	−9.8	$2p$
	2^+	−8.27	−8.2	$2p$
	2^+	−6.94	−4.6	$4p$-$2h$
	2^+		−2.7	$2p$
	3^+	−6.82	−6.6	$2p$
	4^+	−8.64	−8.7	$2p$
	4^+		−1.7	$4p$-$2h$
F^{18}	1^+	−9.74	−9.9	$2p$
	1^+	−8.04	−6.2	$2p$
	1^+		−5.4	$4p$-$2h$
	1^+		−4.4	$2p$
	2^+		−6.6	$2p$
	2^+		−4.5	$4p$-$2h$
	3^+	−8.80	−9.3	$2p$
	3^+		−5.5	$2p$
	3^+		−3.8	$4p$-$2h$
	3^+		−0.8	$2p$
	4^+		−3.8	$2p$
	4^+		−1.8	$4p$-$2h$
	5^+	−8.61	−8.9	$2p$
	5^+		−1.1	$4p$-$2h$

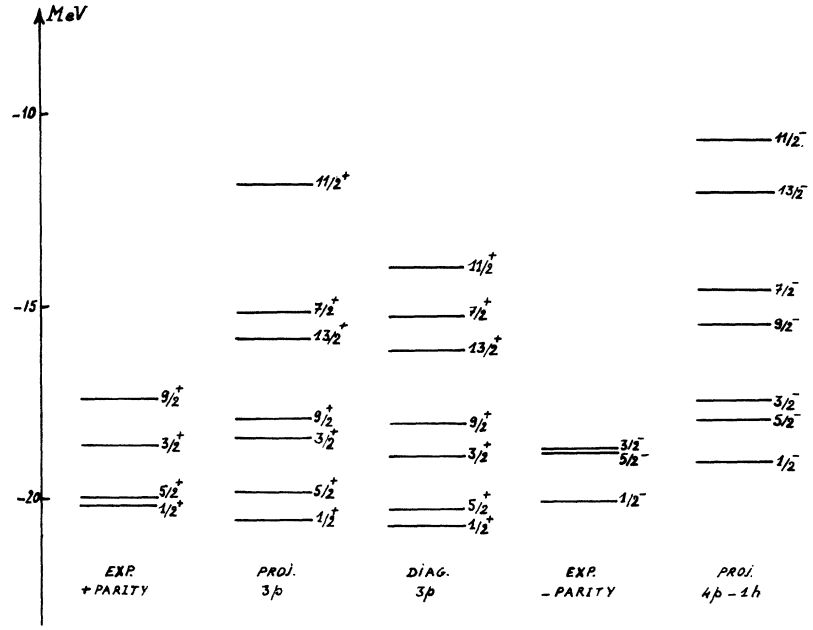

FIG. 2. The calculated positive- and negative-parity levels of F^{19} and a comparsion between the ($3p$) spectrum obtained by projection and diagonalization.

The experimental evidence for $4p$-$2h$ states in F^{18} is not clear. In the table, the results of diagonalizing $2p$ $T=0$ states and projecting angular momentum from a $4p$-$2h$ $K=1, T=0$ intrinsic state are shown. It is seen that in F^{18}, $4p$-$2h$ $T=0$ states also lie very low in energy. Finally, the energies obtained by projecting angular momentum from a $4p$ intrinsic state of Ne^{20} are shown in the table.

All the energies shown in Table I were calculated using a single Rosenfeld force with a 53.25-MeV strength. It should be emphasized that the energies are binding energies relative to O^{16} and not simply relative excitation energies within a band. It may be concluded that with a simple Rosenfeld force and a single interaction strength, over 20 levels arising from seven different types of configurations have been accounted for quite satisfactorily. Thus configuration-mixing calculations are well approximated by projecting angular momentum from the Hartree-Fock Slater determinant and using the same force as in the configuration-mixing calculations.

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