second term on the right-hand side must vanish.[2] By hypothesis, the left-hand side does not vanish. Hence the first term must contribute. Conversely, if the first term does not contribute,[3] the second must, and thus we encounter degenerate vacua, Q.E.D.

We now make the additional assumption that

$$\int d^3x[j^{0*}(\vec{x}t), \varphi_2(y)] = \varphi_1(y). \tag{4}$$

This would be the case, for example, if φ_1 and φ_2 were the 1, 2 components of an isotriplet and $\int d^3x\, j^0$ were the generator of rotations about the 3 axis. Applying the method used above, assuming the vacuum is nondegenerate, we find

$$\langle p_2\beta | \varphi_2(0)|0\rangle = \sum_\alpha \langle p_2\beta | j^0(0)| p_2\alpha\rangle \langle p_2\alpha | \varphi_1(0)|0\rangle,$$

$$\langle p_2\gamma | \varphi_1(0)|0\rangle = \sum_\beta \langle p_2\gamma | j^{0*}(0)| p_2\beta\rangle \langle p_2\beta | \varphi_2(0)|0\rangle,$$

which we combine to obtain

$$\sum_\beta |\langle p_2 | j^0(0)| p_2\beta\rangle|^2 = 1.$$

Then it is easy to show

$$\sum_\alpha |\langle p_1\alpha | \varphi_1(0)|0\rangle|^2 = \sum_\beta |\langle p_2\beta|\varphi_2(0)|0\rangle|^2 \text{ if } p_1^2 = p_2^2, \tag{5}$$

which shows that the Lehmann weights must coincide for all values of the argument. Conversely, if they differ at any point, then we must have degenerate vacua.

It should be noted that we have been careful not to use the terms "zero-mass bosons" or "states of arbitrarily small energy." We do not want to imply that we have proved that which we have not. In fact, let us be more precise as to our use of the term degenerate vacuum: We mean a state of zero total four-momentum which differs from the original vacuum in the values of some quantum numbers. The state $|0'\rangle$ which occurs in the second term of Eq. (3) in the case of degenerate vacuum is such a state. Notice that $|0'\rangle$ cannot be a Lorentz-invariant state, for if it were then we could easily show that $\langle 0'|j^\mu(x)|0\rangle = 0$ which is a contradiction. Therefore, the degenerate state $|0'\rangle$ is not a true vacuum; furthermore, we have infinitely many such states, obtained by applying Lorentz transformations to $|0'\rangle$. Since each such state contributes to the sum over intermediate states, it is questionable whether the sum converges. Even if this difficulty is surmounted, one must resist the impulse to build inequivalent representations using the states $|0'\rangle$ as cyclic vacuum states, since they are not Lorentz-invariant states.

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*Work supported by the U. S. Atomic Energy Commission.

[1]R. F. Streater, Phys. Rev. Letters 15, 475 (1965). There is a misprint: The assumption $c_1 \neq 0$ should be $c_2 \neq 0$.

[2]The matrix element $\langle 0|\int d^3x\, j^0(xt)|0\rangle = 0$ owing to Lorentz invariance. This is seen by observing that the quantity $\langle 0|j^\mu(x)|0\rangle$ must vanish by Lorentz invariance.

[3]The manner in which this occurs is not relevant; however, it is most likely that the first term fails to contribute by virtue of cancellations.

MEASUREMENT OF THE STATISTICAL DISTRIBUTION OF GAUSSIAN AND LASER SOURCES*

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(Received 18 November 1965)

This Letter reports the measurement of the statistical distribution of photons from a Gaussian radiation source synthesized through random superposition of a great number of coherent beams, and a comparison with the distribution of non-Gaussian sources, through a photoelectron counting technique.

