tive masses shown in Table I.

The measurements of the combined heat capacity of $He^3$ and CMN were carried down to a temperature of 2 mdeg K on the $T^*$ scale in the case of the cell with the solid end plug. No strange behavior suggestive of a superfluid transition was observed.

In conclusion we remark that thus far only the heat capacity, the nuclear susceptibility, and the spin diffusion coefficient have been measured down to temperatures below 10 mdeg K. Neither the thermal conductivity nor the viscosity has been measured at sufficiently low temperatures to determine the limiting dependence on temperature. It would be particularly interesting to measure the thermal conductivity, which might also exhibit interesting anomalous behavior as in the case of the specific heat.

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# RADIATION AND ABSORPTION VIA MODE CONVERSION IN AN INHOMOGENEOUS COLLISION-FREE PLASMA*

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In this Letter we outline the small-amplitude wave theory for the conversion of a fast electromagnetic plasma wave into a very slow electrostatic mode (or vice versa) via the commonplace process of propagation across a mildly inhomogeneous plasma (e.g., density change of 2:1). Insofar as the slow wave is subject to collision-free absorption processes (e.g., cyclotron damping at integral multiples of the ion and electron cyclotron frequencies), the mode conversion just described can lead to effective absorption or radiation of electromagnetic energy in or from a plasma.[1]

The mode conversion will be illustrated for wave propagation across a magnetic field into a plasma of increasing or decreasing density. The actual conversion takes place in the vicinity of the surface along which the density value satisfies the condition for a hybrid resonance.[2] We will see that such conversion can be made the basis for an advantageous scheme for ion heating in a plasma at the lower hybrid resonance frequency. At the upper hybrid resonance frequency, such conversion can explain the transferral of wave energy from an extremely short-wavelength electrostatic Bernstein mode[3] into a long-wavelength electromagnetic mode, and thus account for the observability of Landauer radiation[4] by detectors placed well outside the plasma.

We consider a collision-free fully ionized plasma, with electron density $n_e$ increasing slowly in the $x$ direction, immersed in a $z$-directed uniform static magnetic field $B_0$. The appropriate wave equation at any point, $x$, will be based on the cold-plasma, warm-plasma, or electrostatic hot-plasma approximation, respectively, according to whether $k_x \rho_L$ is small, comparable to, or large compared to unity. We use $\rho_L$ to designate Larmor radius.

The dispersion relation for a cold uniform plasma is given by the biquadratic equation

$$\beta n_x^4 - \gamma n_x^2 + \delta = 0, \tag{1}$$

where

$$\beta \equiv K_{xx},$$

$$\gamma \equiv K_{xx}^2 + K_{xy}^2 + K_{zz} K_{xx} - K_{zz} n_z^2 - K_{xx} n_z^2,$$

$$\delta \equiv K_{zz}[(K_{xx} - n_z^2)^2 + K_{xy}^2],$$

and where $n_x \equiv k_x c/\omega$, $n_z \equiv k_z c/\omega$, and $\mathsf{K}$ is the familiar dielectric tensor for a cold plasma.

The branch

$$n_x^2 = [\gamma + (\gamma^2 - 4\beta\delta)^{1/2}]/2\beta \tag{2}$$

will be the one of interest to us, as this choice yields $n_x^2 \approx \gamma/\beta$ as $\beta \to 0$.

In the warm-plasma approximation, first-order finite Larmor radius effects for ions and electrons are added to Eq. (1). A derivation from the Vlasov equations for ions and electrons reveals that the elements of K may be expressed as power series in the quantities $\lambda_i$ and $\lambda_e$, respectively, where

$$\lambda_i \equiv \frac{k_x^2 kT_\perp^{(i)}}{\omega_{ci}^2 m_i}; \quad \lambda_e \equiv \frac{k_x^2 kT_\perp^{(e)}}{\omega_{ce}^2 m_e};$$

and where $kT_\perp$, $\omega_c$, and $m$ denote the temperature for motion perpendicular to $B_0$, the cyclotron frequency, and the particle mass. To first order in $\lambda$, the most important modification of (1) is the appearance of an $n_x^6$ term,

$$-\alpha n_x^6 + \beta n_x^4 - \gamma n_x^2 + \delta = 0. \tag{3}$$

The evaluation of $\alpha$ indicated by Eq. (1) is correct to lowest order in temperature,

$$\alpha = -\lim_{k_x \to 0} \frac{\omega^2}{c^2} \frac{\partial}{\partial k_x^2} K_{xx}. \tag{4}$$

