LOW-TEMPERATURE HEAT CAPACITY OF LIQUID He³†

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The heat capacity of liquid He³ at various pressures was measured as a function of temperature down to about 15 mdeg K by Anderson, Reese, and Wheatley.[1] It was pointed out in their paper that even at low pressures, a limiting constant value for C/n_3RT^*, where C is the heat capacity, n_3 is the number of moles of He³, R is the gas constant, and T^* is the magnetic temperature, was not obtained at low temperatures; but that rather this quantity continued to rise as the temperature dropped according to the law $C/n_3RT^* = \alpha + \beta T^*$. The parabolic dependence $C/n_3RT^* = \alpha' + \beta'T^{*2}$ expected for a Fermi gas[2] or, according to Richard's theory,[3] for a Fermi liquid did not fit the experimental data. The measured heat capacity C contains a contribution from the paramagnetic salt refrigerant and thermometer immersed in the He³ as well as from the heater wires. This contribution is relatively largest at the lower temperatures. If this "calorimeter background" was improperly corrected for, it might account for the anomalously high specific heat. Recently, the low-pressure heat-capacity experiments were repeated by us[4] with results which emphasized even further the departure of C/n_3RT^* from a parabolic law of temperature dependence. Anderson[5] replotted the experimental data up to temperatures over 0.1°K and found that they obeyed approximately a law $C/n_3RT^* = A\ln B/T^*$. He further suggested that if this result were true, it should have reper-

cussions on the usual assumption that He³ is a normal Fermi liquid at sufficiently low temperatures and hence on the possibility of a superfluid state in He³. Anderson's paper stimulated a great deal of interest among theoretical physicists; in particular, Balian and Fredkin[6] studied the effect of an interaction between He³ atoms and zero-sound phonons which, according to their calculation, led to a heat capacity of the form $C/n_3RT = A(\ln B/T)^{1/2}$ and to a possible drastic lowering of the theoretical transition temperature for superfluidity.

In the meantime we completed an experiment in which it was possible to eliminate the effects of calorimeter background on the heat-capacity measurements. Heat-capacity data for pure He³ only were obtained down to 6 mdeg K for a pressure of 0.28 atm and down to 4 mdeg K for a pressure of 27.0 atm. Although a more detailed account of this work is being prepared for publication elsewhere, it is the purpose of the present Letter to set forth the main results of these heat-capacity measurements, which we believe should be used in preference to our previous results in further consideration of the behavior of He³ at very low temperatures.

In order to eliminate the effects of the heat capacity of the calorimeter background, we have performed a series of experiments with a cell which is schematically illustrated in Fig. 1. The cell contained 6.49 g of cerium
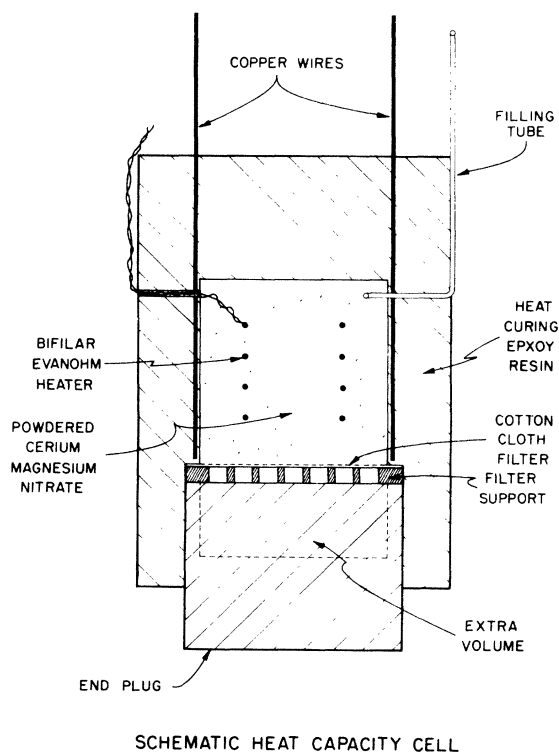
SCHEMATIC HEAT CAPACITY CELL

FIG. 1. Schematic drawing of heat-capacity cell.

magnesium nitrate (CMN) powder packed hard by a hand-held Teflon rod into a cylindrical volume 18.5 mm high and 18.5 mm diam. The Evanohm heater consisted of 13 cm of 0.051-mm diam wire arranged in a bifilar helix inside the salt. The CMN was confined to this region by a circular cotton cloth filter backed by an epoxy support with many 1.5-mm holes. Filter and support were sealed into the main epoxy cell by a room-temperature curing epoxy, the bottom side of the support then being machined off perpendicular to the axis of the cell and to a precisely measured depth below the end of the main cell. The resulting counterbore was then fitted with a solid epoxy end plug and sealed with room-temperature curing epoxy along its cylindrical sides. With these precautions it was possible after the first phase of the experimental measurements to disassemble the cell from the apparatus and to remove the end plug by boring without removing any of the filter support or in any way changing either the amount or configuration of the CMN and heater confined by the filter. Moreover, the dead volume in the filter support was unchanged. A second end plug with a precisely measured cylindrical

dead volume was then fitted to the cell and sealed in so that the volume of He^3 was increased without changing any of the other conditions. These precautions enabled us to compare the measured increase in the number of moles inside the cell with the known additional volume using the density measurements of Sherman and Edeskuty[7] and Boghosian, Meyer, and Rives.[8] The results were in reasonable agreement considering the uncertain thermal contraction of the epoxy cell. The He^3 used in these experiments contained about 10-ppm He^4 impurity in the gas at room temperature.