The Gaussian radiation source is obtained by sending the light of an amplitude-stabilized single-mode He-Ne laser onto a moving ground glass disk, and observing the random superposition of the diffracted contributions within a coherence time and a coherence area[1] following a procedure first introduced by Martienssen and Spiller in connection with a different illuminating light.[2] Actually the 6328Å laser light, fully polarized, with TEM_{00} field cross distribution and 10^{-3}-rad divergence,[3] is focused through a lens of 2-cm focal length onto a spot of 2×10^{-3}-cm size over a glass disk ground with average irregularities size around 3×10^{-4} cm. Diffraction of this field gives rise to a

Table I. Moments and associated errors of the statistical distributions of Fig. 1(a).[a]

	$M_1 = \langle n\rangle$	$M_2' = \langle n^2\rangle - \langle n\rangle^2$	$M_3' = \langle(n-\langle n\rangle)^3\rangle$	Experimental errors (%) M_1	M_2'	M_3'	Deviations from theoretical[b] values (%) $1-M_{2\text{th}}'/M_2'$	$1-M_{3\text{th}}'/M_3'$
G	1.202	2.633	···	0.63	2.55	···	−0.5	···
L	1.286	1.301	1.337	0.43	1.58	4.58	+1.25	+3.92

[a] M_1 referred to the zero line, M_2' and M_3' referred to the center $\langle n\rangle$.
[b] Theoretical values: for Bose distribution $M_{2\text{th}}' = n(1+n)$; for Poisson distribution $M_{2\text{th}}' = M_{3\text{th}}' = \langle n\rangle$.

random distribution of interference peaks with angular size related to the reciprocal spot size.[4] When the disk moves, due to the very high number N of scattering centers, the expected statistical distribution of the field at a given point beyond the disk can be shown by a very classical argument[5] to be a Gaussian distribution, which is characteristic of a thermal source[6]; its coherence time is limited by the transit time of the disk through the focal region of the lens, and can be changed between 50 and 1000 μsec by controlling the velocity of the motor driving the disk; its coherence area is related to the size of the interference peaks, and is of the order of 1 cm^2 at 40-cm distance.

The statistical properties of this source have been measured by a photoelectron counting technique, which has definite advantages over interference (amplitude or intensity) or coincidence techniques insofar as it gives complete information on the light field.[7-9] The measurement consists in counting the photoelectrons arriving in a fixed observation time T and plotting the statistical distribution of counts. A 56 AVP photomultiplier tube, having its photocathode at 40 cm distance from the disk and with a 10^{-2}-cm^2 diaphragm in front of it, yields a current pulse for each photoelectron. The successive pulses, standardized in shape and height, charge a capacitor for a given observation time T, to a voltage proportional to the number of photoelectrons which have arrived in that interval. The voltage is then classified by a 256-channel pulse-height analyzer.

Results are shown in Fig. 1(a) and Tables I and II. Deviations from a Bose distribution are less than $1/10^2$ in the second moment. The standard pulse former has a 20-nsec dead time, and with an average photoelectron rate of 1/μsec, there could be relevant dead-time effects. Therefore, to be free from any error, a zero dead-time technique has been used in alternative; that is, pulses are not standardized, but sent directly to the integrating capacitor, and hence the count distribution is an average over a superposed amplitude distribution associated to the single-photoelectron response [Fig. 1(b)]. Knowledge of the amplitude distribution allows reconstruction of the correct photon distribution[10] and here again a Bose distribution correct to better than $1/10^2$ in the second moment has been obtained. For comparison, the photoelectron distribution of the laser field is reported, and it appears to be Poissonian with an accuracy given in Table I up to the third moment.

These results are explained by the following considerations. The single-mode, amplitude-stabilized, laser field is most generally described by a mixture of coherent states[6] with equal amplitude A_0 and uniformly distributed in phase; but for times short compared with the reciprocal intrinsic bandwidth of the laser mode,[11] it can be represented by a coherent state with negligible "phase diffusion."[7] Using this field and summing up the scattered contributions from N diffracting centers, one calculates the following intensity at a given point,

Table II. Loss of space and time coherence of a Gaussian source.

Position of the lens (mm)	$M_{2\text{th}}'/M_2'$
1	0.99
2	1.07
3	1.13
4	1.26
5	1.69
Measuring time (μsec)	**$M_{2\text{th}}'/M_2'$**
10	1.00
50	1.09
100	1.37

with the disk steady:

$$A_0^2 \sum_{i,j}^{N} \cos k_0 (l_i - l_j), \tag{1}$$

where l_i are the optical paths and k_0 is the propagation constant of the mode. When the disk moves, the different random-walk contributions like (1) give a Gaussian field distribution, provided the scattering centers are really "chaotic."

