Table I. Experimental tests of Eqs. (4) and (5).[a]

E (BeV)	$\sigma_{tot}(\bar{p}p)-\sigma_{tot}(pp)$ (mb)	$5[\sigma_{tot}(\pi^-p)-\sigma_{tot}(\pi^+p)]$ (mb)	$(5/4)[\sigma_{tot}(\bar{p}n)-\sigma_{tot}(pn)]$ (mb)
6	18.7 ± 1.3	11.5 ± 2.0	21.2 ± 5.5
8	16.4 ± 1.0	12.0 ± 2.0	19.4 ± 5.3
12	12.3 ± 1.0	8.5 ± 2.0	16.7 ± 5.1
14	11.6 ± 1.1	7.5 ± 2.0	16.5 ± 5.1
16	10.5 ± 1.0	8.5 ± 2.0	15.6 ± 5.1
18	11.6 ± 4.1	7.5 ± 2.0	6.5 ± 11.5

[a]The data are taken from W. Galbraith, E. Jenkins, T. Kycia, B. Leontić, R. Phillips, A. Read, and R. Rubinstein, Phys. Rev. 138, B913 (1965). The numbers in the last column carry large errors and therefore the test of Eq. (5) is less accurate than that of Eq. (4).

Note added in proof. —We would like to add that upon inclusion of kineton corrections [P. G. O. Freund, Phys. Rev. Letters 14, 803 (1965)], a small departure from unity of the singlet-to-octet ratio in formula (2) can be accounted for.

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[1]K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 48 (1965); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters 14, 404 (1965); see also M. Bég and A. Pais, Phys. Rev. Letters 14, 267 (1965).

[2]K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965); J. M. Cornwall, P. G. O. Freund, and K. T. Mahanthappa, Phys. Rev. Letters 14, 515 (1965); R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters 14, 518 (1965).

[3]R. F. Sawyer, Phys. Rev. Letters 14, 471 (1965).

[4]B. Sakita and K. C. Wali, Phys. Rev. 139, B1355 (1965). It should be pointed out that the relative coefficient (one) of the two terms in (2) is fixed by $M(12)$. Nonrelativistic SU(6) does not fix this coefficient.

[5]We do not make any assumptions concerning the even charge-conjugation states, and thus about the Pomeranchuk shrinking or nonshrinking of diffraction peaks.

[6]In the limit in which only even charge-conjugation SU(3) singlet exchange (in the t channel) is taken into account, one obtains the relations $\sigma_{tot}(K^+p)=\sigma_{tot}(K^-p) =\sigma_{tot}(\pi^+p)=\sigma_{tot}(\pi^-p)=\sigma_{tot}(MB)$ and $\sigma_{tot}(pp)=\sigma_{tot}(\bar{p}p) =\sigma_{tot}(BB)=\sigma_{tot}(\bar{B}B)$ [P. G. O. Freund, M. Ruegg, A. Speiser, and A. Morales, Nuovo Cimento 25, 307 (1962)]. A universality for this $C=+1$ SU(3) singlet exchange can be formulated [P. G. O. Freund, Phys. Letters 2, 136 (1962)] in the form $\sigma_{tot}(MB)=(m/M) \times\sigma_{tot}(BB)$, where m and M are the central meson and baryon masses. In view of the universality prediction $m/M=\frac{2}{3}$ (P. G. O. Freund, Phys. Rev. Letters 14, 1088 (1965); 15, 176(E) (1965); and to be published), this relation takes the form $\sigma_{tot}(MB)\cong\frac{2}{3}\sigma_{tot}(BB)$ in reasonable agreement with experiment.

ANALYSIS OF TOTAL CROSS-SECTION DIFFERENCES AT HIGH ENERGY*

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High-precision determinations of differences in total cross sections for $K^\pm$ and $\pi^\pm$ on protons have recently been made for the momentum interval 6 to 20 BeV/c.[1] Such data on total cross-section differences, $\Delta_{AB}\equiv\sigma_t(\bar{A}B)-\sigma_t(AB)$, at high energy are particularly suitable for the study of symmetry predictions since invariance principles considerably simplify the dynamics involved.

