tons more than required for a pure helion mantle and pure neutron core. Hence it can be predicted that for dysprosium, prolate deformation begins at $N=92$ rather than $N=90$. The energy evidence of Barber et al. shows that the process has not begun at $Dy_{88}$ and is completed by $Dy_{96}$.

A core with two spherons as inner core may contain from 13 to 19 spherons. The 19-spheron core has the symbol 1-5-(1)-5-(1)-5-1; each of the inner-core spherons with its neighbors forms an icosahedral complex. The 19-spheron core corresponds to $N$ about 124 according to Eq. (1), and to $N=118$ according to the highest-$Z$ rule for stable isotones $({}_{80}Hg_{118}{}^{198})$. We draw the conclusion that all of the nuclei showing prolate deformation in the range $N=90$ to 120 ($A=150$ to 190) have a two-spheron inner core, and that is the geometrical properties (the shape) of this inner core that is responsible for the deformation.

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## APPLICATION OF CURRENT COMMUTATION RULES TO NONLEPTONIC DECAY OF HYPERONS*

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Recently, current commutation relations derived from the quark model[1] were successfully applied to get the axial-vector renormalization constant.[2] This success encourages us to speculate that the rules might be useful in other processes also.[3] In this paper we apply the commutation rules $[A_0^{(i)}(x), V_\mu^{(j)}(y)]_{t=0} = if_{ijk}\delta^3(x-y)A_\mu^{(k)}(x)$ and $[A_0^{(i)}(x), A_\mu^{(j)}(y)] = if_{ijk}\delta^3(x-y)V_\mu(x)$, together with other assumptions, to nonleptonic processes. These other assumptions are as follows: (I) Nonleptonic decay is described by a Hamiltonian[4]

$$H=(J_\mu J_\mu^\dagger + J_\mu^\dagger J_\mu),$$

where $J_\mu$ is the Cabibbo current.[5] (II) The commutation relation of $H$ and $A_0(x)$ is that derived from the quark model.[1] (III) $\partial_\mu A_\mu^{(k)}(x) = c\varphi^{(k)}$ ($k=1,2,3$), where $\varphi^{(k)}$ is the pion field. This is called the partial conservation of the axial-vector current (PCAC).[6] (IV) An unsubtracted dispersion relation in the momentum transfer to the baryon is valid for the nonleptonic decay vertex of the hyperons. Note that we do not assume the $|\Delta I| = \frac{1}{2}$ rule[7] nor the octet enhancement hypothesis.[8]

From assumption II we obtain

$$[\int d^3x A_0^{(k)}(x), H^{(i,j)}(0)] = if_{kil}[J_\mu^{(j)}(0)J_\mu^{(l)}(0) + J_\mu^{(l)}(0)J_\mu^{(j)}(0)] + (i \leftrightarrow j), \tag{1}$$

where

$$H^{(i,j)} = J_\mu^{(i)}(0)J_\mu^{(j)}(0) + J_\mu^{(j)}(0)J_\mu^{(i)}(0),$$

and

$$(i,j) = (1,4) \text{ or } (2,5), \quad k = 1,2,3; \quad l \text{ goes from 1 to 8.}$$

The left-hand side of Eq. (1) can be written following Fubini and Furlan[2]:

$$[\int d^3x A_0^{(k)}(x), H^{(ij)}(0)] = \int d^4x [\partial_\mu A_\mu^{(k)}(x), H^{(i,j)}(0)]\theta(-x_0) = c\int d^4x [\varphi^{(k)}(x), H^{(i,j)}(0)]\theta(-x_0). \tag{2}$$

We have used assumption III to go from the second line to the third.

At the same time we have the nonleptonic amplitude,

$$(2k_0)^{1/2}\langle B^{(n)}\pi^{(k)}|H^{(ij)}(0)|B^{(m)}\rangle = i\int d^4x e^{-ikx}\langle B^{(n)}|[j^{(k)}(x), H^{(i,j)}(0)]|B^{(m)}\rangle\theta(-x_0), \tag{3}$$

where $j^{(k)}(x) = (\Box - \mu^2)\varphi^{(k)}(x)$ and $k$ is the four-momentum vector of the pion.

Comparing the absorptive part of (3) with that of the matrix element of (2), we have

$$\text{abs}\langle B^{(n)}|[\int d^3x A_0^{(k)}(x), H^{(ij)}(0)]|B^{(m)}\rangle = c' \lim_{k\to 0} (2k_0)^{1/2}\text{abs}\langle B^{(n)}\pi^{(k)}(k)|H^{(ij)}(0)|B^{(m)}\rangle. \tag{4}$$

In order to extend the equality in Eq. (4) to the real parts of the matrix elements, we make use of assumption IV. The right-hand side of Eq. (4) is not exactly the nonleptonic amplitude but its analytic continuation with respect to the mass of the pion, but we assume that what we get for the latter in the following symmetry consideration is also valid for the former.