We note from (3) that $n_x^2$ no longer becomes infinite as $\beta \to 0$, and also that a new mode of wave propagation has been introduced which, when $\beta$ is large, takes for form $n_x^2 \approx \beta/\alpha$. For a plasma with only a single species of ion, $\beta \to 0$ at the lower and upper hybrid frequencies,[2] $\omega_{\mathrm{LH}}$ and $\omega_{\mathrm{UH}}$, where

$$\frac{1}{\omega_{\mathrm{LH}}^2} = \frac{1}{\omega_{ci}^2 + \omega_{pi}^2} + \frac{1}{\omega_{ci}\omega_{ce}}, \tag{5}$$

$$\omega_{\mathrm{UH}}^2 = \omega_{ce}^2 + \omega_{pe}^2, \tag{6}$$

and where $\omega_{pi}^2 \equiv 4\pi n_i Z_i^2 e^2/m_i$, $\omega_{pe}^2 \equiv 4\pi n_e e^2/m_e$. The new modes are of the form (when $\beta$ is large)

$$\omega^2 = \omega_{\mathrm{LH}}^2 + \gamma_{\mathrm{LH}} k_x^2 kT/m_i, \tag{7}$$

$$\omega^2 = \omega_{\mathrm{UH}}^2 + \gamma_{\mathrm{UH}} k_x^2 kT/m_e, \tag{8}$$

corresponding to the lower and upper hybrid frequencies, respectively. We call the propagation modes described by Eqs. (7) and (8) ion and electron plasma waves, respectively.

Dispersion relations which differ only in the numerical values for $\gamma_{\mathrm{LH}}$ and $\gamma_{\mathrm{UH}}$ may be derived starting from the fluid equations for ions and electrons, using a scalar pressure and an adiabatic law to describe the finite-temperature effects.

For still shorter $x$ wavelengths, the correct dispersion relation must be obtained from the Vlasov equation. The following form, an extrapolation of that first obtained by Bernstein for the case of electron oscillations,[3] is correct for $k_z = 0$ and is still approximately true for $k_z \neq 0$ provided $|k_x/k_z| \gg 1$:

$$1 = \frac{4\pi n_i m_i c^2}{B_0^2} \frac{\alpha(q_i, \lambda_i)}{\lambda_i} + \frac{4\pi n_e m_e c^2}{B_0^2} \frac{\alpha(q_e, \lambda_e)}{\lambda_e}, \tag{9}$$

where

$$\alpha(q, \lambda) \equiv 2\sum_{n=1}^{\infty} e^{-\lambda} I_n(\lambda) \frac{n^2}{q^2 - n^2} \tag{10a}$$

$$= \frac{\lambda}{q^2 - 1^2} + \frac{1 \cdot 3\lambda^2}{(q^2 - 1^2)(q^2 - 2^2)} + \frac{1 \cdot 3 \cdot 5\lambda^3}{(q^2 - 1^2)(q^2 - 2^2)(q^2 - 3^2)} + \cdots, \tag{10b}$$

and where $q_i \equiv \omega/\omega_{ci}$, $q_e \equiv \omega/\omega_{ce}$. For small values of $\lambda$ we may use the first two terms on the right side of (10b) to obtain Eqs. (7) and (8) directly. For large values of $\lambda$, and when $q$ is close to an integer, $j$, the summation in (10a) is dominated by the single resonant term, leading to an approximate dispersion relation for the very short-wavelength ion mode:

$$\frac{\omega^2}{\omega_{pi}^2} = \left[\frac{j^3 e^{-\lambda} I_j(\lambda)}{\lambda}\right]_i \frac{\omega_{ci}}{\omega - j\omega_{ci}}, \tag{11}$$

and a similar relation for electrons, in which the subscript $i$ is replaced by $e$. It may be shown that the quantity in square brackets reaches a maximum value ~0.463 when

$$\lambda = j^2/3 + \tfrac{1}{6} + 5/j^2 + \cdots. \tag{12}$$

A composite representation of these dispersion relations is sketched in Figs. 1 and 2. The regions in an inhomogeneous plasma for validity of the cold-, warm-, and hot-plasma approximations are indicated, and it is shown how these approximations join smoothly on to each other.