A phenomenological study of the Δ_{AB} is greatly facilitated by the following factors:

(i) The Δ_{AB} are linearly related to the imaginary part of the elastic amplitudes by the optical theorem. At high energy the elastic amplitudes are expected to be dominated by Regge meson exchanges in the channel $A+\bar{A}\rightarrow B+\bar{B}$.[2]

(ii) Only trajectories of exchanged mesons with odd charge-conjugation quantum number C contribute to the Δ_{AB}. For meson-nucleon scattering (Δ_{MN}), the exchanged meson must have natural parity $P=(-1)^J$ (i.e., signature $\tau=P$) due to the coupling to the external pseudoscalar meson pair. In nucleon-nucleon and nucleon-antinucleon scattering at high energy, trajectories with $\tau=-P$ and $P=+$ make no contribution to the Δ_{NN}.[3] Furthermore, since no mesons with odd C, even J, and odd parity ($\tau=-P$ and $P=-$) have been established, the Δ_{NN} should also be dominated by exchanges of meson states of natural parity and odd C.

(iii) The observed meson states of odd charge-conjugation quantum number and natural parity are satisfactorily classified as a single SU(3) vector nonet $(K^*, \rho, \omega, \varphi)$.

In this Letter we analyze the Δ_{MN} and Δ_{NN} in terms of the Regge trajectories of the vector-meson nonet. The ratio of the ρ-meson couplings to the conserved $\pi\pi$ and $\bar{K}K$ meson currents is determined, and it agrees within one standard deviation with the SU(3)-invariant coupling prediction. From a statistical fit to the data, the f/d ratio of the $V\bar{B}B$ charge coupling, the ratio of the $V\bar{N}N$ to the VMM coupling at zero momentum transfer, and the trajectory intercepts α_V are evaluated.

The asymptotic contribution of the Regge pole of a vector meson V to the total cross section may be written in natural units ($\hbar=c=1$) as

$$\sigma_t(AB)=\frac{\gamma_{AV}\gamma_{BV}}{s^{1/2}q_{AB}(s)}\pi^{1/2}\frac{\Gamma(\alpha_V+\frac{3}{2})}{\Gamma(\alpha_V+1)}\left(\frac{s-M_A{}^2-M_B{}^2}{s_0}\right)^{\alpha_V}, \quad (1)$$

where we have used the factorization theorem[4] for the dimensionless residue γ_{AB}. In Eq. (1), $q_{AB}(s)$ denotes the center-of-mass momentum; s_0 is an arbitrary scaling factor which we fix at (1 BeV)^2 in our subsequent analysis. For economy of notation we define

$$R_{AVB}(s)\equiv\pi^{1/2}\frac{\Gamma(\alpha_V+\frac{3}{2})}{\Gamma(\alpha_V+1)}\frac{(s-M_A{}^2-M_B{}^2)^{\alpha_V}}{s^{1/2}q_{AB}(s)}. \quad (2)$$

Through invariance under charge conjugation and isotopic-spin rotations, we can isolate the separate (I, G) meson exchange contributions in terms of the Δ_{AB}. We find for the $(1,+)$ contribution

$$\tfrac{1}{2}\Delta_{\pi^+p}=\gamma_{\pi\rho}\gamma_{p\rho}R_{\pi\rho p}(s), \quad (3)$$

$$\tfrac{1}{4}[\Delta_{K^+p}-\Delta_{K^+n}]=\gamma_{K\rho}\gamma_{p\rho}R_{K\rho p}(s), \quad (4)$$

$$\tfrac{1}{4}[\Delta_{pp}-\Delta_{pn}]=\gamma_{p\rho}{}^2R_{p\rho p}(s). \quad (5)$$

