SHELL-MODEL STUDY OF THE CALCIUM ISOTOPES*

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The calcium isotopes should be a good example of the shell model because doubly magic calcium 40 has a high-lying first excited state. The well-isolated $1f_{7/2}$ shell occurs beyond 20 particles so that the regularities expected in j-j coupling should be observed in the seven calcium isotopes, calcium-41 through calcium-47. This is not the case at all, as several theorists have discovered.[1-6]

These regularities which do not occur are that the seniority-0 and -2 states should occur at the same energies in Ca^{42}, Ca^{44}, and Ca^{46}; and Ca^{43} with the configuration of $(f_{7/2})^3$ should have the same spectrum as Ca^{45} with the configuration of $(f_{7/2})^{-3}$.

Even when configuration interaction is included by adding the complete $1f$ and $2p$ shell and by using both central and tensor forces, the levels of Ca^{42}, Ca^{43}, and Ca^{44} have not yet been satisfactorily explained. In fact, this work is the first successful shell-model result for the calcium isotopes. The most detailed study of Ca^{42} and Ca^{43} has been performed by H. Mitler.[6] He used tensor forces and the $1g_{9/2}$ level as well as the f and p levels, and even then he was unable to fit both Ca^{42} and Ca^{43} using the same parameters and conventional potentials.

In this calculation we have included only two single-particle levels, the $1f_{7/2}$ level and the $2p_{3/2}$ levels, and have examined the four calcium isotopes Ca^{42} through Ca^{45}. In this work, harmonic-oscillator wave functions were used throughout and only the $T=1$ part of the two-particle residual interaction was used since we were dealing with only neutrons outside a closed shell. The central potential was broken up into two parts: V_{11}, which is the $T=1$, $S=1$ part of the interaction, and V_R which is the $T=1$, $S=0$ part of the interaction that remains. The matrix elements of V_R can be broken up into a sum of Talmi integrals I_{nl}, which are defined as

$$I_{nl}=\int_0^\infty R_{nl}{}^2(r)V(r)r^2dr.$$

Only the $l=0$ parts of this decomposition make large contributions so we ignore for the most part the $l>0$ terms. A full discussion of this point is given in Pandya and Green.[7] This leaves us with $l=0$, $n=0$, 1, 2, and 3 terms, i.e., I_{0s}, I_{1s}, I_{2s}, and I_{3s}. In order to account for some effect of the higher l values, a term V_{10} was added to V_R, where V_{10} was taken specifically to have a Gaussian radial dependence. For a Gaussian potential

$$I_{0l}=V_0\beta^{l+\frac{3}{2}},$$

$$I_{1s}=(\tfrac{3}{2}-3\beta+\tfrac{5}{2}\beta^2)I_{0s},$$

$$I_{2s}=(\tfrac{15}{8}-\tfrac{15}{2}\beta+\tfrac{65}{4}\beta^2-\tfrac{35}{2}\beta^3+\tfrac{63}{8}\beta^4)I_{0s},$$

$$I_{3s}=(\tfrac{35}{16}-3\times\tfrac{35}{8}\beta+15\times\tfrac{49}{16}\beta^2-11\times\tfrac{35}{4}\beta^3+31\times\tfrac{63}{16}\beta^4$$
$$-11\times\tfrac{63}{8}\beta^5+13\times\tfrac{33}{16}\beta^6)I_{0s},$$

where

$$\beta=\lambda^2/(\lambda^2+1),\ \lambda=r_0/r_1,\ V(r)=V_0e^{-r^2/r_0{}^2},$$

and

$$R_{nl}(r)\approx\exp(-r^2/r_1{}^2).$$

Thus V_{10} adds a contribution to the $l=0$, I_{ns} values which may be easily calculated. In case I (see Table I) these are −17.401, −10.875, −8.564, and −7.289 for I_{0s}, I_{1s}, I_{2s}, and I_{3s}, respectively, while in case II they are −13.934, −8.709, −6.858, and −5.837. The tensor force is included by a term V_T. In V_{11} and V_T the radial form is also taken as a Gaussian. In V_{10} and V_{11}, λ is set equal to 1.0, while for V_T, λ is taken as 1.2 to account for the long-

er range of the tensor part of the two-body interaction.