Substituting (1) in (4) we find

$$\lim_{k\to 0} (2k_0)^{1/2}\langle B^{(\nu_1)}\pi^{(\nu_3)}|H^{(\nu_4\nu_5)}(0)|B^{(\nu_2)}\rangle = \text{const}\left\{\begin{pmatrix} 8 & 8 & 8\text{a} \\ \nu_3 & \nu_4 & \nu_6\end{pmatrix}\langle B^{(\nu_1)}|J_\mu^{(\nu_5)}J_\mu^{(\nu_6)} + J_\mu^{(\nu_6)}J_\mu^{(\nu_5)}|B^{(\nu_2)}\rangle\right.$$
$$\left. + \begin{pmatrix} 8 & 8 & 8\text{a} \\ \nu_3 & \nu_5 & \nu_6\end{pmatrix}\langle B^{(\nu_1)}|J_\mu^{(\nu_4)}J_\mu^{(\nu_6)} + J_\mu^{(\nu_6)}J_\mu^{(\nu_4)}|B^{(\nu_2)}\rangle\right\}, \tag{5}$$

where the $\nu$'s denote $I^2$, $I_z$, and $Y$ and ( | ) are SU(3) Clebsch-Gordan coefficients.[9] One might suspect that the $\underline{27}$ part in $H^{(\nu_4,\nu_5)}(0)$ does not contribute to the right-hand side of Eq. (5), but we found that this is actually not the case. Note that the matrix elements in the right-hand side of (5) are symmetric with respect to $\nu_5$ and $\nu_6$. We can expand these in terms of $\underline{8}$s and $\underline{27}$ tensors, dropping the $\underline{1}$ which does not contribute to the nonleptonic decay:

$$J_\mu^{(\nu_5)}J_\mu^{(\nu_6)} + J_\mu^{(\nu_6)}J_\mu^{(\nu_5)} = \sum_\nu \begin{pmatrix} 8 & 8 & 27 \\ \nu_5 & \nu_6 & \nu\end{pmatrix} T_\nu^{(27)} + \sum_\nu \begin{pmatrix} 8 & 8 & 8\text{s} \\ \nu_5 & \nu_6 & \nu\end{pmatrix} T_\nu^{(8)}. \tag{6}$$

We find for the right-hand side of (5)

$$\text{const}\left[\begin{pmatrix} 8 & 8 & 8\text{a} \\ \nu_3 & \nu_4 & \nu_6\end{pmatrix}\left\{\sum_\nu \begin{pmatrix} 8 & 8 & 27 \\ \nu_5 & \nu_6 & \nu\end{pmatrix}\langle B^{(\nu_1)}|T_\nu^{(27)}|B^{(\nu_2)}\rangle + \sum_\nu \begin{pmatrix} 8 & 8 & 8\text{s} \\ \nu_5 & \nu_6 & \nu\end{pmatrix}\langle B^{(\nu_1)}|T_\nu^{(8)}|B^{(\nu_2)}\rangle + (\nu_4 \leftrightarrow \nu_5)\right\}\right]. \tag{7}$$

For the matrix elements in (7), we then use the Wigner-Eckart theorem:

$$\langle B^{(\nu_1)}|T_\nu^{(27)}|B^{(\nu_2)}\rangle = \begin{pmatrix} 8 & 27 & 8 \\ \nu_2 & \nu & \nu_1\end{pmatrix}\langle\langle 8|T^{(27)}|8\rangle\rangle \langle B^{(\nu_1)}|T_\nu^{(8)}|B^{(\nu_2)}\rangle$$

$$= \begin{pmatrix} 8 & 8 & 8\text{s} \\ \nu_2 & \nu & \nu_1\end{pmatrix}\langle\langle 8|T^{(8)}|8\rangle\rangle_\text{s} + \begin{pmatrix} 8 & 8 & 8\text{a} \\ \nu_2 & \nu & \nu_1\end{pmatrix}\langle\langle 8|T^{(8)}|8\rangle\rangle_\text{a}.$$

Table I. Contribution of three independent amplitudes to various processes.