The experiments performed were as follows. First, for the cell with a solid end plug, the number of moles of He^3 in the cell at low pressure was measured using a standard system of calibrated volume, Toepler pump, and manometer. Then the heat capacity was measured as a function of T^* at a pressure of 0.28 atm. Next a pressure of 27.0 atm was applied and the heat capacity again measured. In this latter measurement the additional number of moles in the cell may be calculated using the data of references 7 and 8. Hence, by differences with the former measurement, the increase due to pressure in heat capacity per mole of pure He^3 (without calorimeter background) over that for pure He^3 at low pressure may be calculated directly. After these measurements the apparatus was disassembled as indicated above and the solid end plug replaced by the one with an extra volume for He^3. The new number of moles of He^3 in the cell was measured and the heat capacity as a function of T^* also measured at the same low pressure as in the first series of measurements. The ratio of the increase of heat capacity of 0.28 atm to the increase of the number of moles of He^3 at this pressure gives directly the heat capacity per mole of pure He^3 only. These results may then be combined immediately with the pressure data to obtain the heat capacity per mole of He^3 at 27.0 atm.

The results of the measurements are shown in Fig. 2, where a measured or difference heat capacity has been divided by $n_3 R T^*$ to remove most of the temperature dependence. Three sets of curves are shown at the bottom. The open circles, crosses, and solid circles represent, respectively, the 0.28-atm data for the cell with the solid end plug, the cell with the extra volume in the end plug, and the difference data, the complete data at low tempera-
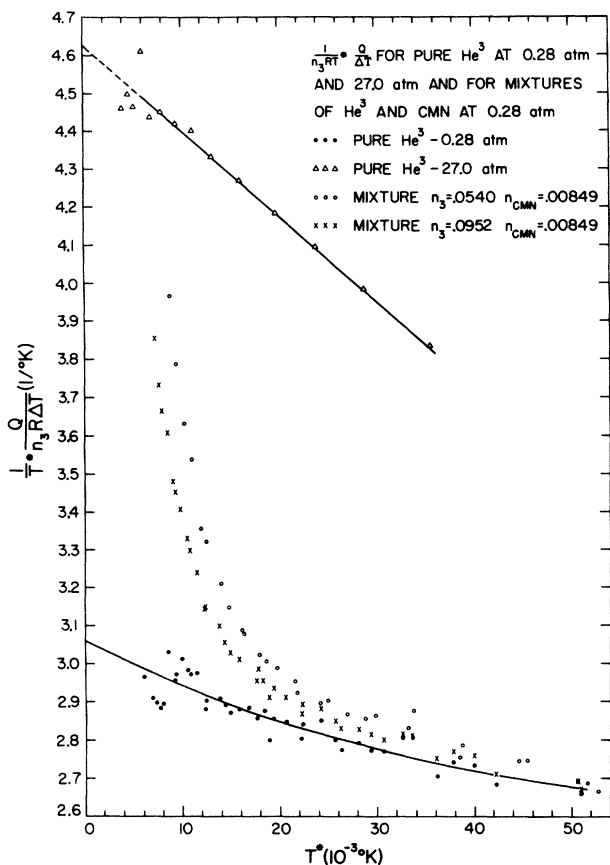
FIG. 2. Values of C/n_3RT^* at 0.28 and 27.0 atm for He^3 and for mixtures of He^3 and CMN. The smooth curves represent He^3 without the effect of calorimeter-background heat capacity.

tures not being shown for the former two. In the absence of calorimeter background all three sets of points should be the same. One sees that for $T^* \gtrsim 30$ mdeg K, the points for the different runs are in good agreement with one another, testifying to the consistency of the measurements and the absence of systematic error in the calibration of the temperature scale and the number of moles in the cell. The solid line is a fit to the solid circles and represents our best estimate of the heat capacity of pure He^3 at 0.28 atm. It curves slightly upward, but does tend to straighten out on being plotted versus $\ln T^*$. The solid line should be considered a bulk measurement since the heat capacity of He^3 mixed with CMN powder has been subtracted. The open triangles represent the 27.0-atm data, which quite accurately obey a linear law for C/n_3RT^* down to below 10 mdeg

K. The solid line averaging the 0.28-atm data was used in obtaining the points given as the 27.0-atm data. At low temperatures the scatter increases because the He^3 heat capacity becomes relatively a smaller proportion of the total heat capacity.