The experimental results show that the assumption of a great number of completely uncorrelated scattering centers was correct within the above mentioned accuracy, insofar as a Bose distribution of photoelectrons arises from a Gaussian field distribution observed within a coherence time.[8,9]

If the incident field corresponds to M fully

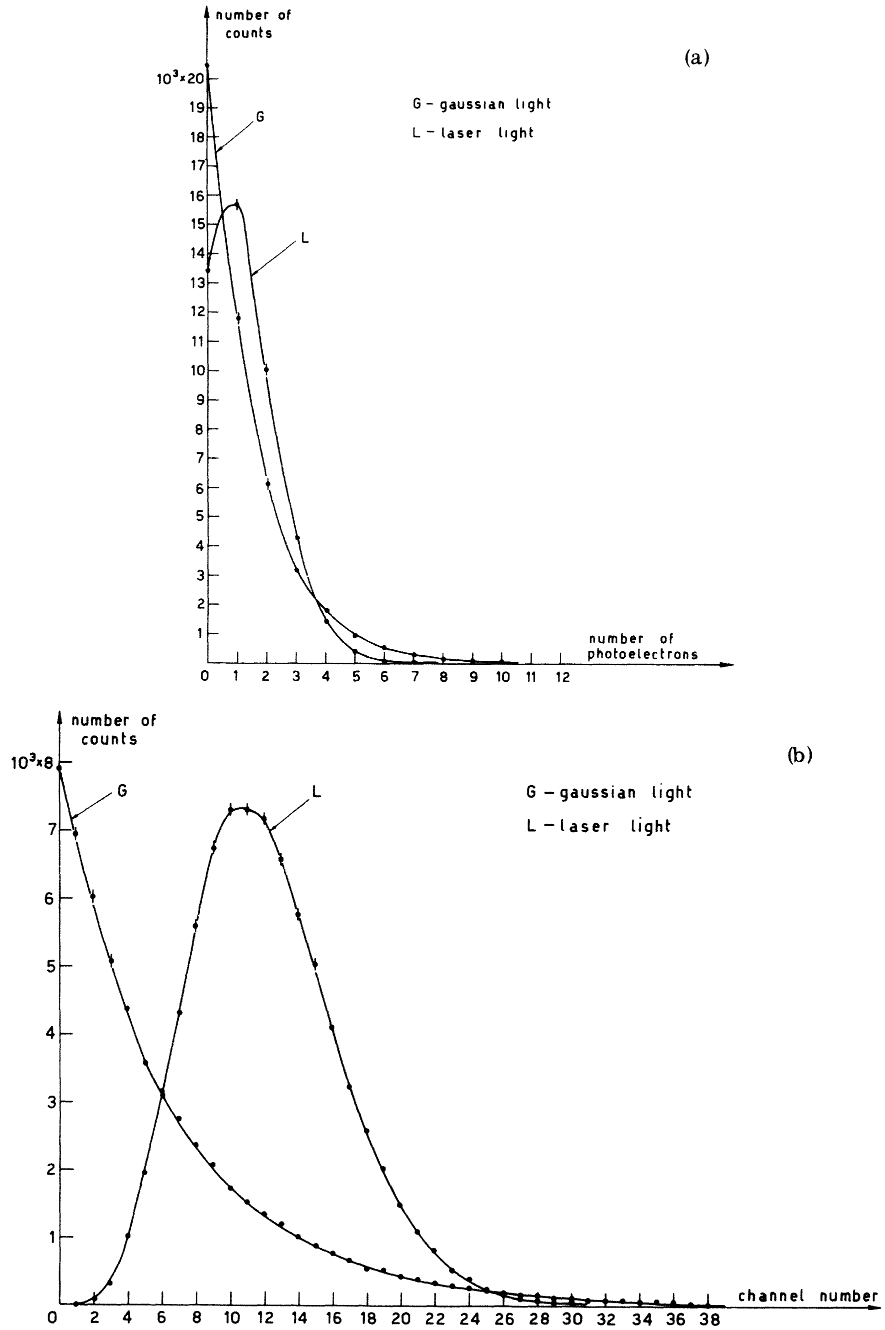

FIG. 1. (a) Photoelectron distribution of a Gaussian source and a laser source. Single-photoelectron pulse standardized. (b) The same, without standardization.

