Table I. Upper limits on the experimental cross section for the production of a heavy electron with mass $m_e{}^*$. The theoretical cross section and lower limits on λ are also shown.

| $m_e{}^*$[MeV] | $(\partial^2\sigma/\partial\Omega_e{}^{\text{Lab}}\partial\Omega_p{}^{\text{Lab}})_{\text{exp}}$ $(10^{-33}\ \text{cm}^2/\text{sr}^2)$ | $(\partial\sigma/\partial\Omega_{e^*}{}^{\text{cm}})_{\text{exp}}$ $(10^{-33}\ \text{cm}^2/\text{sr})$ | $(\partial\sigma/\partial\Omega_{e^*}{}^{\text{cm}})_{\text{theo}}$ $(10^{-30}\ \text{cm}^2/\text{sr})$ | $\lambda\times 10^2$ |
|---|---|---|---|---|
| 500 | 0.41 | 1.53 | 7.44 | $\lesssim 1.4$ |
| 600 | 0.41 | 1.07 | 5.32 | $\lesssim 1.4$ |
| 700 | 0.41 | 0.80 | 3.52 | $\lesssim 1.5$ |
| 800 | 0.41 | 0.65 | 2.48 | $\lesssim 1.7$ |
| 900 | 0.41 | 0.52 | 1.84 | $\lesssim 1.7$ |
| 1000 | 1.10 | 1.59 | 2.60 | $\lesssim 2.5$ |

between 0.2 and 1 GeV does not exist. The limits on λ also imply that the pair production results must be explained in a different way.[7,9]

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## RENORMALIZATION EFFECTS AND THE CABIBBO ANGLE

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We report in this note calculations of the renormalization of the semileptonic weak coupling constants by the U(3) symmetry-breaking interactions. Our consideration is based on a phenomenological field-theoretical model of U(3) symmetry breakdown, which is an extension of the idea that the symmetry breakdown is dominated by the nonvanishing expectation value of the scalar field $S_{33}$.[1,2] The conclusions of this paper depend essentially on meson-pole dominance approximations. Details and other related results will be submitted for publication elsewhere.

In this model, we assume that the phenomenological Lagrange function for the $0^-$ meson fields $\Phi_{ab}$, $1^-$ meson fields $U_{ab}$, and $\frac{1}{2}^+$ baryon fields $\Psi_{ab}$ is of the form

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_{\text{int}}\,\mathcal{L}_{33}, \tag{1}$$

where $\mathcal{L}_0$ is the free-field Lagrange function,[3] $\mathcal{L}_{\text{int}}$ represents the U(3)-symmetric strong interactions, and $\mathcal{L}_{33}$ effectively represents the symmetry-breaking effects resulting from the nonvanishing scalar vacuon, $\langle S_{33}\rangle \neq 0$. $\mathfrak{L}_{33}$ is of the form

$$\mathfrak{L}_{33}(\Psi, U, \Psi)$$
$$=\alpha_\Phi(\Phi^\mu\Phi_\mu)_{33}-\beta_\Phi m_0^{\ 2}(\Phi\Phi)_{33}$$
$$+\tfrac{1}{2}\alpha_U(U^{\mu\nu}U_{\mu\nu})_{33}-\beta_U m_1^{\ 2}(U^\mu U_\mu)_{33}$$
$$-\beta_\Psi M(\overline{\Psi}\Psi)_{33}-\beta_\Psi^{\ 1}M(\Psi\overline{\Psi})_{33}, \qquad (2)$$

where $m_0$, $m_1$, and $M$ are the masses of the pseudoscalar octuplet, vector nonuplet, and the baryon octuplet, respectively, in the U(3)-symmetry limit. In (2), we have assumed that the nine vector mesons are degenerate in the U(3)-symmetry limit. This has been justified by Schwinger from U(6)⊗U(6) considerations.

The $\alpha$ terms in (2) can be absorbed into the structure of $\mathfrak{L}_0$ by appropriately redefining the field variables and thus introducing renormalization constants for these fields. For example, (2) contains terms for $K$ and $K^*$, but not for $\pi$ and $\rho$. As a result, the field variables associated with $K$ and $K^*$ are renormalized by $\mathfrak{L}_{33}$, while those associated with $\pi$ and $\rho$ are not. Thus, we redefine the field variables $K_\mu$ and $K_{\mu\nu}^*$ according to

$$K_\mu \to (1+\alpha_\Phi)^{-1/2}K_\mu, \qquad (3)$$

$$K_{\mu\nu}^* \to (1+\alpha_U)^{-1/2}K_{\mu\nu}^*, \qquad (4)$$

and similarly for other fields that appear in $\mathfrak{L}_{33}$. In order to preserve the kinematical structure of the phenomenological Lagrange function, the structure that characterizes the propagation of physical excitations, we simultaneously make transformations on the field variables $K$ and $K_\mu^*$ according to

$$K \to (1+\alpha_\Phi)^{1/2}K, \qquad (5)$$

$$K_\mu^* \to (1+\alpha_U)^{1/2}K_\mu^*, \qquad (6)$$

and similarly for others. There are no $\alpha$-type terms for baryons in (2). These terms are excluded by charge conservation. The baryon fields are therefore not renormalized by the symmetry-breaking interactions.