At Argonne National Laboratory, a very extensive computer program (called CLMS) has been developed for doing shell-model calculations. This has been the work of S. Cohen, R. Lawson, M. Macfarlane, and one of the authors (M.S.). The CLMS program was used to compute the matrix elements for a unit strength for each of the seven terms in our two-body potential. Then a search routine program was used to vary the strength of these seven parameters to determine the best fit to the known energy levels of Ca^{42} through Ca^{45}. The position of the $\frac{7}{2}$ ground state of Ca^{41}, which has a spectroscopic factor of about 1, determined the position of the single-particle $1f_{7/2}$ state, and the mean energy, weighted by the reduced widths, of the two $\frac{3}{2}$ levels of Ca^{41} at 1.95 and 2.47 MeV determined the energy of the $2p_{3/2}$ state. These values were fixed and the other seven parameters were allowed to

Table I. Theoretical predictions for levels with known spin in Ca^{42} to Ca^{45} for the two-single-particle-level ($1f_{7/2}$, $2p_{3/2}$) shell model using harmonic-oscillator wave functions.

			Energy (MeV)		
Nucleus	J	Experiment	Theory I	Theory II	Theory III
Ca^{42}	0	−19.832[a]	−19.832[a]	−19.788[a]	−19.792[a]
	2	1.520	1.659	1.533	1.516
	0	1.840	2.347	3.203[b]	3.146[b]
	2	2.430	2.180	2.483	2.516[b]
	4	2.750	3.083	2.956	2.957
	6	3.190	3.254	3.164	3.156
Ca^{43}	$7/2^-$	−27.762[a]	−27.777[a]	−27.756[a]	−27.757[a]
	$5/2^-$	0.347	0.351	0.295	0.288
	$3/2^-$	0.593	0.613	0.663	0.636
	$3/2^-$	2.048	2.191	2.065	2.096[b]
Ca^{44}	0	−38.898[a]	−38.873[a]	−38.820[a]	−38.823[a]
	2	1.160	1.254	1.258	1.237
	0	1.880	2.123	2.950[b]	2.915[b]
	4	2.280	2.176	2.217	2.190[b]
	2	2.660	2.390	2.571	2.612
	6	3.300	3.201	3.226	3.217
	2	3.351	3.043	3.370	3.374
Ca^{45}	$7/2^-$	−46.317[a]	−46.323[a]	−46.376[a]	−46.371[a]
	$5/2^-$	0.176	0.291	0.202	0.199
	$3/2^-$	1.432	1.278	1.325	1.318
	$3/2^-$	1.900	1.965	1.939	1.973[b]
	$3/2^-$	2.250	2.292	2.275	2.314
σ^2 (MeV^2)			0.775	0.107	0.093
Number of levels in search			22	20	16
Mean deviation (MeV)			0.187	0.073	0.075
		λ	Parameter values (MeV)		
$E_{7/2}$			− 8.360	− 8.360	− 8.360
$E_{3/2}$			− 6.254	− 6.254	− 6.254
V_{10}		1.0	−49.219	−39.412	−38.822
V_{11}		1.0	+21.610	+19.755	+19.828
V_T		1.2	−13.143	−13.114	−13.241
I_{0s}			− 5.981	− 8.292	− 8.885
I_{1s}			+ 8.081	+ 3.290	+ 3.095
I_{2s}			+ 2.400	+ 0.555	+ 0.786
I_{3s}			+ 5.291	+ 9.007	+ 8.628

[a]Binding energy of ground state relative to Ca^{40} ground state.
[b]Level not included in least-square search routine.