In the vicinity of the hybrid resonance [i.e., in the vicinity of the plane $\beta = 0$, which we call

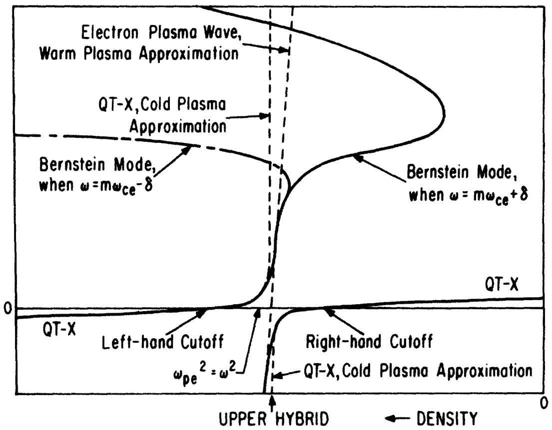


FIG. 1. The square of the index of refraction ($n_x^2$) is plotted against density. The frequency, which is fixed, is equal to the hybrid frequency at the value of density indicated. For an inhomogeneous plasma with linearly increasing density, the abscissa is proportional to distance ($x$). Dotted lines represent extrapolations of approximate dispersion relations beyond their regions of validity. Thus the extrapolated cold-plasma hybrid resonance ($n_x^2 \to \infty$) point is seen to coincide with the cut-off ($n_x^2 \to 0$) point for the warm-plasma-approximation plasma wave. QT-X designates the quasitransverse extraordinary (electromagnetic) mode. The parallel wave number, $k_z$, is assumed small or zero. At the top of the figure the bulge out to the right corresponds to the maximum of the right-hand side in Eq. (11) which occurs under the conditions of Eq. (12).

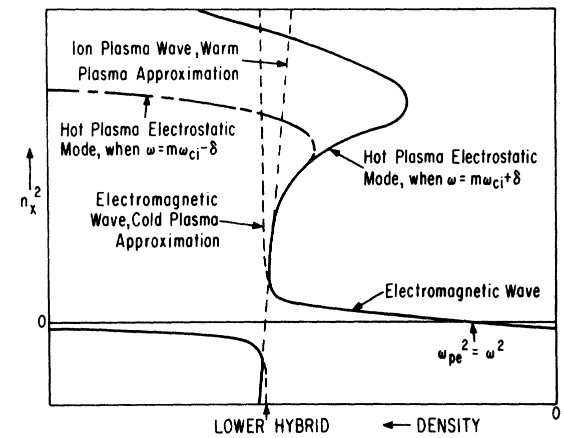


FIG. 2. Same as Fig. 1, except that it is assumed that $k_z^2c^2/\omega_{pi}^2 \gtrsim 2$, in accordance with the criterion [T. H. Stix, The Theory of Plasma Waves (McGraw-Hill Book Company, Inc., New York, 1962), p. 65] for accessibility to the lower hybrid resonance.

the "critical layer"] there is a confluence of the cold-plasma electromagnetic mode and the plasma wave. It is germane to ask whether energy propagating toward the critical layer in one of the two modes will be reflected back in the same mode, or whether part or all of the energy will be reflected (or transmitted) into the second mode. To examine this question of mode conversion in the vicinity of the critical layer we set $\beta = x(d\beta/dx)_{x=0}$, and the plane $x=0$ now defines the critical layer. Undoing the Fourier transformation indicated in Eq. (3), we obtain the wave equation relevant to this problem,

$$\frac{d^4E}{du^4} + u\frac{d^2E}{du^2} + \mu E = 0, \tag{13}$$

where, to obtain a dimensionless form, we have substituted $u = \alpha^{-1/3}(d\beta/dx)^{1/3}$ and $\mu = \lambda\alpha^{1/3} \times (d\beta/dx)^{-4/3}$, and where $\alpha$, $d\beta/dx$, and $\gamma$ are to be evaluated at the point $\beta = 0$, or equivalently, $x=0$. In obtaining Eq. (14) we have neglected the $\delta$ term in (3).

