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PROPOSED EXPERIMENT FOR OBSERVATION OF NONLINEAR COMPTON WAVELENGTH SHIFT*

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Within the past three years there has been published a considerable body of theoretical work[1,2] on the subject of free-electron–photon interactions. The particular focus of interest has been on the nonlinearities[3] inherent in familiar processes such as Compton scattering. Unfortunately, in all cases the nonlinearities make insignificant contributions to the cross sections until the photon density is of the order $\rho \sim \omega m^2/e^2$, which is nearly $10^{27}/\mathrm{cm}^3$ at optical frequencies. However, we believe that, in the experiment described below, it might be possible, using available lasers, to observe the most interesting of all the nonlinearities: a new wavelength shift proposed for Compton scattering.[2,4]

We discuss the wavelength shift briefly first, and then the experimental setup. Finally, a calculation of the effect of the shift on the electron scattering angles is given, and the importance of possible competing processes is estimated.

The Compton shift is most conveniently expressed, for our purposes, by the formula for the frequency of the scattered photon. In the rest frame of the incident electron this has the familiar form

$$\omega' = \frac{\omega}{1 + (\omega/m)(1-\cos\theta)}, \tag{1}$$

where θ is the photon scattering angle. It is well known that this formula is a direct consequence of the conservation of energy and momentum in the collision

$$p_\mu' + k_\mu' = p_\mu + k_\mu. \tag{2}$$

Recently Brown and Kibble[2] and Goldman[2] have proposed that Eq. (2) must be rewritten as

$$p_\mu' + k_\mu' = p_\mu + \gamma k_\mu, \tag{3}$$

where $\gamma = 1 - \frac{1}{2}\epsilon[(m^2/p'\cdot k) - (m^2/p\cdot k)]$, and ϵ is a dimensionless parameter, $\epsilon = e^2\rho/\omega m^2$, which, as we remarked above, is of order unity when $\rho \sim 10^{27}/\mathrm{cm}^3$. It is not surprising that there is some controversy[4] surrounding this rewriting of the energy-momentum-conservation relation. It implies that the electron acquires energy and momentum (in the form of added mass in the amount $\epsilon^{1/2}m$) while in the photon beam by interacting simultaneously with all of the photons present, and that each photon gives up an infinitesimal amount of energy and momentum to the electron as it leaves the beam. Needless to say, it is no longer possible to view Compton scattering as a two-particle billiard-ball collision. Nor is it possible to compute this effect in any finite order of perturbation theory. For our present purposes, the main result is to change Eq. (1) to

$$\omega' = \frac{\omega}{1 + (\omega/m + \frac{1}{2}\epsilon)(1-\cos\theta)}. \tag{4}$$

The experiment which we suggest is of the Kapitza-Dirac[5] type, in which a low-energy

(10- to 100-eV) electron beam is directed through a laser cavity; and, when the laser is flashed, scattering of the electrons by the photons is observed. Several factors combine to make the experiment feasible, despite the fact that ϵ, the nonlinearity parameter, is only of order 10^{-6}. In the first place, the stimulated nature of the photon emissions enhances the counting rate enormously. A straightforward perturbation calculation[5] leads to the conclusion that virtually every electron in the incident beam will be Compton scattered. Also, since the standing electromagnetic waves in the laser cavity serve as a Bragg lattice for the electrons, scattering occurs only at a finite number of very sharply defined angles.[6] Most important of all, though, is the fact that these allowed scattering angles are much more sensitive to ϵ than any other observable quantity. This can be inferred from the fact that in Eq. (4) ϵ appears on the same footing not with unity, but with $\omega/m \sim 10^{-6}$.

The formula which shows the distinction between Eqs. (2) and (3) in an experimentally useful way is the relation between $\sin\theta$ and $\sin\theta'$, the incident and scattered electron angles shown in Fig. 1. One finds the equation

$$\frac{\sin\theta'}{\sin\theta}=p\frac{E+p\sin\theta}{W-p\sin\theta}\left[\frac{W-p\sin\theta}{W+p\sin\theta}-\frac{\epsilon m^2}{2\omega(E+p\sin\theta)}\right]$$
$$\times\left\{(E+p\sin\theta)^2\left[\frac{W-p\sin\theta}{W+p\sin\theta}+\left(\frac{W-p\sin\theta}{W+p\sin\theta}-\frac{\epsilon m^2}{2\omega(E+p\sin\theta)}\right)\frac{p\sin\theta}{W-p\sin\theta}\right]^2-m^2\right\}^{-1/2}, \tag{5}$$

where E and p are the incident-electron energy and momentum, and $W=E+\epsilon m^2/2\omega$. (Notice that the "classical" result $\theta=\theta'$ is obtained, as it must be, by setting $\epsilon=0$.) In the nonrelativistic limit appropriate to 10- to 100-eV electrons and optical photons this reduces to

$$\frac{\sin\theta'}{\sin\theta}\cong\frac{\omega/m-\epsilon/2}{\omega/m+\epsilon/2}. \tag{6}$$

If we assume $p^2/2m\cong 30$ eV, $\omega\cong 3$ eV, and $\rho\sim 10^{21}/\mathrm{cm}^3$, we find $\theta\sim\frac{1}{2}\times 10^{-3}$, and $(\theta'-\theta)/\theta \cong 16\%$. Thus it is necessary to be able to detect a 16% deviation in the allowed scattering angle, a shift of about 10^{-4} rad. According to Bartell,[5] it is apparently not unusually difficult in a Kapitza-Dirac experiment to obtain angular resolution of this quality.