The corresponding results for the imaginary part of the $(0,-)$ amplitudes are slightly more complicated due to the presence of both φ and ω exchanges:

$$\tfrac{1}{4}[\Delta_{K^+p}+\Delta_{K^+n}]=\gamma_{K\varphi}\gamma_{p\varphi}R_{K\varphi p}(s)+\gamma_{K\omega}\gamma_{p\omega}R_{K\omega p}(s), \quad (6)$$

$$\tfrac{1}{4}[\Delta_{pp}+\Delta_{pn}]=\gamma_{p\varphi}{}^2R_{p\varphi p}(s)+\gamma_{p\omega}{}^2R_{p\omega p}(s). \quad (7)$$

The signs of the residues in Eqs. (3) to (7) are the signs at the physical poles, $t=-m_V{}^2$.

The general SU(3)-invariant interaction Lagrangians for the vector-meson nonet[5] residues at $t=0$ are

$$L_{VMM}=\sqrt{2}\gamma_M\langle M[V_\mu, \partial_\mu M]\rangle, \quad (8)$$

$$L_{V\bar{B}B}=\sqrt{2}\gamma_N(f\langle\bar{B}\gamma_\mu[V_\mu, B]\rangle+d\langle\bar{B}\gamma_\mu\{V_\mu, B\}\rangle+\beta\langle V_\mu\rangle\langle\bar{B}\gamma_\mu B\rangle), \quad (9)$$

where $\langle\,\rangle$ denotes trace over SU(3) indices. For our purposes the relevant members of the nonet are $V_1{}^1=(\rho^0+\omega)/\sqrt{2}$, $V_2{}^2=(-\rho^0+\omega)/\sqrt{2}$, and $V_3{}^3=-\varphi$. The residues of Eqs. (3) to (7) may be expressed in terms of the SU(3) parameters of Eqs. (8) and (9):

$$\tfrac{1}{2}\gamma_{\pi\rho}=\gamma_{K\rho}=\gamma_{K\varphi}/\sqrt{2}=\gamma_{K\omega}=\gamma_M, \quad (10)$$

$$\gamma_{p\rho}=(f+d)\gamma_N,$$

$$\gamma_{p\varphi}=\sqrt{2}[f-d-\beta]\gamma_N,$$

$$\gamma_{p\omega}=[f+d+2\beta]\gamma_N. \quad (11)$$

We adopt the conventional normalization $f+d=1$.

The ρ-meson trajectory.—The contribution of the ρ-meson trajectory to meson-nucleon total cross sections serves as a test of exact SU(3) symmetry for the residues. According to Eqs. (3) and (4), the cross-section ratio

$$\frac{[\Delta_{K^+p}-\Delta_{K^+n}]}{\Delta_{\pi^+p}}=\frac{2\gamma_{K\rho}}{\gamma_{\pi\rho}}\frac{R_{K\rho p}(s)}{R_{\pi\rho p}(s)} \quad (12)$$

is determined up to a known kinematic factor by the couplings of the ρ meson to the conserved pseudoscalar meson current at $t=0$. From

Eq. (10) the exact SU(3) symmetry prediction is $\gamma_{K\rho}/\gamma_{\pi\rho}=\frac{1}{2}$.[6]

The measured values of Δ_{π^+p} and $[\Delta_{K^+p}-\Delta_{K^+n}]$ were simultaneously used in Eqs. (3) and (4) to determine the three parameters $\gamma_{\pi\rho}$, $\gamma_{K\rho}$, and α_ρ. With 25 measurements[1,7] the least-squares fit gave

$$\gamma_{\rho K}/\gamma_{\rho\pi}=0.57\pm0.06, \tag{13}$$

$$\alpha_\rho=0.48\pm0.05, \tag{14}$$

$$\gamma_{\rho K}\gamma_{\rho\pi}=2.61\pm0.05, \tag{15}$$

with $\chi^2=16$. The ratio of the residues is in good agreement with the exact SU(3) couplings.[8]