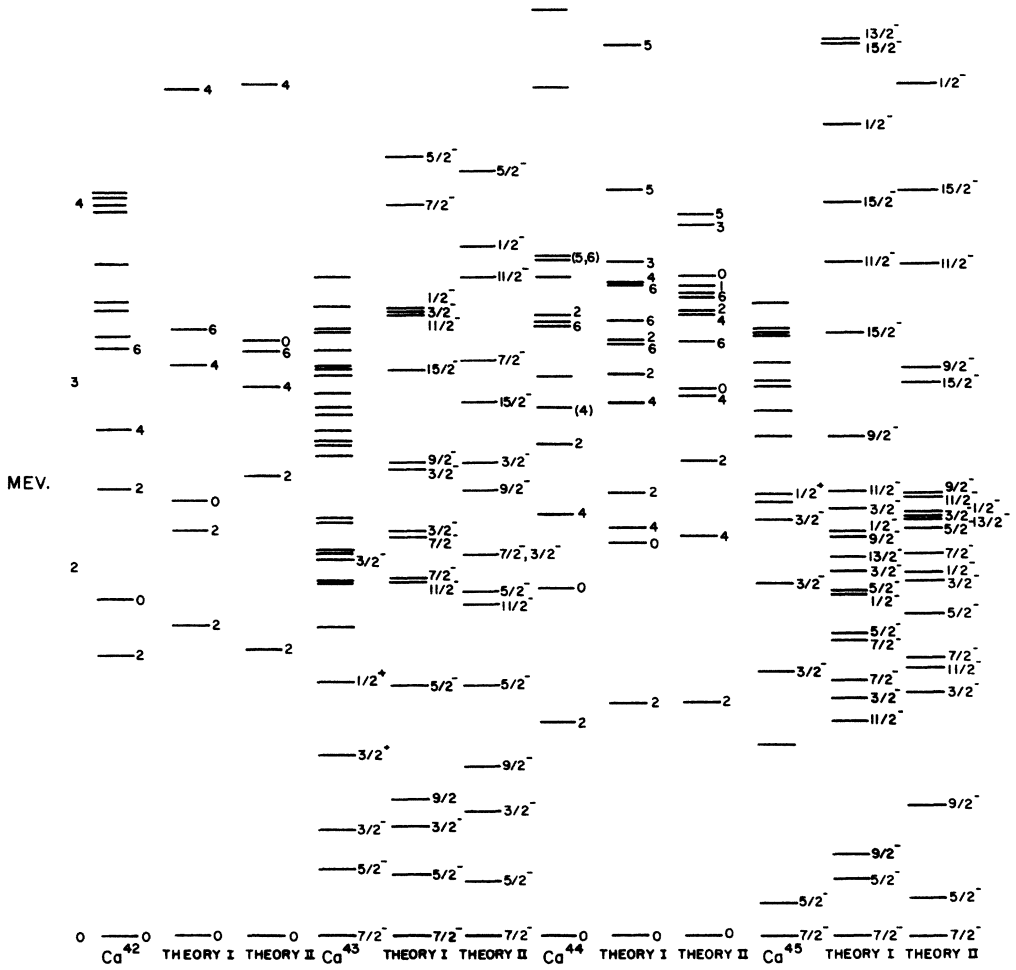

FIG. 1. Experimental energy spectra; theory-I and theory-II predictions for Ca^{42}, Ca^{43}, Ca^{44}, and Ca^{45}. The results that are plotted do not include some high excited states or the results for theory III. These are contained in Table II of the supplementary material.[9]

vary for fixed values of λ. Table I lists the experimental values and spins that were used for the four different nuclei and gives the theoretical predictions for two cases. Case I is where all the known levels were weighted equally, and the search routine was used to determine the smalles value of σ^2, which is equal to the sum of the squares of the difference in the energy between the experimental and theoretical values. In case II the second spin-zero states in both Ca^{42} and Ca^{44} were omitted from the search routine and the remaining 20 levels were included in the search. One immediately sees that σ^2 is dramatically lowered to a very small value as soon as one allows these two 0^+ states to be omitted from the search. This indicates quite convincingly that these levels come from some other shell-model configuration such as core excitation of $1d_{3/2}$ nucleons. The $\frac{3}{2}^+$ and $\frac{1}{2}^+$ levels in Ca^{41}, Ca^{43}, and Ca^{45} also come from such core excitation. In case III four arbitrarily chosen known levels in addition to the 0^+ states were omitted from the input data and the entire search repeated. It is most gratifying that σ^2 only decreases a small amount and the parameters are essentially the same. Thus the predictions seem to be well founded and do not depend on the particular levels chosen for the search routine. This of course does not apply to the two 0^+ states discussed above.