Inverting the Laplace transformation of (13), one finds[5] that four linearly independent solutions may be obtained by carrying out the integral

$$E(u) = \int dw \exp\left[-i\left(\frac{1}{3w^3} - \frac{u}{w} - \mu w\right)\right] \tag{14}$$

along four properly chosen separate paths. The integrand must go to zero at the start and finish of each unclosed path. That (14) is a solution of (13) may be easily verified by direct substitution. The integration in (14) may be carried out by the method of steepest descents (to traverse the saddle points closest to the origin in the $w$ plane) together with the integral representations for Bessel functions of order $-1$ (to traverse the outer saddle points). An integration along one such path typifies this method and leads to an important result for the case where $\alpha$ and $\gamma$ are of the same sign, as in Fig. 2. For $|u| \gg 1$, we obtain

$$i\left(\frac{-2u}{\pi\mu}\right)^{1/2} K_1[2(-\mu u)^{1/2}] \longleftrightarrow -\left(\frac{\pi u}{2\mu}\right)^{1/2} H_1^{(1)}[2(\mu u)^{1/2}] + \frac{\exp i(\frac{2}{3}u^{3/2} - \pi/4)}{2^{1/2}u^{5/4}}. \tag{15}$$

$K_1$ is the modified Bessel function, $H_1$ the Hankel function, $\mu$ is positive, and (15) gives the connection between asymptotic solutions for negative $u$ (on the left-hand side) and those for positive $u$ (right-hand side). Not shown on the left-hand side are solutions decaying faster than the $K_1$ function [e.g., $\sim\exp(-\frac{2}{3}|u|^{3/2})$].

Given a time dependence for $E$ as $\exp-i\omega t$, the right side of (15) represents a slow wave and a fast wave both with phase velocities directed away from the $u=0$ plane, connected, on the left side of (15), to an evanescent (spatially decaying) mode. Examination shows that the fast wave is actually a backward wave, so that its group velocity is directed toward the $u=0$ plane. The physical interpretation of (15) is that a fast-wave packet moving toward the critical layer is completely converted into a slow-wave packet upon reflection at the critical layer.

A second path of integration through the $w$ plane yields the result converse to (15), namely, the complete conversion of an inward-moving fast-wave packet into an outward-moving slow-wave packet. The other two independent paths of integration concern solutions with spatial decay toward the $u=0$ plane rather than away from it.

The physical process just described may be utilized to heat a plasma. One would shine electromagnetic radiation on a plasma, exciting the fast electromagnetic mode. The energy in this wave travels inward, is reflected at the lower hybrid critical layer, and converted to the slow ion plasma wave. Traveling outward, this wave slows down even more, its velocity becomes comparable to the ion thermal velocity, and its wavelength comparable to the electron Larmor radius. If the frequency of the wave is adjusted to be close to an integral multiple of the ion cyclotron frequency, the wave energy will be absorbed via the collision-free process of ion cyclotron damping.

There are several major benefits accompanying such a plasma-heating scheme: (a) The absorption of wave energy takes place well away from the walls, deep within the plasma; (b) even though the frequency may be equal to a high multiple of the ion cyclotron frequency, the energy is still absorbed by the ions; (c) as the lower hybrid frequency is well below the electron cyclotron frequency, and as the phase velocity along $B_0$ may be made comparable to the velocity of light, collision-free absorption by electron cyclotron damping and by ordinary electron Landau damping should be negligible; and (d) the problem of instabilities due to perturbations of the velocity distribution created by the rf heating fields may be less serious for this method than for lower frequency heating schemes.

It seems plausible that some portion of the absorption observed in "turbulent heating" experiments may have, in fact, been due to the above-described linear process. Babykin et al.[6] in fact state that especially great absorption is observed around the lower hybrid resonance condition.

Radiation from a reflex discharge with spectral peaks in the vicinity of integral multiples of the electron cyclotron frequency was first observed by Landauer. Canobbio and Croci[7] have related his observations to the Bernstein modes, Eq. (9), while Dreicer[8] has demonstrated experimentally that the upper hybrid frequency condition must be satisfied somewhere along the plasma density profile in order that radiation be observed. It has been proposed[9] that Landauer radiation could be due initially to excitation of the Bernstein modes by thermal or suprathermal electrons,[7] followed in turn by propagation of this wave energy first inward to the upper hybrid critical layer, reflection and mode conversion at this layer, tunneling outward past the right-hand cutoff, and propagation from there to the detecting apparatus outside the plasma via the electromagnetic QT-X mode. (See Fig. 1.)

