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# DIRECT MEASUREMENT OF THE SIZE OF CHARGED QUANTIZED VORTEX RINGS IN He II*

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A direct measurement has been made of the size of charged quantized vortex rings in liquid $He^4$ at about 0.3°K. The method consisted in studying the current-voltage characteristics of a tetrode tube, in which the size of holes in the last grid was varied. The data indicate a cutoff voltage which is determined by the size of the grid openings and implies that the size of these vortex rings can be correctly calculated by classical hydrodynamics.

Existence of charged vortex rings in liquid $He^4$ was first experimentally recognized by Rayfield and Reif.[1,2] They found an inverse relationship between the velocity, $v$, and the energy, $E$, of charged complexes at temperatures below about 0.6°K. In seeking an explanation for their data, they noted that classical vortex rings exhibit such a dependence of $v$ on $E$. The following equations describe the behavior of vortices in a "perfect" fluid[3]:

$$E = \tfrac{1}{2}\rho\kappa^2 R\left[\ln\left(\frac{8R}{a}\right) - \frac{7}{4}\right], \tag{1}$$

$$v = \frac{\kappa}{4\pi R}\left[\ln\left(\frac{8R}{a}\right) - \frac{1}{4}\right], \tag{2}$$

where $\rho$ is the density of the fluid, $R$ the radius of the ring, $a$ the core radius, and $\kappa$ the circulation. Rayfield and Reif used these equations to eliminate $R$ and obtain an equation relating $v$ and $E$, which was then found to represent their data extremely well.

In our experiment, we have studied the dependence of $R$ on $E$. The measurements are somewhat analogous to those of Parks, Mochel, and Surgent,[4] who have observed size effects of quantized vortices in a superconductor.

The apparatus used for this measurement is shown in Fig. 1. A $Po^{210}$ alpha source, $S$, produces ionization between the source and grid $G_1$. The voltage $V_S$ controls the current density. $G_1$ has large openings (170 $\mu$), while the opening size in $G_2$ was varied from about 4 to 10 $\mu$.[5] The voltage, $V_{12}$, applied between $G_1$ and $G_2$ was varied continuously. The total energy, $E$, of a vortex at $G_2$ is then $e(V_S + V_{12})$, where $e$ is the charge of the ion (assumed to have the magnitude of the electronic charge). Throughout the temperature range in which data were taken (0.29-0.40°K), frictional energy losses can be neglected.[2] The current, $I$, was measured on a collector situated behind $G_2$. Some typical current-voltage curves are

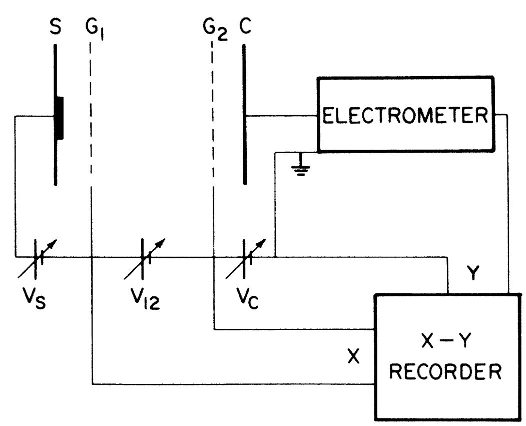


FIG. 1. Schematic diagram of the apparatus. The collector is actually guarded. Data were taken with both "attractive" and "repulsive" signs for $V_C$.