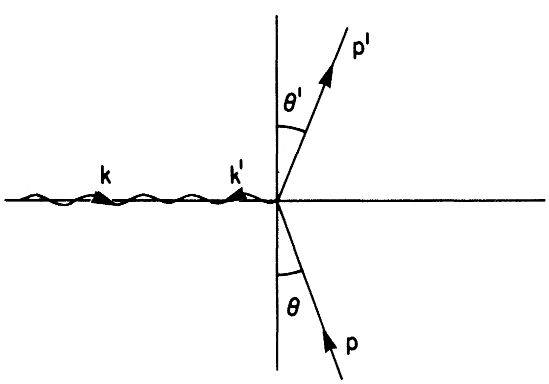

FIG. 1. Stimulated Compton scattering within the laser cavity. The photon is backscattered at an angle of 180°.

Thus, tentatively, we find that the effect of the altered energy-momentum-conservation relation can be seen in photon beams for which $\epsilon\sim 10^{-6}$ instead of $\epsilon\sim 1$. The photon densities required are still very high, but on the verge of feasibility with the aid of focusing or backward telescoping. Of course, if electrons with smaller momenta than we have assumed can be made available, then θ and θ' are that much larger (since $\theta\cong\omega/p$), and a correspondingly smaller beam intensity would lead to the same magnitude of the difference $\theta'-\theta$.

In closing, it should be pointed out that, while we have ignored the difficulties such as electron beam collimation, magnetic-field screening, and laser intensity measurement that are inherent in any such experiment, we have not neglected the possibility of other electron-photon interactions which might be thought to obscure the Compton scattering. Three-photon events, such as

$$e^-+\gamma\to e^-+\gamma'+\gamma',$$
$$e^-+\gamma+\gamma\to e^-+\gamma',$$

have non-negligible probability in very intense photon beams, but are kinematically forbidden in a Kapitza-Dirac-type experiment with nonrelativistic electrons. The most favorable competing allowed process is the four-photon event

$$e^- + \gamma + \gamma \rightarrow e^- + \gamma' + \gamma',$$

which satisfies the second-order Bragg reflection condition $\theta \cong 2\omega/p$. It should thus easily be distinguishable from Compton scattering.

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[2]L. S. Brown and T. W. B. Kibble, Phys. Rev. 133, A705 (1964); I. I. Goldman, Phys. Letters 8, 103 (1964); Z. Fried and J. H. Eberly, Phys. Rev. 136, B871 (1964).

[3]Here we term a process inherently nonlinear if the cross section is a nontrivial function of the density of incident photons.

[4]T. W. B. Kibble, Phys. Rev. 138, B740 (1965); P. J. Redmond, Proceedings of the Conference on Quantum Electrodynamics of High-Intensity Photon Beams, Durham, North Carolina, August 1964 (unpublished); P. Stehle and P. G. De Baryshe, to be published.

[5]P. L. Kapitza and P. A. M. Dirac, Proc. Cambridge Phil. Soc. 29, 297 (1933). Lately several reminders of this paper have appeared: A. C. Hall, Nature 199, 683 (1963); I. R. Gatland, L. Gold, and J. W. Moffat, Phys. Letters 12, 105 (1964). Very recently the first successful observation of the Kapitza-Dirac effect has been reported: L. S. Bartell, H. B. Thompson, and R. R. Roskos, to be published.

[6]For each electron the relative width $\Delta\theta/\theta$ of the allowed Bragg scattering angle is given by the relative breadth in frequency of the laser radiation: $\Delta\omega/\omega \sim 10^{-7}$.

FLUX AND ENERGY SPECTRUM OF PRIMARY COSMIC-RAY ELECTRONS*

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Since the discovery of primary electrons in the cosmic radiation,[1,2] attempts have been made to measure the flux of these particles at various energies.[3-6] The low intensity of the electron component causes great difficulties in obtaining enough events for good statistics and energy resolution and has so far prevented the determination of a reliable energy spectrum. Yet the knowledge of the electron spectrum in the vicinity of the earth is important for several reasons. At the present time, it appears likely that the primary electrons observed near the earth are of galactic origin. A determination of their flux and spectrum makes it possible to investigate in detail their relation to the nonthermal galactic radio emission. Furthermore, the modulation of their intensity and spectrum due to solar-controlled mechanisms may be different from the modulation of heavier primary particles. Parker[7] has discussed the possibility of velocity-dependent modulation, which can be tested by investigating the primary cosmic-ray electrons.

The experiment which we shall discuss here gives an approximate energy spectrum of the primary electron component. It was carried out in the summer of 1964 at a period near solar activity minimum. This period is most advantageous for investigations of low-energy galactic particles since the effects of solar modulation are approaching their minimum. Two balloon flights were made at Ft. Churchill on 22 and 29 July 1964, both of them floating under about 4 g/cm^2 of residual atmosphere, in order to measure the flux and energy spectrum of the electron component. A schematic cross section of the instrument which was used is shown in Fig. 1. Vertically incident particles trigger a counter telescope consisting of the scintillation counters T and I and a gas Čerenkov counter C. The geometry factor of the telescope is 1.08 cm^2 sr. The gas counter is filled with 2.4 atm of SF_6 and has a threshold of about 8 MeV for electrons and 15 BeV for protons. The energy loss of the particles in Counter I is measured in order to discriminate between singly and multiply charged particles. After passing through counter I, the particles penetrate a layer of 10.8 g/cm^2 of lead, a plastic scintillation counter S, and en-