Calculations similar to those leading to Eq. (15) verify that efficient mode conversion takes place in this case also. For the usual conditions under which Landauer radiation is seen, the attenuation due to tunneling is small.[10]

Quantitative analysis for the Landauer problem is complicated by the presence of the two cutoffs close to the critical layer. Calculations of the coefficients of transmission and reflection which are based on the connection, at five joints, of piecewise solutions of the inhomogeneous plasma-wave equation will be presented in a subsequent paper.

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# PLASMA RADIATION FROM THIN SILVER FOILS EXCITED BY LIGHT

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If electrons (some 10-keV energy) traverse a thin metal film (some 100 Å) they excite a radiative plasma oscillation of energy $\hbar\omega_p$ ($\omega_p$ = volume plasma frequency), which emits a peak of electromagnetic radiation around the frequency $\omega_p$. This radiation, predicted by Ferrell,[1] has been observed by Steinmann[2] as well as by Brown, Wessel, and Trounson[3] in silver and later by Arakawa, Herickhoff, and Birkhoff in aluminum and some other metals.[4]

In the following an experiment in silver is described in which this radiative plasma mode is not excited by electrons, but by electromagnetic radiation. Since the radiative plasma oscillation consists of electrons vibrating perpendicular to the surface, a light emission is expected with an intensity peak around $\omega_p$ as in the electron experiment, if the electric field of the exciting radiation has a component normal to the foil. The emitted radiation should be polarized, its electric field vector vibrating in the plane given by the foil normal and the direction of observation, and show the angular dependence of intensity calculated by Ferrell.

These considerations are verified by the experiments: If one irradiates a silver foil with polarized white light at a nonzero angle of incidence ($\alpha$), one observes at the angle $\theta$ relative to the foil normal (see Fig. 1) a strong intensity maximum at $\lambda_p = 3275 \pm 15$ Å ($\hbar\omega_p = 3.77 \pm 0.02$ eV) (see Fig. 2), if the electric vector of the exciting light lies in the plane given by the direction of the incoming beam and the foil normal. This maximum does not appear if the incoming light is polarized perpendicularly to this plane. The wavelength of the intensity maximum agrees with $\lambda_p = 3300$

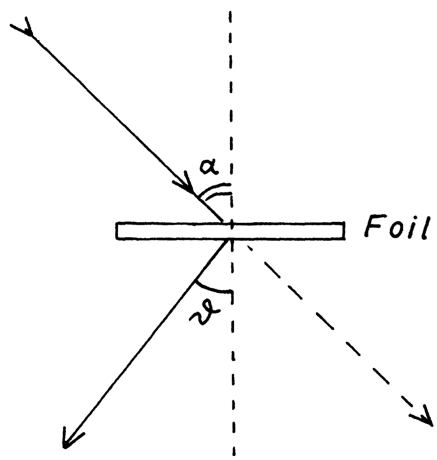


FIG. 1. A polarized light beam hits a thin silver foil at the angle $\alpha$. The light emitted by the excited surface plasma oscillation at the angle $\theta$ relative to the foil normal is detected by a photomultiplier.

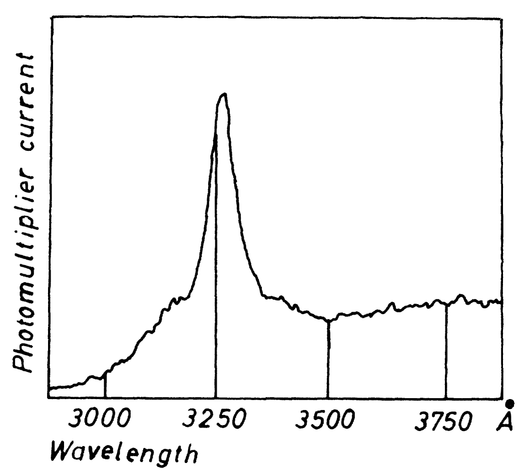


FIG. 2. Recorder traces of the photon emission from Ag foil (at room temperature) at $\alpha = 30°$ and $\theta = 45°$. The peak intensity is observed at $\lambda_p = 3275$ Å corresponding to $\hbar\omega_p = 3.77$ eV.