momentum and total baryon number. This minimization procedure makes sense when (1) a substantial barrier separates the desired minimum from any configurations of lower energy which might lead to collapse and when, in addition, (2) the desired configuration and the collapsing configurations differ so greatly that one is in no danger of choosing trial functions that contain appreciable admixture of the properties of such collapsing configurations.

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[1]B. K. Harrison, M. Wakano, and J. A. Wheeler, in La Structure et l'evolution de l'univers (R. Stoops, Brussels, 1958).

[2]B. K. Harrison, K. S. Thorne, M. Wakano, and J. A. Wheeler, Gravitation Theory and Gravitational Collapse (University of Chicago Press, Chicago, Illinois, 1965).

[3]See for example, R. A. Lyttleton, The Stability of Rotating Liquid Masses (University Press, Cambridge, England, 1953).

[4]R. Kerr, Phys. Rev. Letters 11, 237 (1963).

[5]We follow the notation (with $c = G = 1$) of L. Landau and E. M. Lifshitz, Classical Theory of Fields (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962) for those equations which pertain to general relativity.

[6]For a recent application see P. J. Roberts, Astrophys. J. 137, 1129 (1963).

[7]A form equivalent to this is given by W. J. van Stockum, Proc. Roy. Soc. Edin. 57, 137 (1937).

[8]See A. Trautman, in Gravitation, edited by L. Witten (John Wiley & Sons, Inc., New York, 1962).

[9]See C. W. Misner and D. H. Sharp, Phys. Rev. 136, B571 (1964).

[10]The present variational principle for stationary configurations is to be distinguished from the general action principle of A. H. Taub, Phys. Rev. 94, 1468 (1954). Appreciation is expressed to R. H. Lindquist for a discussion of this point and other aspects of the present problem.

EXTENSION OF A THEOREM OF STREATER*

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In a recent Letter, Streater[1] has proven a theorem on broken symmetry: If a conserved current generates a transformation $\varphi_1(x) \to \varphi_2(x)$ between fields corresponding to particles of different mass, then there must be states in the theory degenerate with the vacuum (in the absence of states of negative norm). Streater's proof made use of certain assumptions on the Lehmann weights for the two fields. In this paper we show that by dropping these assumptions we still may obtain a result which is essentially the same as the above.

The first theorem we prove is the following: Let $\varphi_1(x)$ and $\varphi_2(x)$ be quantized fields (not necessarily spin zero). Suppose there exists a conserved current $j^\mu(x)$ such that

$$\int d^3x[j^0(\vec{x},t),\varphi_1(y)] = \varphi_2(y). \qquad (1)$$

Let $|p_2\rangle$ be a state with energy-momentum $p_2{}^\mu$ for which $\langle p_2|\varphi_2(0)|0\rangle \neq 0$. Then, if the vacuum is nondegenerate, there must exist a state $|p_1\rangle$ with energy-momentum $p_1{}^\mu = p_2{}^\mu$ for which $\langle p_1|\varphi_1(0)|0\rangle \neq 0 \neq \langle p_2|j^0(0)|p_1\rangle$. Conversely, if no such partner $|p_1\rangle$ for $|p_2\rangle$ exists then there must be states degenerate with the vacuum.

This is essentially the result of Streater, except that we do not impose his restrictions on the Lehmann weights; in particular, we do not demand that the propagators for the fields have poles: We allow the fields to describe only "composite" particles.

Consider the matrix element of Eq. (1):

$$\int d^3x \langle p_2|[j^0(\vec{x}t),\varphi_1(y)]|0\rangle = \langle p_2|\varphi_2(y)|0\rangle. \qquad (2)$$

We now take the Fourier transform with respect to time t and the four-vector y, and insert a complete set of states:

$$\begin{aligned}\langle p_2|\varphi_2(0)|0\rangle\, \delta^{(4)}(p_1-p_2)\delta(q^0) &= \sum_{k\alpha}\langle p_2|j^0(0,q^0)|k\alpha\rangle\langle k\alpha|\varphi_1(p_1)|0\rangle \\ &\quad -\sum_{l\beta}\langle p_2|\varphi_1(p_1)|l\beta\rangle\langle l\beta|j^0(0,q^0)|0\rangle. \qquad (3)\end{aligned}$$

Now, if the vacuum is nondegenerate, the second term on the right-hand side must vanish.[2] By hypothesis, the left-hand side does not vanish. Hence the first term must contribute. Conversely, if the first term does not contribute,[3] the second must, and thus we encounter degenerate vacua, Q.E.D.