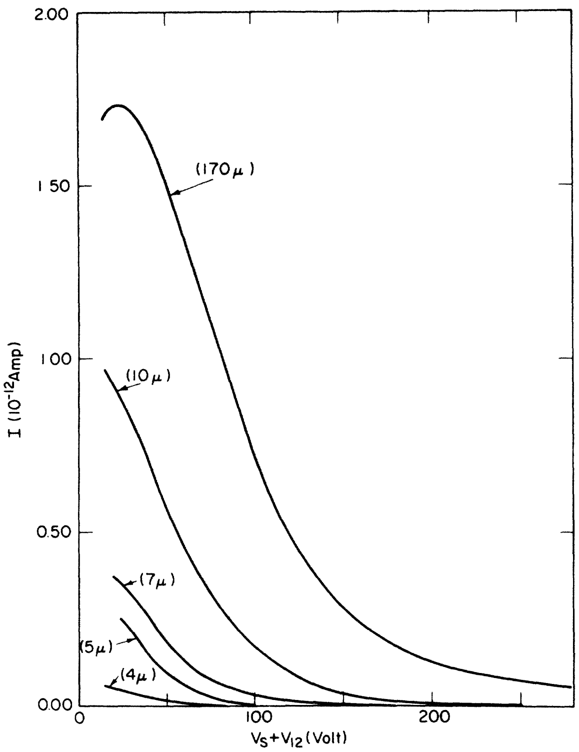


FIG. 2. Current-voltage characteristics of the apparatus for various grids in location $G_2$. The opening sizes given are nominal values used only for purposes of labeling.

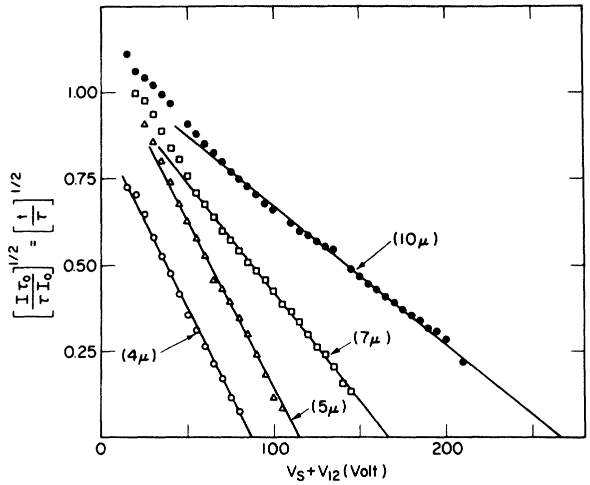


FIG. 3. Plots of square root of ratio of vortex transmission coefficient to geometrical transmission coefficient for the small grids.

shown in Fig. 2. Although it is evident from the data that there is a cutoff voltage, $V_{\rm CO}$, which depends on grid size, a direct determination of $V_{\rm CO}$ from this graph is rather difficult since the curves approach cutoff with zero slope. We also show in Fig. 2 a characteristic curve taken with a grid whose openings (170 $\mu$) are much too large to show size effects. The shape of this curve is apparently determined by interactions between vortex rings. We will show below that a plot of $I^{1/2}$ vs $V$, or better still $(I/I_i)^{1/2}$ vs $V$ ($I_i$ is the current incident on the last grid), is much more useful in determining the cutoff voltage. We take the current transmitted by the 170$\mu$ grid, $I_0$, to be equal to the current incident on a small grid under identical operating conditions, except for the fact that the geometrical transmission coefficient $\tau_0$ of the large grid is less than one. Thus $I_i\tau_0 = I_0$, and $t \equiv (I/I_0)\tau_0$ is the transmission coefficient of a small grid for vortices. In Fig. 3 we have plotted $(t/\tau)^{1/2} = (I\tau_0/I_0\tau)^{1/2}$ vs $V$ for the four small grids. The ordinate is the square root of the vortex transmission coefficient divided by the geometrical transmission coefficient $\tau$ (ratio of open area to total area) for each grid.

The usefulness of this type of plot is suggested by the following argument. Let us assume that a ring can pass through an aperture only if its core does not overlap an edge. Then for our case of square openings of side $l$, the transmission is proportional to the quantity $(\frac{1}{2}l-R)^2$ if $R \leq \frac{1}{2}l$, and the transmission should be zero for $R > \frac{1}{2}l$. Equation (1) predicts $R$ very nearly proportional to $V$. Thus this picture predicts that a plot of $(t/\tau)^{1/2}$ versus $V$ should yield a straight line intersecting the voltage axis at the point $V_{\rm CO}$ where $R(V_{\rm CO}) = \frac{1}{2}l$ and approaching $(t/\tau)^{1/2} = 1$ for small voltages.

