is negative and is almost the same in both cases. The first radial integral I_{0S} is attractive while the other three radial integrals are repulsive. Figure 1 gives the entire spectrum for both the first case and the second case. One immediately notices in Fig. 1 that in Ca^{43} and Ca^{45} there are predicted low-lying $\frac{9}{2}^-$ states that have not been observed experimentally.[8] This region of the spectrum is obscured by strong C^{12} and O^{16} contaminant in the reaction $Ca^{43}(p, p')Ca^{43*}$. It would be very interesting to reexamine this region of the experimental spectrum to see whether these levels do indeed exist. The general features are predicted very well, especially the positions of the $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$ states in Ca^{43} and Ca^{45}.[9]

The over-all agreement is very gratifying and we are now in the process of examining the details of the wave functions and intend to calculate predictions of reduced widths, gamma-ray transitional probabilities, and moments. From a preliminary examination of the wave functions produced for Ca^{42} and Ca^{43}, we see that (a) in Ca^{42} the two spin-2 states are mixed very strongly, which agrees with the observed results from $Ca^{43}(d,t)Ca^{42}$ experiment,[10] and (b) in Ca^{43} the 595-keV $\frac{3}{2}^-$ level would be very weakly excited in the $Ca^{42}(d, p)Ca^{43}$ experiment, in agreement with the observed results.[4] Future results will be published along with the predictions for the spectrum of Ca^{46}-Ca^{49} for these parameters.

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K_{e3} DECAY AND UNIVERSALITY IN CABIBBO'S THEORY OF LEPTONIC DECAYS*

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Within the framework of Cabibbo's theory[1] of leptonic decays, the bare coupling constant for beta decay is $G\cos\theta$ and that for $|\Delta S|=1$ decays is $G\sin\theta$, where G is the coupling constant for muon decay. Cabibbo determined $\sin\theta$ from the ratio of the rates for K_{e3} and π_{e3} decay, getting $\theta=0.26$, which implies $\cos\theta=0.966$, whereas the value of $\cos\theta$ determined from beta decay after inclusion of radiative corrections is $\cos\theta=0.978\pm0.0015$.[2] This discrepancy was discussed by Sakurai,[3] who pointed out that in the computation of the K_{e3} decay rate, $G\sin\theta$ should be replaced by $(G\sin\theta)C$. Here $C=Z_1^{-1}(K\pi)Z_2^{1/2}(K)Z_2^{1/2}(\pi)$ is a renormalization factor which can be expected to differ from unity because the $\Delta S=1$ current, $j_\mu^{(4)}+ij_\mu^{(5)}$, is not conserved in the presence of an SU(3)-symmetry-breaking interaction H_{ms}'. Sakurai estimated C from the departure of the ratio $\Gamma(K^*\to K+\pi)/\Gamma(\rho\to\pi+\pi)$ from the SU(3) symmetry prediction, obtaining $C=(0.81)^{-1}$, and $\cos\theta=0.979$, in agreement with the value determined from β decay. However, it was subsequently shown by Ademollo and Gatto[4] that the correction to $G\sin\theta$ is only of second order in $H_{\rm ms}'$, which suggests that C^2 is unity to within $(0.1)^2$, i.e.,

$$|C^2-1| \lesssim 1\text{-}5\,\%. \tag{1}$$

If the estimate (1) is accepted, the value of $\cos\theta$ as determined by Cabibbo is hardly affected and the discrepancy appears to be reinstated.

We wish to point out that the use of Eq. (1) and the most recent "best value" of the K_{e3} decay rate, together with the inclusion of hitherto neglected form-factor effects in the determination of $G\sin\theta$ from this rate, may yield a value of $\cos\theta$ which is in quite close agreement with that determined from beta decay.

The hadronic part of the matrix element for K_{e3}^+ decay may be written in the form

$$\langle\pi^+|j_\mu^{(4)}+ij_\mu^{(5)}|K^+\rangle$$
$$=(1/\sqrt{2})[(p_K+p_\pi)_\mu f^{(+)}(q^2)+(p_K-p_\pi)_\mu f^{(-)}(q^2)],$$

where $q=p_K-p_\pi$ and the $f^{(\pm)}(q^2)$ are form factors. A phenomenological form for $f^{(\pm)}(q^2)$, used in recent analyses of K_{e3} decay, is

$$f^{(+)}(q^2)=CM^2/(M^2-q^2), \tag{2}$$

where $C=f^{(+)}(0)$ and M is a parameter. If the electron mass is neglected the decay rate is given by

$$\Gamma(K\to\pi+e+\nu)=(\text{const})G^2\sin^2\theta C^2 I(M), \tag{3}$$

where

$$I(M)=\int_{m_\pi}^{E_{\max}}[M^2/(M^2-q^2)]^2(E_\pi^2-m_\pi^2)^{3/2}dE_\pi,$$

with $q^2=m_K^2+m_\pi^2-2m_KE_\pi$.

In the absence of symmetry breaking we have $C=1$, since then $j_\mu^{(4)}+ij_\mu^{(5)}$ is conserved. Let θ_0 denote the value of θ determined by Eq. (3) with $f^{(+)}(q^2)\equiv 1$, i.e., with $C=1$ and $M=\infty$. The "true" value of θ is then given by

$$\sin^2\theta=C^{-2}x\sin^2\theta_0, \tag{4}$$

where $x=I(\infty)/I(M)$ is the correction factor resulting from the variation of $f^{(+)}(q^2)$ with q^2. From Table I, x is seen to vary from 0.93 to 0.71 for 1000 MeV $\geq M \geq$ 500 MeV, which corresponds to a 10 to 30% effect on $\sin^2\theta$ and which is therefore of the same order of magnitude as that attributed in reference 3 to $C^2\neq 1$, if M is in the indicated range. (Electromagnetic corrections to K_{e3} decay are not expected to be larger than ~1 %.)