We now make the additional assumption that

$$\int d^3x[j^{0*}(\vec{x}t), \varphi_2(y)] = \varphi_1(y). \quad (4)$$

This would be the case, for example, if φ_1 and φ_2 were the 1, 2 components of an isotriplet and $\int d^3x\, j^0$ were the generator of rotations about the 3 axis. Applying the method used above, assuming the vacuum is nondegenerate, we find

$$\langle p_2\beta|\varphi_2(0)|0\rangle = \sum_\alpha \langle p_2\beta|j^0(0)|p_2\alpha\rangle\langle p_2\alpha|\varphi_1(0)|0\rangle,$$

$$\langle p_2\gamma|\varphi_1(0)|0\rangle = \sum_\beta \langle p_2\gamma|j^{0*}(0)|p_2\beta\rangle\langle p_2\beta|\varphi_2(0)|0\rangle,$$

which we combine to obtain

$$\sum_\beta |\langle p_2|j^0(0)|p_2\beta\rangle|^2 = 1.$$

Then it is easy to show

$$\sum_\alpha |\langle p_1\alpha|\varphi_1(0)|0\rangle|^2 = \sum_\beta |\langle p_2\beta|\varphi_2(0)|0\rangle|^2 \text{ if } p_1^2 = p_2^2, \quad (5)$$

which shows that the Lehmann weights must coincide for all values of the argument. Conversely, if they differ at any point, then we must have degenerate vacua.

It should be noted that we have been careful not to use the terms "zero-mass bosons" or "states of arbitrarily small energy." We do not want to imply that we have proved that which we have not. In fact, let us be more precise as to our use of the term degenerate vacuum: We mean a state of zero total four-momentum which differs from the original vacuum in the values of some quantum numbers. The state $|0'\rangle$ which occurs in the second term of Eq. (3) in the case of degenerate vacuum is such a state. Notice that $|0'\rangle$ cannot be a Lorentz-invariant state, for if it were then we could easily show that $\langle 0'|j^\mu(x)|0\rangle = 0$ which is a contradiction. Therefore, the degenerate state $|0'\rangle$ is not a true vacuum; furthermore, we have infinitely many such states, obtained by applying Lorentz transformations to $|0'\rangle$. Since each such state contributes to the sum over intermediate states, it is questionable whether the sum converges. Even if this difficulty is surmounted, one must resist the impulse to build inequivalent representations using the states $|0'\rangle$ as cyclic vacuum states, since they are not Lorentz-invariant states.

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[1]R. F. Streater, Phys. Rev. Letters 15, 475 (1965). There is a misprint: The assumption $c_1 \neq 0$ should be $c_2 \neq 0$.

[2]The matrix element $\langle 0|\int d^3x j^0(xt)|0\rangle = 0$ owing to Lorentz invariance. This is seen by observing that the quantity $\langle 0|j^\mu(x)|0\rangle$ must vanish by Lorentz invariance.

[3]The manner in which this occurs is not relevant; however, it is most likely that the first term fails to contribute by virtue of cancellations.

MEASUREMENT OF THE STATISTICAL DISTRIBUTION OF GAUSSIAN AND LASER SOURCES*

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This Letter reports the measurement of the statistical distribution of photons from a Gaussian radiation source synthesized through random superposition of a great number of coherent beams, and a comparison with the distribution of non-Gaussian sources, through a photoelectron counting technique.

The Gaussian radiation source is obtained by sending the light of an amplitude-stabilized single-mode He-Ne laser onto a moving ground glass disk, and observing the random superposition of the diffracted contributions within a coherence time and a coherence area[1] following a procedure first introduced by Martienssen and Spiller in connection with a different illuminating light.[2] Actually the 6328Å laser light, fully polarized, with TEM_{00} field cross distribution and 10^{-3}-rad divergence,[3] is focused through a lens of 2-cm focal length onto a spot of 2×10^{-3}-cm size over a glass disk ground with average irregularities size around 3×10^{-4} cm. Diffraction of this field gives rise to a