ENTROPY TRANSPORT BETWEEN TWO SUPERCONDUCTORS BY ELECTRON TUNNELING*

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If an electronic heat current $\langle Q\rangle$ is made to flow from one metal to another across a thin insulating barrier (for example, an oxide layer), there will appear a temperature drop δT and an associated surface thermal resistance ($R \equiv \delta T/\langle Q\rangle$, if the cross section of the metals is unity). In this Letter, we report some preliminary calculations of this tunneling heat current. We find that R depends very much on whether the metals are in the superconducting or normal phases. In addition there is an oscillatory heat flux due to fluctuations in the quasiparticle tunneling between two superconductors. We might also add that the change in chemical potential due to the temperature difference δT can give rise to the ac and dc Josephson supercurrents.[1,2] However, our calculations show that no entropy is carried by these currents, which is in agreement with one's physical expectations.

As implied above, we assume that the heat energy carried by the electrons is transferred by a tunneling process through the interface barrier. For this purpose, we shall use an effective tunneling Hamiltonian such as[3]

$$\hat{H}_V = \sum_{\vec{k}\vec{k}'}\{V_{\vec{k}\vec{k}'}c_{\vec{k}}^{\,*}d_{\vec{k}'} + V_{\vec{k}\vec{k}'}^{\,*}d_{\vec{k}'}^{\,*}c_{\vec{k}}\}, \tag{1}$$

where $c_{\vec{k}}$ and $d_{\vec{k}'}$ are the usual destruction operators for the single-particle states in the two different sides of the barrier. We remark that the matrix element V has a weak dependence[4] on the initial and final energies in the important region. Since we neglect completely the phonon heat current as well as the electron-phonon coupling (by which the electrons might give their energy to phonons in the insulating barrier[5]), our computation will slightly underestimate $\langle Q\rangle$. For typical oxide layers (10-20 Å thick), we believe a major mechanism of heat transfer is by tunneling at all but the lowest temperatures.

The perturbation $\hat{H}_V$ couples metal 1 and metal 2 [assumed for simplicity to be in thermal equilibrium at temperatures T_1 and T_2 ($\leq T_1$), respectively], and it is a straightforward matter to calculate the resulting heat current ($1 \to 2$), with $\hat{Q} = -i[\hat{E}_1, \hat{H}_V] = i[\hat{E}_2, \hat{H}_V]$ and $\hat{E}_1 = \sum_{q_1}\epsilon_{q_1}\hat{n}_{q_1}$. According to first-order perturbation theory, the quasiparticle heat current is given by[6] (see reference 2 for a similar calculation)

$$\langle Q\rangle_t = \langle |V|^2\rangle \int\frac{d\vec{k}_1}{(2\pi)^3}\int\frac{d\vec{k}_2}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}[f^-(\omega/T_1) - f^-((\omega+\Omega_0)/T_2)]\omega$$
$$\times[A_1(\vec{k}_1,\omega)A_2(\vec{k}_2,\omega+\Omega_0) + B_1(\vec{k}_1,\omega)B_2(\vec{k}_2,\omega+\Omega_0)\cos(2\Omega_0 t+\Phi)]. \tag{2}$$

Above, A and B are the usual spectral densities of the normal (G) and anomalous (F) Green's functions, respectively; $f^-(x)$ is the Fermi function (we set $k_B = \hbar = 1$); Φ is the phase difference between the two metals before the heat current is introduced; and (reinstating $\hbar$)

$$\Omega_0 = [\mu_2(T_2) - \mu_1(T_1)]/\hbar$$
$$= [\int_{T_0}^{T_1} dT S_1(T) - \int_{T_0}^{T_2} dT S_2(T)]/\hbar, \tag{3}$$

where[7] S_j is the electronic entropy per electron in metal j and T_0 is defined by $\mu_1(T_0) = \mu_2(T_0)$.

If the metals are identical and $\delta T = T_1 - T_2$ is small, then we can write down the more suggestive result

$$\Omega_0 \simeq S(T)\delta T/\hbar. \tag{3'}$$

It is important to remember that all energies are measured with respect to the chemical potential μ_1. Parenthetically we might note that there are no vertex corrections to (2), and thus it is correct even in the presence of impurities. For simplicity, we assume that both metals are simple BCS superconductors,[8] and for definiteness, take $\Delta_2(T_2) \geqslant \Delta_1(T_1)$. Then (2) reduces to