polarized laser modes, the corresponding intensity at a given point is given by

$$\sum_{l,m}^{M} \sum_{i,j}^{N} A_l A_m \cos(\Phi_l - \Phi_m + k_l l_i - k_m l_j), \qquad (2)$$

where A_l are the mode amplitudes, $\Phi_l = \omega_l t - \theta_l(t)$ are the mode phases, and $\theta_l(t)$ represent random functions of time related to the "phase-diffusion" process. Equation (2) reduces to (1) when $M=1$. For observation times of the order of a beat period $2\pi(\omega_l - \omega_m)^{-1}$, provided $\theta_l - \theta_m$ can be taken constant, a time fluctuation modifies the average distribution[12]; for much longer T's, only the terms with $l=m$ will be nonzero and (2) reduces to a set of independent contributions like (1). If they are approximately isofrequencial, one has again the same spatial pattern, and hence a Guassian amplitude distribution. If $(k_l - k_m)/k_l$ is not negligible (nonmonochromatic incident field), one has nonoverlapping interference patterns on the observation screen and therefore the distribution deviates from a Gaussian one, as shown in Fig. 2 by using an argon laser going on modes belonging to three different lines.

The experiments here reported show that measurement of the photoelectron distribution yields a complete statistical information on a radiation field. For times short compared to the coherence time, when the first-order correlation function $G^{(1)}(r_1, t; r_1, t+\tau)$ can be taken as constant, the moments $M_r = \sum_n n^r p(n)$ of the reported distributions yield direct information on "every order" correlation functions[8,9] $G^{(r)}$ within the limits of accuracy allowed by the measurement, and therefore this method seems a much more powerful tool than a single interference or coincidence experiment. For times of the order of, of larger than, the coherence time, relationships between G's and M's are not so straightforward, but can still be worked out case by case.

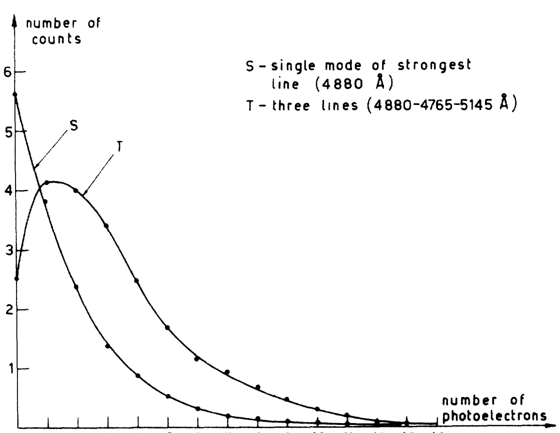

FIG. 2. Effect of the frequency spread of the incident light on the statistical distribution of the "randomized" source.

I am glad to acknowledge very useful discussions with E. Gatti, I. De Lotto, and A. Sona, and help in measurements from A. Berné, P. Burlamacchi, C. Cottini, and V. Pellegrini.

*This research was partially supported by the Comitato Nazionale per le Ricerche Nucleare (Italian National Research Council).

[1]The theory of light diffraction through a random distribution of scattering centers acting as a random diffraction grating is due to M. von Laue [A. Sommerfeld, Optics (Academic Press, Inc., New York, 1954), p. 194] and has been reformulated in connection with radiofrequency propagation [J. A. Ratcliffe, Repts. Progr. Phys. 19, 188 (1956)]. Recently similar arguments have been used to explain the granularity of scattered laser light [see, e.g., L. Allen and D. G. C. Jones, Phys. Letters 7, 321 (1963)].

[2]W. Martienssen and E. Spiller, Am. J. Phys. 32, 919 (1964). It should be mentioned, however, that their quasithermal source relies on the spatial relationships of the diffracting screen whereas the statistical features of the incident beam are filtered out by the use of a slow detector. Here, as shown later below, the use of a single-mode laser does not introduce fluctuations averaged out by the measuring procedure.

[3]Single axial and transverse mode is easily isolated using a cavity of 20 cm length with a Fresnel number near to 1; control is made through Fabry-Perot and beat-frequency observations.

[4]J. W. S. Rayleigh, Theory of Sound (Dover Publications, New York, 1945), p. 36.

[5]See, e.g., M. Born and E. Wolf, Principles of Optics (Pergamon Press, New York, 1959), p. 404.

[6]R. J. Glauber, Phys. Rev. 131, 2766 (1963).

[7]E. Wolf and C. L. Mehta, Phys. Rev. Letters 13, 705 (1964).

[8]R. J. Glauber, in Quantum Optics and Electronics, edited by C. De Witt, A. Blandin, and C. Cohen-Tannoudji (Gordon and Breach Publishers, Inc., New York, 1964).