The cross-section second difference $[\Delta_{pp}-\Delta_{pn}]$ appearing in Eq. (5) is valuable for evaluation of γ_N/γ_M independent of the parameters associated with the (0, −) exchanges. Unfortunately, this second difference is at present not too well determined experimentally.[9] With the available data for Eqs. (3) to (5), we find

$$\gamma_N/\gamma_M=0.1\pm1.2. \tag{16}$$

We note that Eq. (5) indicates that $[\Delta_{pp}-\Delta_{pn}]$ must be positive.

ω, φ, and ρ trajectories.—For the vector meson nonet, the general mass-splitting Lagrangian has the form

$$m^2=a\langle VV\rangle+b\langle\lambda VV\rangle+c\langle V\rangle^2+d\langle V\rangle\langle\lambda V\rangle, \tag{17}$$

where the 3×3 matrix λ is $\lambda_{\alpha\beta}=\delta_{\alpha3}\delta_{\beta3}$. With an "Ansatz" requiring the nonappearance of $\langle V\rangle$ in the mass Lagrangian (i.e., $c=d=0$), Okubo obtained the mass formulas[5]

$$m_\omega^2=m_\rho^2, \tag{18}$$

$$m_\varphi^2=2m_{K^*}^2-m_\rho^2. \tag{19}$$

The implication of Eqs. (18) and (19) for approximately linear nonet trajectories is $\alpha_\omega\simeq\alpha_\rho$ and $\alpha_\varphi<\alpha_\rho$. As a working hypothesis we take $\alpha_\omega=\alpha_\rho$. Nevertheless, we find that our solutions are quite insensitive to the precise value of α_ω for $\alpha_\varphi\leq\alpha_\omega\leq\alpha_\rho$.

The suggestion that nonet couplings should also not include terms which involve $\langle V\rangle$ as a factor[10] implies that $\beta=0$ in Eq. (9). We are now in a position to investigate this speculation about β in a simultaneous fit to Eqs. (3) through (7).

Since the coupling ratio of Eq. (13) is consistent with exact SU(3) symmetry, we henceforth use the couplings of Eqs. (10) and (11) in our over-all statistical fit. Then Eqs. (3) to (7) contain six independent parameters: β, γ_M, γ_N, f/d, α_ρ, α_φ $(\alpha_\omega=\alpha_\rho)$. We adopt the following procedure for the analysis. For a fixed value of β, we determine the remaining five parameters by minimizing chi square. In order to obtain a reasonable fit to the data, we find that β must be constrained to the interval $-1.0\leq\beta\leq1.0$. In Fig. 1 the solutions for the parameters are plotted versus β. We note the following conclusions from the statistical fit to the 58 measurements[1,7,10] in the momentum interval 5 to 20 BeV/c:

(i) The f/d ratio of the $V\bar{B}B$ charge coupling is insensitive to β. The value is[11]

$$f/d=-2.0\pm0.4. \tag{20}$$

This shows a significant deviation from the prediction of universality[12] or SU(6) and $\tilde{U}(12)$[13]