| SU(3) coefficients for \ Process | $\Lambda \to p+\pi^-$ | $\Lambda \to n+\pi^0$ | $\Xi^- \to \Lambda+\pi^-$ | $\Xi^0 \to \Lambda+\pi^0$ | $\Sigma^+ \to n+\pi^+$ | $\Sigma^+ \to p+\pi^0$ | $\Sigma^- \to n+\pi^-$ |
|---|---|---|---|---|---|---|---|
| $\langle\langle T^{(27)}\rangle\rangle$ | $-(\sqrt{6})/90$ | $-\sqrt{3}/90$ | $(\sqrt{6})/90$ | $-\sqrt{3}/90$ | $-\frac{1}{9}$ | $\sqrt{2}/45$ | $1/15$ |
| $\langle\langle T^{(8)}\rangle\rangle$ | $-(\sqrt{5})/20$ | $-(\sqrt{10})/40$ | $-(\sqrt{5})/20$ | $(\sqrt{10})/40$ | 0 | $(\sqrt{15})/60$ | $-(\sqrt{30})/60$ |
| $\langle\langle T^{(8)}\rangle\rangle$ | $-1/20$ | $-\sqrt{2}/40$ | $1/20$ | $-\sqrt{2}/40$ | 0 | $\sqrt{3}/20$ | $(\sqrt{6})/20$ |

We thus have found that all the nonleptonic amplitudes of hyperons can be expressed in terms of three independent parameters. This is remarkable since we have not assumed the octet enhancement. Substituting $\nu_4 \equiv (I, I_2, Y) = (1, -1, 0)$ and $\nu_5 = (\frac{1}{2}, \frac{1}{2}, 1)$ in (7), we can easily calculate the SU(3) coefficients explicitly. The results are summarized in Table I.

As only three amplitudes out of seven are independent, we have four relationships among them. We find from Table I

$$T(\Lambda \to p+\pi^-) = \sqrt{2}T(\Lambda \to n+\pi^0), \tag{A}$$

$$T(\Xi^- \to \Lambda+\pi^-) = -\sqrt{2}T(\Xi^0 \to \Lambda+\pi^0), \tag{B}$$

$$-\sqrt{2}T(\Sigma^+ \to p+\pi^0) = T(\Sigma^+ \to n+\pi^+) + T(\Sigma^- \to n+\pi^-), \tag{C}$$

$$2T(\Xi^- \to \Lambda+\pi^-) - T(\Lambda \to p+\pi^-) = \sqrt{\tfrac{3}{2}}T(\Sigma^- \to n+\pi^-). \tag{D}$$

Let us first look at Eqs. (A), (B), and (C). One might at first sight think that these are exactly the same as obtained from the $|\Delta I| = \frac{1}{2}$ rule. However, they are not. To obtain the $|\Delta I| = \frac{1}{2}$ results, it would be necessary to reverse the signs in (A) and (B) and also the sign of $T(\Sigma^+ \to n+\pi^+)$ in (C). Nevertheless (A), (B), and (C) are consistent with all experiments since they do not give the relative signs of the amplitudes. Thus, we have been unable to obtain the $|\Delta I| = \frac{1}{2}$ rule as a consequence of assumptions I-IV, but it is quite interesting that we have obtained a solution which cannot be distinguished from the $|\Delta I| = \frac{1}{2}$ rule by experiment. It is easy to show that in (A), (B), and (C) we have fairly large contributions from $|\Delta I| = \frac{3}{2}$ part of the Hamiltonian.

Equation (D) is equivalent only when $T(\Sigma^+ \to n+\pi^+) = 0$ to the triangular relationship obtained by the author and independently by Lee[10] assuming octet enhancement and $R$ symmetry. If both equations are valid (the triangular relationship has been proved on more general grounds by Gell-Mann for $s$ waves[4]), then the $s$-wave part of $T(\Sigma^+ \to n+\pi^+)$ is zero. Relation (C) and experiment would then imply that the $p$-wave part of $T(\Sigma^- \to n+\pi^-)$ is zero, in which case relation (D) disagrees with experiment.

This is the only defect in our theory. This point should be further investigated.

Finally, we note that our formulation gives a method for calculating the magnitude of the nonleptonic decay amplitudes. This method is as follows: Put a complete set of the intermediate states between two currents in the right-hand side of Eq. (5). If we neglect all but the lowest lying baryon states we can then express this right-hand side of (5) in terms of electric and magnetic form factors and also the axial and induced pseudoscalar form factors with over-all strengths determined by the Fermi constant. All these points will be discussed in another paper.

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## ERRATUM

EFFECTS OF HIGH MAGNETIC FIELDS ON THE ULTRASONIC VELOCITY AND ATTENUATION IN Nb-25% Zr. Y. Shapira and L. J. Neuringer [Phys. Rev. Letters 15, 724 (1965)].

Equation (2) should read

$$\beta = c^2\omega/4\pi\sigma\mu V_s^2.$$