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NEW DETERMINATION OF THE PION-NUCLEON COUPLING CONSTANT AND *s*-WAVE SCATTERING LENGTHS*

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The most accurate determination of the pion-nucleon coupling constant published to date is that of Woolcock[1] using the dispersion relation for the invariant amplitude B_+ in the forward direction. (The subscript + refers to π^+p elastic scattering.) This method gave

$$f^2 = 0.081 \pm 0.003.$$

Although we were satisfied that the method was reliable and that the quoted error was realistic, we thought that a more accurate result could now be obtained by a more direct method. This is simply to re-evaluate the familiar dispersion relations for the $\pi^\pm p$ elastic-scattering amplitudes in the forward direction, taking account of the large amount of experimental data now available. This calculation also yields the s-wave scattering lengths. Since the results are interesting (and surprising), we give here a brief account of the method used, and of the values obtained for the constants.

Instead of writing the dispersion relations in the usual form, first written down by Goldberger, Miyazawa, and Oehme,[2] we take advantage of the sum rule discussed in the same paper to eliminate one of the scattering-length combinations. The most convenient form for evaluation is to write $D_\pm$, the real parts of the $\pi^\pm p$ forward-scattering amplitudes in the laboratory system, in the form

$$D_\pm(\omega_L) = A \mp \frac{2f^2}{\omega_L \mp 1/2M} + \frac{\omega_L}{4\pi^2} \mathrm{P}\int_0^\infty d\omega_L' \times \frac{q_L'}{\omega_L'}\left[\frac{\sigma_\pm(\omega_L')}{\omega_L'-\omega_L} - \frac{\sigma_\pm(\omega_L')}{\omega_L'+\omega_L}\right]. \tag{1}$$

$\hbar$, c, and μ, the charged pion mass, are chosen as basic units; q_L, ω_L are the momentum and total energy, respectively, of the incident pion in the laboratory system; and $M = M_p/[1-(M_n^2-M_p^2)]$, where M_n and M_p are the neutron and proton masses, respectively. $\sigma_\pm$ are the total cross sections for $\pi^\pm p$ scattering. The constants A and f^2 are to be determined.

Using the language of charge independence, if a_1 and a_3 are the usual s-wave pion-nucleon scattering lengths for $I=\frac{1}{2}$ and $\frac{3}{2}$, respectively, we have from (1)

$$D_+(1) + D_-(1) = \left(1+\frac{1}{M_p}\right)\frac{2}{3}(a_1+2a_3) = 2A - \frac{1}{M}\frac{2f^2}{1-1/4M^2} + \frac{1}{2\pi^2}\int_1^\infty \frac{d\omega_L'}{\omega_L' q_L'} \times[\sigma_+(\omega_L') + \sigma_-(\omega_L')], \tag{2}$$

$$D_-(1) - D_+(1) = \left(1+\frac{1}{M_p}\right)\frac{2}{3}(a_1-a_3) = \frac{4f^2}{1-1/4M^2} + \frac{1}{2\pi^2}\int_1^\infty \frac{d\omega_L'}{q_L'} \times[\sigma_-(\omega_L') - \sigma_+(\omega_L')]. \tag{3}$$

Thus, having determined A and f, evaluation of two further integrals gives (a_1+2a_3) and (a_1-a_3).

The values of $\sigma_\pm$ required to evaluate the dispersion integrals in Eqs. (1), (2), and (3) were obtained as follows: Up to 400 MeV, the phase shift α_{33} for the $I=\frac{3}{2}$ $p_{3/2}$ state was parametrized using the form

$$q^3 \cot\alpha_{33} = \sum_{n=0}^{n} a_n q^{2n},$$

where q is the center-of-mass momentum. The best least-squares fit was obtained for $N = 6$, and corresponds to a scattering length of 0.217 ± 0.004. The contribution from the $I = \frac{3}{2}$ $p_{3/2}$ state to $\sigma_\pm$ was calculated and added to the contribution from the other partial waves. The small phase shifts are known sufficiently well now for their contribution to be calculated with good accuracy, and very well-fitting curves to the experimental values of $\sigma_\pm$ were obtained.

Between 400 MeV and 10 BeV all one can do is to draw smooth curves to fit the experimental values of $\sigma_\pm$ as well as one can. At the lower energies in the range, we relied mainly on the Saclay data.[3] The Princeton data[4] were published too late to be used. Some aspects of these data are puzzling, and it will be necessary to do some extensive recalculations. We hope to report on these when giving a more detailed account of this work. In view of the good fit reported on in this note, one would hope that the value of f^2 will be little altered in a fit using the alternative data. Höhler and Baacke[5] have already pointed out that the integral in Eq. (3) is hardly altered by substituting the Princeton data for the Saclay data.