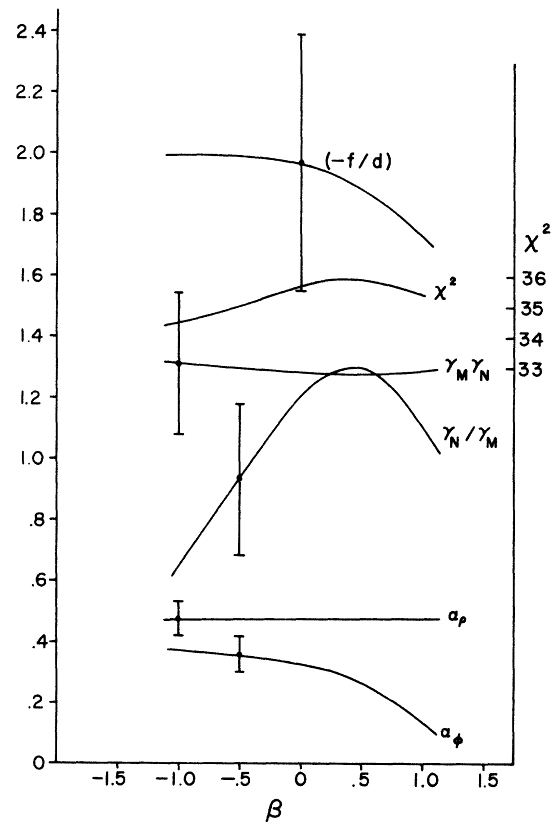

FIG. 1. Plot of most likely solutions for the parameters $\gamma_M\gamma_N$, γ_N/γ_M, f/d, α_ρ, α_φ versus the parameter β. Cf. Eqs. (3)-(7) and Eqs. (10)-(11). Representative errors are given for each parameter. The values of χ^2 for the fits are indicated on the right ordinate.

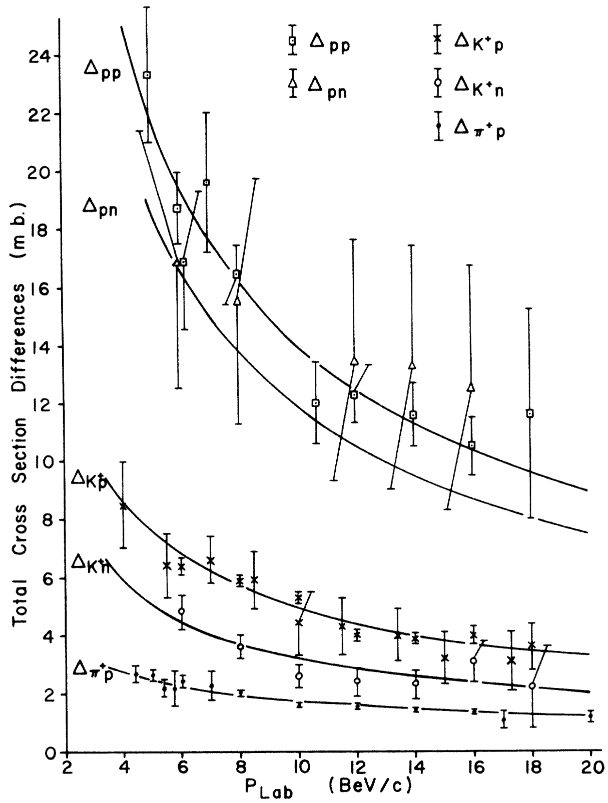

FIG. 2. Experimental measurements[1,7,10] of the cross-section differences Δ_{pp}, Δ_{pn}, Δ_{K^+p}, Δ_{K^+n}, and Δ_{π^+p}, where $\Delta_{AB} \equiv \sigma_t(\bar{A}B)-\sigma_t(AB)$. The curves shown were calculated using the parameters of Fig. 1 for $\beta=0$.

which give $d=0$. Universality predicts in addition that $\gamma_M=\gamma_N$.

(ii) The limits on γ_N/γ_M from the data are

$$0.5<\frac{\gamma_N}{\gamma_M}<1.5.$$

More precise data on the Δ_{NN} are needed to improve the value for this ratio. Similar conclusions apply to the determination of β.

(iii) α_ρ is unchanged for all choices of β,

$$\alpha_\rho=0.48\pm0.05.$$

In Fig. 2 we show the fit with $\beta=0$ for the Δ_{MN} and Δ_{NN}.