Table I. $x=I(\infty)/I(M)$ for the decay $K^+\to\pi^0+e^++\nu_e$.

M (MeV)	x
500	0.71
600	0.80
700	0.86
800	0.89
900	0.92
1000	0.93
1500	0.97
2000	0.98
∞	1

The most recent "best" value for $\Gamma(K^+\to e^+ +\pi^0+\nu_e)$ is, according to Trilling,[5] $(3.61\pm0.20)\times10^6\ \text{sec}^{-1}$. From Eq. (3), with $C=1$ and $M=\infty$, we then get, using $G=(1.4350\pm0.0011)\times10^{-49}$ erg cm^3 as deduced from muon decay,[2] that $\sin\theta_0=0.222\pm0.006$. Equation (4) then implies

$$\cos\theta\cong 1-C^{-2}x(0.0246\pm0.001). \tag{5}$$

The experimental situation regarding the value of M is still unclear at the present time. Apparently a value of $M\sim 1000$ MeV is consistent with the K_{e3}^+ data.[5] On the other hand, a value $M=(480^{+100}_{-30})$ MeV obtained from K_2^0 decay has been reported, for instance, by Fisher et al.[6] If we take, for the sake of definiteness a value $M=M_{K^*}=880$ MeV, as suggested by dispersion theory, we get $x=0.90$, so that from Eq. (5), with $C^2=1$,

$$\cos\theta=0.978\pm0.001.$$

More generally, with $C^2=1$, $\cos\theta$ varies from 0.980 for $M=500$ MeV to 0.974 for $M=1000$ MeV to 0.972 for $M=\infty$. These numbers should be compared with the value of $\cos\theta$ determined from an unweighted average over a variety of beta decays, $\cos\theta=0.978\pm0.0015$, or that determined from the O^{14} decay alone, $\cos\theta=0.975\pm0.003$.

In conclusion, the present experimental situation seems to be quite compatible with Cabibbo's version of universality and with $C^2=1$ to within a few percent, as suggested by the theorem of Ademollo and Gatto. A more definitive test of Cabibbo's type of universality from this point of view will require further experimental clarification of the values of the form factors and branching ratios for K_{e3} decay, as well as a reliable determination of C on an independent basis, perhaps via the leptonic decays of the hyperons.[7] These latter are, however, complicated by the presence of the axial vector current.

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RELATION BETWEEN πp, pp, AND $\bar{p}p$ SCATTERING AT HIGH ENERGIES*

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It is well known that $M(12)$ invariance[1] relates the cross sections for the scattering of mesons on baryons by the Johnson-Treiman relation[2]

$$\sigma_{\text{tot}}(K^-p)-\sigma_{\text{tot}}(K^+p)=2[\sigma_{\text{tot}}(\pi^-p)-\sigma_{\text{tot}}(\pi^+p)]. \quad (1)$$

The same relation can be derived assuming the odd charge-conjugation amplitude in the crossed (t) channel to be dominated by F-type coupled vector mesons as suggested by universality.[3]

Universality and $M(12)$ invariance further relate the zero momentum-transfer (q) coupling of the nonet of vector mesons $[V=V_{\text{octet}}+(1/\sqrt{3})V_{\text{singlet}}1]$ to the eight $\frac{1}{2}^+$ baryons (B) and eight 0^- mesons (P). In particular, $M(12)$ tells us that at $q=0$ the VBB coupling is of the form[4]

$$f_1(\langle\bar{B}\gamma_\mu[V_\mu,B]\rangle+\langle\bar{B}\gamma_\mu B\rangle\langle V_\mu\rangle), \quad (2)$$

and the VPP coupling is

$$f_2\langle V_\mu[P,\partial_\mu P]\rangle. \quad (3)$$

In (2) and (3) $\langle\ \rangle$ stands for SU(3) trace. Universality further tells us

$$f_1=f_2 \quad (g_{\rho^0\pi^-\pi^+}=2g_{\rho^0\bar{p}p}). \quad (4)$$

Assuming the odd charge-conjugation objects exchanged at high energies between hadrons (vector-meson Regge poles?) to have couplings (residua) to mesons and baryons obeying the same SU(3) relations (2)-(4), one readily finds the relation

$$\sigma_{\text{tot}}(\bar{p}p)-\sigma_{\text{tot}}(pp)=5[\sigma_{\text{tot}}(\pi^-p)-\sigma_{\text{tot}}(\pi^+p)]. \quad (5)$$

This relation agrees with experiment to the same accuracy as relation (1) (see Table I). It thus substantiates the $M(12)$ relation between the F-type coupling of baryons to the vector-meson octet and their coupling to the vector-meson singlet,[4] as well as the universality principle (4).[6] We finally wish to observe that (1) further implies

$$\sigma_{\text{tot}}(\bar{p}p)-\sigma_{\text{tot}}(pp)=(5/4)[\sigma_{\text{tot}}(\bar{p}n)-\sigma_{\text{tot}}(pn)], \quad (6)$$

which also agrees with the existing experimental data (see Table I).

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