The over-all features for both cases are that the V_{10} is strong and attractive. V_{11} is about half as strong as the V_{10} and is repulsive. V_T

is negative and is almost the same in both cases. The first radial integral I_{0s} is attractive while the other three radial integrals are repulsive. Figure 1 gives the entire spectrum for both the first case and the second case. One immediately notices in Fig. 1 that in Ca^{43} and Ca^{45} there are predicted low-lying $\frac{9}{2}^-$ states that have not been observed experimentally.[8] This region of the spectrum is obscured by strong C^{12} and O^{16} contaminant in the reaction $Ca^{43}(p, p')Ca^{43*}$. It would be very interesting to reexamine this region of the experimental spectrum to see whether these levels do indeed exist. The general features are predicted very well, especially the positions of the $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$ states in Ca^{43} and Ca^{45}.[9]

The over-all agreement is very gratifying and we are now in the process of examining the details of the wave functions and intend to calculate predictions of reduced widths, gamma-ray transitional probabilities, and moments. From a preliminary examination of the wave functions produced for Ca^{42} and Ca^{43}, we see that (a) in Ca^{42} the two spin-2 states are mixed very strongly, which agrees with the observed results from $Ca^{43}(d, t)Ca^{42}$ experiment,[10] and (b) in Ca^{43} the 595-keV $\frac{3}{2}^-$ level would be very weakly excited in the $Ca^{42}(d, p)Ca^{43}$ experiment, in agreement with the observed results.[4] Future results will be published along with the predictions for the spectrum of Ca^{46}-Ca^{49} for these parameters.

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[9]The basis wave function, matrices, eigenvalues, and eigenfunctions for all the states have been computed. This supplementary material has been deposited as Document No. 8643 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington, D. C. 20036. A copy may be secured by citing the Document number and by remitting $31.25 for photoprints or $9.00 for 35-mm microfilm. Advance payment is required. Make checks or money orders payable to: Chief, Photoduplication Service, Library of Congress.

K_{e3} DECAY AND UNIVERSALITY IN CABIBBO'S THEORY OF LEPTONIC DECAYS*

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Within the framework of Cabibbo's theory[1] of leptonic decays, the bare coupling constant for beta decay is $G\cos\theta$ and that for $|\Delta S| = 1$ decays is $G\sin\theta$, where G is the coupling constant for muon decay. Cabibbo determined $\sin\theta$ from the ratio of the rates for K_{e3} and π_{e3} decay, getting $\theta = 0.26$, which implies $\cos\theta = 0.966$, whereas the value of $\cos\theta$ determined from beta decay after inclusion of radiative corrections is $\cos\theta = 0.978 \pm 0.0015$.[2] This discrepancy was discussed by Sakurai,[3] who pointed out that in the computation of the K_{e3} decay rate, $G\sin\theta$ should be replaced by $(G\sin\theta)C$. Here $C = Z_1^{-1}(K\pi)Z_2^{1/2}(K)Z_2^{1/2}(\pi)$ is a renormalization factor which can be expected to differ from unity because the $\Delta S = 1$ current, $j_\mu^{(4)} + ij_\mu^{(5)}$, is not conserved in the presence of an SU(3)-symmetry-breaking interaction H_{ms}'. Sakurai estimated C from the departure of the ratio $\Gamma(K^* \to K + \pi)/\Gamma(\rho \to \pi + \pi)$ from the SU(3) symmetry prediction, obtaining $C = (0.81)^{-1}$, and $\cos\theta = 0.979$, in agreement with the value de-