$$\langle Q\rangle_t = -2\pi\langle |V|^2\rangle N_1(0)N_2(0)\left\{\int_{\Delta_2}^{\infty} d\omega\, \omega\left[\frac{\omega}{(\omega^2-\Delta_1^2)^{1/2}}\cdot\frac{\omega}{(\omega^2-\Delta_2^2)^{1/2}}\right][\tanh(\omega/2T_1)-\tanh(\omega/2T_2)]\right.$$
$$\left. -\cos(2\Omega_0 t+\Phi)\int_{\Delta_2}^{\infty} d\omega\,\omega\left[\frac{\Delta_1}{(\omega^2-\Delta_1^2)^{1/2}}\cdot\frac{\Delta_2}{(\omega^2-\Delta_2^2)^{1/2}}\right][\tanh(\omega/2T_1)-\tanh(\omega/2T_2)]\right\} \quad (4)$$

Setting $\omega+\Omega_0 \simeq \omega$, as we have done, produces negligible error [as an example, for two normal metals (3′) gives $\hbar\Omega_0 \sim (T_1/\epsilon_F)\delta T$, with $\delta T \ll T_1$ and ϵ_F the Fermi energy]. The second term in (4) gives rise to an oscillatory heat flux, and is the analog of a similar term in ordinary tunneling.[1,2] While it does not contribute to the time-averaged quantity R, one might be able to detect it by using[1,9] an alternating temperature difference $\delta T(t) = \delta T + a\cos\omega_0 t$. If one could overcome the technical difficulties related to the rapid damping of temperature oscillations, one would expect to find that $\langle Q\rangle$ as a function of δT would have resonances whenever $n\omega_0 = 2\Omega_0$, where $n = 1, 2, 3, \ldots$.

One can also compute the heat carried by the tunneling of condensed pairs, which is given by

$$\langle Q_J\rangle_t = -\langle |V|^2\rangle N_1(0)N_2(0)\int_{-\infty}^{\infty} d\omega_1\,\omega_1\int_{-\infty}^{\infty} d\omega_2\,\mathrm{Re}\left[\frac{\Delta_1}{(\omega_1^2-\Delta_1^2)^{1/2}}\right]$$
$$\times\,\mathrm{Re}\left[\frac{\Delta_2}{(\omega_2^2-\Delta_2^2)^{1/2}}\right]\left(\frac{\tanh(\omega_2/2T_1)-\tanh(\omega_2/2T_2)}{\omega_1-\omega_2}\right)\sin(2\Omega_0 t+\Phi) \quad (5)$$

for BCS superconductors. This double integral is identical to that occurring in the theory of the usual Josephson supercurrent, except for the extra factor of ω_1. As a result of this, the ω_1 integrand is effectively odd, and $\langle Q_J\rangle_t$ vanishes. Of course, a Josephson supercurrent (of frequency $2\Omega_0$) will appear if the two metals are connected in a circuit. Its observation will probably be difficult as a result of thermal fluctuations and other complications.

In closing, we briefly discuss the asymptotic expansion of the first term in (4) for high and low temperatures, although the integral is easily done numerically. We shall first consider the case when metal (1) is normal ($\Delta_1 = 0$) and the other side (2) has an energy gap Δ. The integral in (4) is equal to $I(T_2)-I(T_1)$, with

$$I(T) = (\pi T)^2/6 + \tfrac{1}{2}\Delta\left\{\ln\left(\frac{\pi T}{\gamma\Delta}\right) - \tfrac{1}{2} + \frac{3\times 7}{4\times 8}\zeta(3)\left(\frac{\Delta}{\pi T}\right)^2 - O\left(\frac{\Delta}{\pi T}\right)^4\right\} \quad (6)$$

for $\Delta \ll \pi T$, where $\zeta(3) = 1.202$ is a zeta function and $\ln\gamma = 0.577$ is Euler's constant. For low temperatures, we find

$$I(T) = 2\Delta^2\left(\frac{\pi T}{2\Delta}\right)^{1/2} e^{-\Delta/T}\left\{1 + \tfrac{7}{8}\left(\frac{T}{\Delta}\right) + O\left(\frac{T}{\Delta}\right)^2\right\} \quad (7)$$

for $\Delta \gg \pi T$. If both sides are normal (intrinsically or by application of a magnetic field H), then $I(T) = (\pi T)^2/6$ holds for all temperatures. We remark that according to (6), the total heat current first increases[10] when the temperature of one metal is lowered through its transition temperature. In the limit $\delta T \gg (T_1 + T_1)/2$, the surface thermal resistance R is independent of δT. Finally, we call attention to the fact that $R_{NS}(H > H_c)/R_{NS}$ is not given by the usual Bardeen-Rickayzen-Tewordt expression for the thermal conductivity in impure superconductors.