[9]L. Mandel and E. Wolf, Rev. Mod. Phys. 37, 231 (1965).

[10]F. T. Arecchi, A. Berné, and P. Burlamacchi, to be published.

[11]The reciprocal bandwidth for a carefully mounted laser has been shown by beating two independent gas lasers to be much longer than the maximum observation times used in these measurements [A. Javan, E. W. Ballik, and W. L. Bond, J. Opt. Soc. Am. 52,

96 (1962)].
[12]This beat effect has been observed in a different way (F. T. Arecchi, E. Gatti, and A. Sona, to be published) using a 2-m cavity going on two adjacent axial modes and with observation times of the order of the beat period, i.e., 13 nsec.

ESR MEASUREMENTS OF METASTABLE ATOMIC NITROGEN IN HELIUM-NITROGEN AFTERGLOWS

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(Received 15 November 1965)

The electron spin resonance (ESR) spectra of the $^2D_{3/2}$ and $^2D_{5/2}$ metastable levels of atomic nitrogen were observed in the afterglow of a helium-nitrogen microwave discharge. Appreciable amounts of metastable nitrogen were previously measured by mass spectrographic means by Foner and Hudson[1] in such afterglows. The spectra, which are the first spectra of a gaseous metastable atom to be observed with an ESR spectrometer, are shown in Fig. 1. A 100-kHz and 100-Hz double-modulation ESR spectrometer was used with an output time constant of 10 sec. This spectrometer records the second derivative of the absorption curve. The lower curve is a superposition of three tracings taken at twice the gain and twice the modulation amplitude of those of the upper curve.

The gas-flow system is shown in Fig. 2. It is a moderately high-speed system, pumped by a 7-liter/sec mechanical vacuum pump. A faster pump was tried, but it gave no increase in signal. The capillary nozzle increases the flow velocity and forms a beam of gas which reduces wall de-excitation of the metastable atoms. The position of the discharge cavity relative to the end of the nozzle is critical, best results being obtained when the visible discharge region extends just to the end of the nozzle. Correct operating conditions are indicated by the presence of a bright orange beam, which emanates from the nozzle and passes completely through the ESR cavity. Maximum

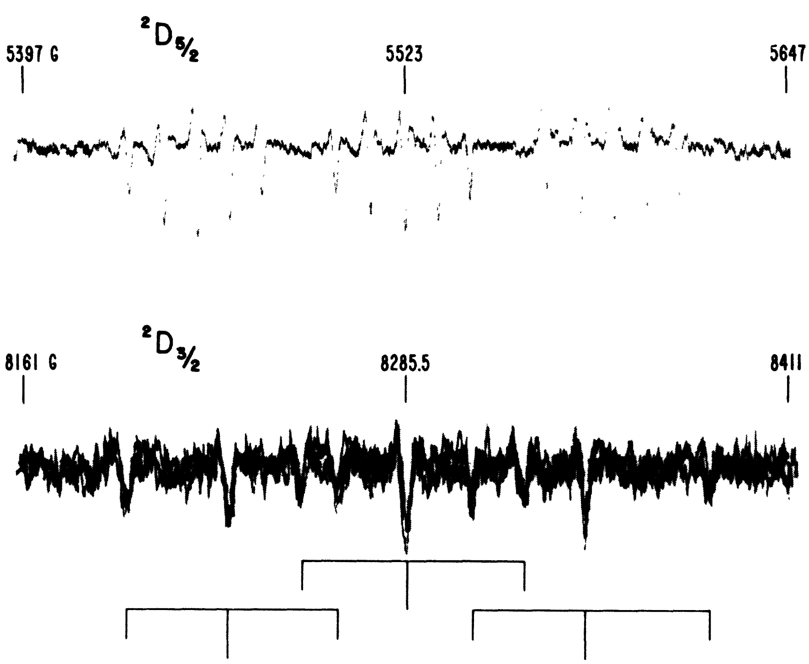

FIG. 1. Tracings of the ESR spectra of the metastable ^2D levels of atomic nitrogen, as observed, taken in a helium-nitrogen afterglow. The upper trace shows the $^2D_{5/2}$ spectrum and the lower trace, the $^2D_{3/2}$ spectrum. The microwave frequency was 9.280 GHz. The 250-G scans were made in 50 minutes with a 10-sec time constant.