For the purpose of evaluating the Johnson-Treiman SU(6) predictions[14,15]

$$\Delta_{\pi^+p}=\Delta_{K^+n}=\tfrac{1}{2}\Delta_{K^+p}, \qquad (21)$$

we can regard our statistical fit as an interpola-

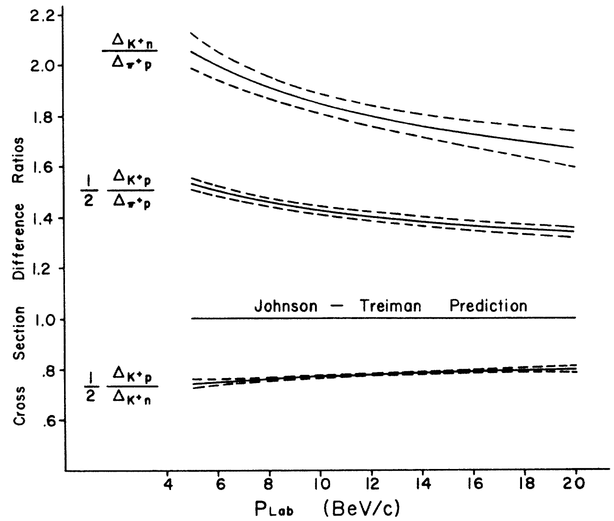

FIG. 3. Evaluation of the Johnson-Treiman SU(6) predictions $\Delta_{\pi^+p}=\Delta_{K^+n}=\frac{1}{2}\Delta_{K^+p}$. The error corridors are the result of our statistical fit to the experimental data. The ratios shown should equal one if the J-T relations are satisfied.

tion formula for the Δ_{MN}. In Fig. 3 we compare the fit to the Δ_{MN} with Eq. (21). It appears that the SU(3) sum rule of Eqs. (10) and (12) gives a much better representation of the data.[16] This is consistent with the value of f/d found in Eq. (20).

We thank Professor C. Goebel and Professor R. F. Sawyer for conversations.

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[1]W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontić, R. H. Phillips, A. L. Read, and R. Rubinstein, Brookhaven National Laboratory Report No. BNL-9410, 1965 (unpublished); also R. H. Phillips, private communication.

[2]A number of references on the phenomenology based on Regge poles can be found in R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965); B. M. Udgaonkar, in Strong Interactions and High Energy Physics (Plenum Press, New York, 1963), p. 223.

[3]W. G. Wagner, Phys. Rev. Letters 10, 202 (1963).

[4]M. Gell-Mann, Phys. Rev. Letters 8, 263 (1962).

[5]S. Okubo, Phys. Letters 5, 165 (1963). We use the Okubo approach in dealing with the vector meson nonet. The physical particles (φ,ω) are related to the representation members (φ_8,ω_1) by $\varphi=(\sqrt{2}\varphi_8-\omega_1)/\sqrt{3}$ and $\omega=(\varphi_8+\sqrt{2}\omega_1)/\sqrt{3}$. This identification forbids the decay mode $\varphi+\rho+\pi$. Henceforth, V will designate the 3×3 matrix for the vector meson nonet.

[6]In the limit of degenerate mass $m_\pi=m_K$ and exact SU(3) couplings, Eq. (12) reduces to the sum rule

$\Delta_{K^+p} = \Delta_{K^+n} + \Delta_{\pi^+p}$ obtained by V. Barger and M. H. Rubin, Phys. Rev. 140, B1365 (1965).

[7]A. Citron et al., Phys. Rev. Letters 13, 205 (1964); also R. H. Phillips, private communication; W. F. Baker et al., in Proceedings of the Sienna International Conference on Elementary Particles (Società Italiana di Fisica, Bologna, Italy, 1963), Vol. I, p. 634; A. N. Diddens et al., Phys. Rev. Letters 10, 262 (1963); G. von Dardel et al., Phys. Rev. Letters 7, 127 (1961); S. J. Lindenbaum et al., Phys. Rev. Letters 7, 352 (1961); G. von Dardel et al., Phys. Rev. Letters 8, 173 (1962).