The case of two superconductors with the same transition temperature is particularly simple. One finds that

$$I(T) \simeq \Delta^2 f(\Delta/T)\ln(1/\epsilon), \quad (8)$$

where

$$\epsilon \simeq |1-[\Delta(T_1)/\Delta(T_2)]|^{1/2},$$

and thus there is a sharp peak just below T_c and then an exponential decrease.

In setting up experiments to test some of the predictions of this paper, care must be taken so that the tunneling surface resistance is smaller than that due to all other mechanisms.[5] The former increases exponentially with the thickness of the barrier, and thus one wants very thin oxide layers. The heat current carried by the phonons in metals is generally negligible near the transition temperature, which makes this region preferable from our point of view. The use of "dirty" metals may be to advantage if the impurities scatter the phonons more strongly than the electrons. The surface thermal resistance between two metals has been studied by several workers,[11] but it appears that the conditions needed for our calculation to be valid were not present. In particular, the interface in some of these experiments may be better described by a finite potential barrier, in which case the matrix element V in (1) is energy dependent and our calculation will underestimate the heat current.

As is well known, the Gor'kov theory of superconductivity is formally very similar to Beliaev's description[12] of He II. In particular, one expects[13,14] that the coherent phase coupling between two weakly connected reservoirs of He II at different chemical potentials will give rise to the analog of the ac Josephson effect. The direct parallel of a tunneling junction seems difficult to realize.[13] However, a dynamical analog of a junction is possible, if the singularity associated with moving vortices can disrupt the order parameter at a small orifice between the He II reservoirs. This is suggested by the recent work of Richards and Anderson,[14] who used a difference in gravitational head and observed an ac matter flow through a small orifice. A temperature difference should give rise to the same effect. A suitable generalization of Tsuneto's type of discussion[13] indicates that, in addition, a temperature difference will give rise to an oscillatory heat current at the orifice [the analog of the second term in (4)]. This should generate an easily observable second sound wave [of low frequency Ω_0 given by (3)].

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[1]B. D. Josephson, Phys. Letters 1, 251 (1962).

[2]V. Ambegaokar and A. Baratoff, Phys. Rev. Letters 10, 486 (1963). For a general review of ordinary electronic tunneling which is in the same spirit as the present paper, see also G. Rickayzen, The Theory of Superconductivity (John Wiley & Sons, Inc., New York, 1965), Chap. 10.

[3]M. H. Cohen, L. M. Falicov, and J. C. Phillips, Phys. Rev. Letters 8, 316 (1962).

[4]J. Bardeen, Phys. Rev. Letters 6, 57 (1961).

[5]This mechanism is important between a metal and dielectric, and gives rise to the Kapitza surface resistance. If the heat current is largely carried by phonons in two metals, the surface resistance may be due to "acoustic" mismatch at the interface. For a review of this subject, see R. C. Johnson and W. A. Little, Phys. Rev. 130, 596 (1963).

[6]We note that the validity of (2), (4), and (5) is not restricted to the linear region $\langle Q\rangle \propto \delta T$. The results in this region coincide with those found by considering the linear response to δT, such as in deriving Kubo formulas for thermal conductivity.

[7]Since $\mu \equiv \mu(N,T)$, it is clear that the anomalous effects peculiar to the coherent nature of two coupled superconductors[1] may result from temperature differences as well as number differences. This was independently noted by P. W. Anderson, N. R. Werthamer, and J. M. Luttinger, Phys. Rev. 138, A1157 (1965). We might remark that typically, Ω_0 will be in the kilocycle region.

[8]However, one may easily generalize the discussion to include gapless and strongly coupled superconductors.

[9]S. Shapiro, Phys. Rev. Letters 11, 80 (1963).

[10]This increase may be traced back to the absence of any coherence factor, or more precisely, the vanishing of the second term in (4).

[11]L. J. Barnes and J. R. Dillinger, Phys. Rev. Letters 10, 257 (1963); L. J. Challis and J. D. N. Cheeke, Phys. Letters 5, 305 (1963); see also L. J. Barnes and J. R. Dillinger, Phys. Rev., to be published.

[12]S. T. Beliaev, Zh. Eksperim. i Teor. Fiz. 34, 417 (1958) [translation: Soviet Phys.—JETP 34, 289 (1958)].

[13]T. Tsuneto, Progr. Theoret. Phys. (Kyoto) 31, 516 (1964).

[14]P. L. Richards and P. W. Anderson, Phys. Rev. Letters 14, 540 (1965).