On the basis of the conservation of a generalized charge $Q_{33}$ ($Q_{11}$ being electrical charge), we can prove that

$$\beta_U = 0. \qquad (7)$$

The proof is based on the effective meson-pole model of Schwinger.[3] Essentially, the reason for (7) is that the effects arising from mass renormalization and field renormalization of $U_{33}$, due to $\mathfrak{L}_{33}$, must cancel with each other while the vector field $U_{33}^{\ \mu}$ serves as the intermediary for interactions with $Q_{33}$.

The result (7) has the following consequences:

(i) To the first order of the symmetry breaking, the $|\Delta Y|=1$ vector coupling constants are not renormalized. This is the Ademollo-Gatto theorem.[4] Our consideration is again based on the meson-pole model. The reason for the absence of a first-order renormalization effect is not only that the transmitting vector field $U_{13}^{\ \mu}$ introduces no such effect, because of (7), but also that the $|\Delta Y|=1$ vector current itself (at zero momentum transfer) undergoes no renormalization to the first order of the symmetry breaking. Typically, as a result of the transformations (3) and (5),

$$K_\mu\pi - K\pi_\mu \to \frac{2+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}} \times\left[(K_\mu\pi-K\pi_\mu)-\frac{\alpha_\Phi}{2+\alpha_\Phi}(K_\mu\pi+K\pi_\mu)\right].$$

The combination $(2+\alpha_\Phi)2^{-1}(1+\alpha_\Phi)^{-1/2}$ is of second order in the symmetry-breaking parameter $\alpha_\Phi$.

(ii) With $\beta_U=0$, $\mathfrak{L}_{33}$ implies the following masses for the $1^-$ mesons:

$$m_\omega^{\ 2}=m_\rho^{\ 2}=m_1^{\ 2}, \qquad (8)$$

$$m_{K^*}^{\ 2}=(1+\alpha_U)m_\rho^{\ 2}, \qquad (9)$$

$$m_\varphi^{\ 2}=(1+2\alpha_U)m_\rho^{\ 2}, \qquad (10)$$

and consequently the mass relations

$$m_\omega=m_\rho, \qquad (11)$$

$$m_\varphi^{\ 2}+m_\rho^{\ 2}=2m_{K^*}^{\ 2}. \qquad (12)$$

These formulas are very well satisfied experimentally. They were first conjectured by Okubo[5] and later derived by Kuo and Yao[6] on the basis of SU(6).

We adopt the Cabibbo scheme[7] of the semileptonic weak interactions and assume a single "bare" Cabibbo angle $\theta$, in the U(3) limit, for both the vector and axial-vector currents of the strongly interacting particles. Because of the renormalization of $K_\mu$, in accordance with (3), the apparent Cabibbo angle $\theta_A$ determined from $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$ is given by

$$\tan\theta_A = (1+\alpha_\Phi)^{-1/2}\tan\theta. \qquad (13)$$

Based on the Goldberger-Treiman type relations, (3) implies the replacement

$$\sin\theta \to (1+\alpha_\Phi)^{-1}\sin\theta, \qquad (14)$$

for $|\Delta Y|=1$ axial-vector transitions of the leptonic decays of the baryons. The symmetry-breaking effects thus appear as a universal multiplicative factor for all $|\Delta Y|=1$ axial-vector weak vertices of the baryons. This factor can be absorbed as a renormalization of the Cabibbo angle, allowing the "apparent" axial-vector vertex functions to be expressed in terms of two reduced matrix elements as in the U(3)-symmetry limit. The corresponding apparent Cabibbo angle $\theta_A'$ is given by

$$\tan\theta_A' = (1+\alpha_\Phi)^{-1}\tan\theta. \qquad (15)$$

From (13) and (15), we further obtain

$$\tan\theta_A' = (1+\alpha_\Phi)^{-1/2}\tan\theta_A. \qquad (16)$$

To determine the parameter $\alpha_\Phi$, we make use of the decay[8] $K^* \to K+\pi$. Since $\alpha_U$ is already known from (9), the decay rate ratio $\Gamma(K^*\to K+\pi)/\Gamma(\rho\to 2\pi)$ can determine the parameter $\alpha_\Phi$. We shall adopt the following U(3) symmetry interactions[3] that are relevant for the decays $K^*\to K+\pi$ and $\rho\to 2\pi$:

$$ig\,\mathrm{Tr}\Phi^\mu[U_\mu,\Phi]-i(g/2m_1^{\,2})\,\mathrm{Tr}U^{\mu\nu}[\Phi_\mu,\Phi_\nu]. \qquad (17)$$