At 0.28 atm, C/n_3RT^* increases by about 10% between 50 and 6 mdeg K. At 27.0 atm, C/n_3RT^* increases by about 14% between 30 and 5 mdeg K. On the other hand, in an earlier work[9] it was found that within experimental error the self-diffusion coefficient had the Fermi-liquid temperature dependence of T^{*-2} below 50 mdeg K at low pressure and below 30 mdeg K at high pressure in the liquid.

One objection to the present experiments may be that the temperature scale is incorrect. However in the work of reference 4, it was shown that if the self-diffusion coefficient was used to determine the Kelvin temperature, then a thermometer of CMN similar to the present one had a magnetic temperature only about 0.1 mdeg K different from the Kelvin temperature at 3.5 mdeg K. Moreover, if C/n_3RT were really constant and the experimentally observed deviations from constancy were produced by the temperature scale, then one would expect C/n_3RT^* to change fractionally in the same way for both 0.28 and 27.0 atm. This is not observed.

We thus conclude that even in the temperature region where the diffusion coefficient obeys the T^{*-2} temperature dependence, the value of C/n_3RT^* continues to rise, though the effect is not great down to the lowest temperatures measured. The data are not sufficiently accurate at low temperatures to allow a precise determination of the temperature dependence, i.e., to distinguish with precision between a linear or a logarithmic dependence on T^* of C/n_3RT^*, though a parabolic dependence on T^* of C/n_3RT^* seems definitely ruled out. If a linear plot is used to extrapolate C/n_3RT^* to zero, one finds the limiting values and effec-

Table I. Limiting value of C/n_3RT^* and effective mass at two pressures.

P (atm)	$(C/n_3RT^*)_{\lim}$ (°K^{-1})	m^*/m
0.28	3.05	3.08
27.0	4.62	5.78

tive masses shown in Table I.

The measurements of the combined heat capacity of He^3 and CMN were carried down to a temperature of 2 mdeg K on the T^* scale in the case of the cell with the solid end plug. No strange behavior suggestive of a superfluid transition was observed.

In conclusion we remark that thus far only the heat capacity, the nuclear susceptibility, and the spin diffusion coefficient have been measured down to temperatures below 10 mdeg K. Neither the thermal conductivity nor the viscosity has been measured at sufficiently low temperatures to determine the limiting dependence on temperature. It would be particularly interesting to measure the thermal conductivity, which might also exhibit interesting anomalous behavior as in the case of the spe-

cific heat.

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RADIATION AND ABSORPTION VIA MODE CONVERSION IN AN INHOMOGENEOUS COLLISION-FREE PLASMA*

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In this Letter we outline the small-amplitude wave theory for the conversion of a fast electromagnetic plasma wave into a very slow electrostatic mode (or vice versa) via the commonplace process of propagation across a mildly inhomogeneous plasma (e.g., density change of 2:1). Insofar as the slow wave is subject to collision-free absorption processes (e.g., cyclotron damping at integral multiples of the ion and electron cyclotron frequencies), the mode conversion just described can lead to effective absorption or radiation of electromagnetic energy in or from a plasma.[1]

The mode conversion will be illustrated for wave propagation across a magnetic field into a plasma of increasing or decreasing density. The actual conversion takes place in the vicinity of the surface along which the density value satisfies the condition for a hybrid resonance.[2] We will see that such conversion can be made the basis for an advantageous scheme for ion heating in a plasma at the lower hybrid resonance frequency. At the upper hybrid resonance frequency, such conversion can explain the transferral of wave energy from an extremely short-wavelength electrostatic Bernstein mode[3] into a long-wavelength electromagnetic mode, and thus account for the observability of Lan-

dauer radiation[4] by detectors placed well outside the plasma.

We consider a collision-free fully ionized plasma, with electron density n_e increasing slowly in the x direction, immersed in a z-directed uniform static magnetic field B_0. The appropriate wave equation at any point, x, will be based on the cold-plasma, warm-plasma, or electrostatic hot-plasma approximation, respectively, according to whether $k_x \rho_L$ is small, comparable to, or large compared to unity. We use ρ_L to designate Larmor radius.

The dispersion relation for a cold uniform plasma is given by the biquadratic equation

$$\beta n_x^{\ 4} - \gamma n_x^{\ 2} + \delta = 0, \tag{1}$$

where

$$\beta \equiv K_{xx},$$

$$\gamma \equiv K_{xx}^{\ 2} + K_{xy}^{\ 2} + K_{zz} K_{xx} - K_{zz} n_z^{\ 2} - K_{xx} n_z^{\ 2},$$

$$\delta \equiv K_{zz} [(K_{xx} - n_z^{\ 2})^2 + K_{xy}^{\ 2}],$$

and where $n_x \equiv k_x c/\omega$, $n_z \equiv k_z c/\omega$, and K is the familiar dielectric tensor for a cold plasma.