Although this is undoubtedly an oversimplified account of an encounter of a vortex ring with a grid, the graphs shown in Fig. 3 do have the expected characteristics and, in particular, yield a well-defined cutoff voltage for each grid. We have determined the value of the cutoff voltage for each grid for a range of values of source voltage and collector voltage, and for both signs of the charge. No systematic dependence of $V_{\rm CO}$ on any of these variables was found, provided the current density was kept low. Thus we appear to have a unique cutoff voltage for each grid. In Fig. 4 we plot the cutoff energy, $E_{\rm CO} = eV_{\rm CO}$, versus the grid size. The error bars on the cutoff energy represent the average deviation of a number of independent determinations of $E_{\rm CO}$ using dif-

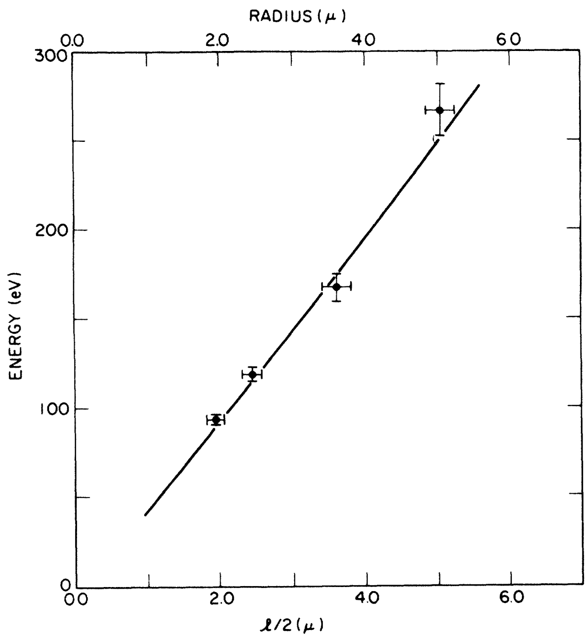


FIG. 4. Cutoff energy versus opening size. The points are experimental. The curve is theoretical, calculated from Eq. (1) using the cutoff condition $R(E_{\rm CO}) = \frac{1}{2}l$.

ferent values of $V_S$, $V_C$, and polarity. The error bars on the opening size are also average deviations and are based on measurements of the size of the grid openings on photomicrographs taken from different regions of each grid.

We also show in Fig. 4 the value of $R(E_{\rm CO})$ calculated from Eq. (1) assuming one quantum of circulation and Rayfield and Reif's value of the core radius ($a = 1.28$ Å). It will be noted that excellent agreement is obtained with the cutoff condition $R(E_{\rm CO}) = \frac{1}{2}l$ with $R(E)$ calculated in this manner. This result confirms the (perhaps surprising) hypothesis that the properties of the quantized vortex ring can be calculated from the classical equations (1) and (2).

Finally we may observe that experiments we have performed on the transmission of vortex rings in water through holes confirm (approximately) the cutoff condition $R = \frac{1}{2}l$. It should be noted that in the liquid-helium experiments either collection of the charge at the grid or failure of the ring to penetrate the hole will result in reduction of the collected current.

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## VELOCITY OF SOUND AND λ LINES

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Measurements of the velocity of first sound very near the λ line of liquid helium have been made by Chase[1] and more recently by Rudnick and Shapiro.[2] These later measurements were interpreted partly with the aid of the thermodynamic relations of Pippard[3] to infer a discontinuity in the velocity of sound. Assuming such a discontinuity, Revzen, Ron, and Rudnick[4] have postulated a new phase for liquid helium. In this note we show that small departures from the Pippard relations can have a large influence on the velocity of sound near the λ line, and the measurements of Rudnick and Shapiro can be interpreted in terms of these small departures instead of a discontinuity of the velocity of sound.

A λ line may be generally defined as the locus of points on a thermodynamic surface for which the specific heat at constant pressure $C_p$, the thermal expansion coefficient $\beta$, or the compressibility $K$ becomes infinite. In addition, there is supposed to be no latent heat or volume change (i.e., the order of the transition is higher than first). The velocity of