Above 10 BeV we used the form

$$\tfrac{1}{2}(\sigma_- - \sigma_+) = 0.425/q_L^{0.5}$$

as given by Höhler,[6] together with a fit to the data of Galbraith et al.,[7] which has the form

$$\tfrac{1}{2}(\sigma_+ + \sigma_-) = 1.104 + 8.30/q_L^{0.9}.$$

The dispersion integral over σ_- was corrected at low energies for the mass difference between the $\pi^- p$ and $\pi^0 n$ systems in charge-exchange scattering and the contribution to σ_- from the radiative capture process.[8] When these corrections are calculated, their contribution proves to be small; even though there is uncertainty concerning the unphysical-region contribution, the error arising from this source is very small.

The values of D_+ used in the fit were calculated, either directly from phase-shift analyses of $\pi^\pm p$ scattering data, or by extrapolating measured differential cross sections for elastic $\pi^\pm p$ scattering to the forward direction. We carefully scrutinized the values of $D_\pm$ quoted in the literature which were obtained by the latter method, and usually increased the quoted errors, especially at low energies where the reliability of the Coulomb corrections employed is doubtful. It is worth noting that, in the final fit, the goodness of fit is not significantly different for the sets of values of $D_\pm$ obtained by the two methods.

Having evaluated $D_\pm(\omega_L)$ (there were 286 values in all) and the corresponding integrals in Eq. (1), the data were fed into a minimization program which found the best fit to Eq. (1) on varying the parameters A and f^2. The result of the fit was

$$f^2 = 0.0822 \pm 0.0018,$$

$$A = -0.116 \pm 0.002,$$

with a value of χ^2 of 254 for 283 degrees of freedom. The value of f^2 is in excellent agreement with the final value given in Eq. (4.49) of Hamilton and Woolcock.[1]

On evaluating the integrals in (2) and (3), we find for the scattering-length combinations

$$(a_1 - a_3) = 0.292 \pm 0.020,$$

$$(a_1 + 2a_3) = -0.035 \pm 0.012.$$

These values are in bad disagreement with those given in Hamilton and Woolcock.[1] The reason for the difference seems to lie in the improvement of our knowledge of the total cross sections, rather than in the new data on $D_\pm$.

The result for $(a_1 - a_3)$ is so surprising that we look for confirmatory evidence. A calculation by Höhler and Baacke[5] using the sum rule (3) with $f^2 = 0.081 \pm 0.002$ gives $(a_1 - a_3) = 0.291 \pm 0.011$. We cannot understand this error, since an error of 0.011 arises from the f^2 term alone. However, there is clearly agreement in the evaluation of the integral. Also Donald et al.,[9] from an analysis of a number of measurements of the total cross section for the charge-exchange process $\pi^- + p \rightarrow \pi^0 + n$ at low energies, obtain $(a_1 - a_3) = 0.291 \pm 0.015$. This is support for the hypothesis of charge independence for low-energy pion-nucleon scattering, but the old discrepancy with the results of pion photoproduction is raised once again. Donnachie and Shaw,[10] from a careful analysis of pion photoproduction from nucleons, have concluded, from the well-known connection between low-energy charge-exchange scattering and threshold pion photoproduction via the Panofsky ratio, that $(a_1 - a_3)$ should be 0.252 ± 0.004, using the best measurement of the Panofsky ratio to date. Perhaps the error is optimistic, but the discrepancy is large nevertheless. It is easy (and tempting) to suspect the principle of detailed balance, especially in view of the recent suggestion[11] of the possibility of a strong violation of C and T invariance in the electromagnetic interaction of strongly interacting particles. We believe, however, that a great deal of careful work remains to be done before other explanations of the discrepancy can be rigorously excluded.

A more detailed account of this work will be published later, together with an analysis of the pion-nucleon s-wave scattering amplitudes.

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PROTON COMPTON SCATTERING MEASUREMENT FROM 450 TO 1350 MeV NEAR 90° IN THE CENTER-OF-MASS SYSTEM*†

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We have performed an experiment to measure the 90° c.m. differential cross section for γ,p scattering (proton Compton scattering), for incident laboratory photon energies (k_L) between 450 and 1350 MeV. It is our ultimate goal to obtain enough information in the form of angular distributions to contribute to the understanding of resonances. Our present data, at a single angle, are not of great value for that purpose, although we do see the effects of the first, second, and third π,p resonances, and possibly a higher one. It remains to be seen whether our data agree with theoretical predictions. In the region of the first resonance, previous experimental results[1] agree well with a dispersion-theory treatment.[2] One feature of this process which simplifies the analysis is the smallness of the elastic cross section in comparison with the various inelastic pion-photoproduction cross sections. The effect of this is that the reactive effect of the elastic-scattering process on itself, through the unitarity condition, is negligible. At the same time, the reactive effects of the inelastic channels are not only important but are, in principle, known in terms of measured processes. This situation prevails throughout the region of our data.

Our apparatus is shown in Fig. 1. The recoil proton is detected in the scintillation counters $S2$, $S3$, and $S4$ with its track being recorded in the thin spark chambers $SC3$, $SC4$, and $SC5$. The scattered photon is converted with 0.7 probability in the lead spark chamber $SC1$ (1.96 radiation lengths), and counted in the scintillator $S1$ and in the 2-in. lead glass Cherenkov counter C; the shower development is observed in the lead chamber $SC2$ (4 radiation