With the renormalization of the $K$ and $K^*$ field variables as given by (3), (4), (5), and (6), the decay widths are calculated to be

$$\Gamma(K^*\to K+\pi)=\frac{g^2}{4\pi}\frac{|\vec{p}|^3}{m_{K^*}^{\,2}}\left[\frac{m_{K^*}}{m_\rho}\frac{3+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}}\right]^2, \qquad (18)$$

$$\Gamma(\rho\to 2\pi)=\frac{4}{3}\frac{g^2}{4\pi}\frac{|\vec{p}|^3}{m_\rho^{\,2}}\left(\frac{3}{2}\right)^2, \qquad (19)$$

where $|\vec{p}|$ is the center-of-mass momentum of the daughter particles in each reaction, and use has been made of (8) and (9). Experimentally,[9]

$$\Gamma(K^*)=50\pm 2\ \mathrm{MeV},$$

$$\Gamma(\rho)=112\pm 4\ \mathrm{MeV},$$

from which we obtain

$$1+\alpha_\Phi = 0.76^{+0.17}_{-0.11}. \qquad (20)$$

Experimentally,[10]

$$\theta_A = 0.264\pm 0.004. \qquad (21)$$

Using (20) and (21), we obtain from (13) and (16) that

$$\theta = 0.238^{+0.022}_{-0.026}, \qquad (22)$$

and

$$\theta_A' = 0.302^{+0.030}_{-0.022} \qquad (23)$$

Willis et al.[11] have carried out an analysis of existing experimental data concerning the branching ratios of baryon leptonic decays. They found that the best values of $\theta$ and $\theta_A'$ are equal within a few percent to 0.26.[12] Our results (22) and (23) are not consistent with their finding. However, the experimental errors on the rates of the leptonic decays of hyperons are rather large; their conclusions may not be reliable. One such indication comes from the experimental $G_A/G_V$ ratio of lambda beta decays. Solution $A$ of Willis et al.[13] gives rise to the ratio

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu}_{\text{Willis et al.}} = F+D/3 = 0.68. \qquad (24)$$

However, there is evidence from polarized lambda beta decays[14] that

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu}_{\text{exp.}} \cong 1 \qquad (25)$$

is most consistent with the experimental data. The type of analysis carried out by Willis et al. should be repeated when more accurate experimantal data become available.

In our model, the $G_A/G_V$ ratio for all $|\Delta Y|=1$ baryon leptonic decays is uniformly "renormalized" by the U(3) symmetry-breaking interactions with a single renormalization constant

$(1+\alpha_\Phi)^{-1}$. Thus,

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu} = (1+\alpha_\Phi)^{-1}(F+D/3), \text{ etc.,} \quad (26)$$

where $F$ and $D$ are the usual reduced matrix elements. With[15]

$$-\left(\frac{G_A}{G_V}\right)^{n\to p+e^-+\nu}_{\text{exp.}} = F+D = 1.18\pm0.02 \quad (27)$$

and the experimental branching ratio[11]

$$B(\Sigma^-\to\Lambda+e^++\nu) = (0.75\pm0.28)\times10^{-4} \quad (28)$$

as input, $F$ and $D$ are determined to be[16]

$$F = 0.38\pm0.17, \quad D = 0.80\pm0.15. \quad (29)$$

It follows from (20), (26), and (29) that

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu}_{\text{theoretical}} = 0.86^{+0.48}_{-0.42}, \quad (30)$$

which is consistent with (25). More accurate experimental data, of course, are required to make a definite test of our model. A reliable determination of the $D/F$ ratio from $\Delta Y=0$ baryon leptonic decays[17] and the $G_A/G_V$ ratios of $|\Delta Y|=1$ hyperon leptonic decays can best serve the purpose.

With the "bare" Cabibbo angle $\theta$ given by (22), the vector coupling constant of nuclear beta decay is related to that of the muon decay by

$$G_\beta = \cos\theta G_\mu = (0.972\pm0.006)G_\mu. \quad (31)$$

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