[8]The most likely value of 0.57 for $\gamma_{\rho K}/\gamma_{\rho\pi}$ could be interpreted as a 15% deviation from the exact symmetry value 0.5.

[9]W. Galbraith et al., Phys. Rev. 138, B913 (1965); W. F. Baker et al., Phys. Rev. 129, 2285 (1963); S. J. Lindenbaum et al., Phys. Rev. Letters 7, 185 (1961); G. von Dardel, Phys. Rev. Letters 5, 333 (1960).

[10]S. L. Glashow and R. H. Socolow, Phys. Rev. Letters 15, 329 (1965).

[11]The value of the f/d ratio obtained in reference 6 was based upon degenerate masses, exact SU(3) couplings, and degenerate trajectory intercepts $\alpha_\rho = \alpha_\omega = \alpha_\varphi$.

[12]J. J. Sakurai, in Proceedings of the International School of Physics (Academic Press, Inc., New York, 1963), p. 41.

[13]B. Sakita and K. C. Wali, Phys. Rev. 139, B1355 (1965).

[14]K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).

[15]R. F. Sawyer, Phys. Rev. Letters 14, 471 (1965).

[16]Similar remarks apply to a recent preprint by P. Freund in which the $d = 0$ value of $\tilde{U}(12)$ was assumed.

REMARKS ON THE CONNECTION BETWEEN EXTERNAL AND INTERNAL SYMMETRIES

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Recently a series of articles[1,2] was published, concerning the impossibility of explaining mass splitting within the context of finite-order Lie algebras, containing the Poincaré (inhomogeneous Lorentz) Lie algebra L. It is the aim of this Letter to analyze the results of the quoted articles, and to show that—lacking mathematical rigor—the author did not prove what he intended to.

To begin with, let us discuss the problem of mass differences.[2] The author claims that if the operator P^2 representing $P_\mu P^\mu$ is self-adjoint on the Hilbert space H on which a representation of a Lie algebra $E \supset L$ is defined, and if its spectrum contains a (real) eigenvalue m^2, then the (closed) eigenspace H_m of P^2 belonging to m^2 is invariant with respect to the operators representing E. We remark that all that is used about m^2 is the fact it is a point in the discrete (also[3] called point) spectrum, and it may, or may not, be isolated. Let us analyze the demonstration. We shall denote by $D(X)$ the domain of the operator X on H; obviously $H_m \subset D(P^2)$. Let e represent on H any element of E, and $H_m{}^e$ be the subspace of those h in $H_m \cap D(e)$ such that

$$eh \in \bigcap_{n=1}^{N} D(P^{2n}).$$

From the nilpotency of $P_\mu P^\mu$ in the enveloping algebra of E, we know that $ad^N P^2 = 0$. We can then consider $(P^2 - m^2)^N eh$ and, since $(P^2 - m^2)h = 0$ for $h \in H_m{}^e$, we have

$$(P^2 - m^2)^N eh = (P^2 - m^2)^{N-1}[P^2, e]h,$$

from the definition of the commutator, an expression which is well defined (notice that we cannot replace the commutator by its value before checking that the obtained expression is defined). Thus

$$(P^2 - m^2)^N eh = [(ad^N P^2)e]h = 0,$$

and therefore $eH_m{}^e \subset H_m$. We can infer therefrom that the space H_m is invariant only if $H_m{}^e$ is dense in H_m, for every e in the representation of E. Now, the set

$$H^e = \{h \in H; h \in D(e), eh \in \bigcap_{n=1}^{N} D(P^{2n})\}$$

is in general a dense subspace of H (not coinciding with H), and $H_m{}^e = H_m \cap H^e$. Instead of all the H^e's, we can also consider a dense subspace D of H, on which all operators representing finite-order elements in the enveloping algebra of E are defined (this is usually the case[4]); if $H_m{}' = H_m \cap D$, we shall have $eH_m{}' \subset H_m$ for all the e's.

But, in a Hilbert space H, the intersection of a closed subspace F with a dense subspace