ston), or $H_{c2}^*$ (Maki[6]) is hindered by (a) experimental uncertainties in $H_{u0}$, $H_{r0}$, $H_c$, and $\gamma$, and (b) the fact that Maki presents explicit upper critical field values away from $T=0$, $T_c$ only for $\alpha^2 \leq 2$. Magnetization measurements at higher $H$ are now in progress to investigate Maki's predictions[6] of (1) marked increase of $(dM_s/dH)$ at $H_u$ with decrease of $T$, and (2) first-order transitions to the normal state at low $T$ and high $\alpha^2 > 2$.

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# STRUCTURAL BASIS OF THE ONSET OF NUCLEAR DEFORMATION AT NEUTRON NUMBER 90

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Nuclei in the mass-number range $150 < A < 190$ show a permanent oblate deformation.[1] It was recognized by Mottelson and Nilsson[2] that the onset of the change from spherical to ellipsoidal shape occurs between 88 and 90 neutrons. Evidence about the significance of the value 90 for $N$ was reported by Brix and others.[3] Barber et al.[4] have made a careful study of the change in mass accompanying the addition of pairs of neutrons to nuclei in this region, and have found that the abnormal mass effect is about equally divided between the $88 \rightarrow 90$ and $90 \rightarrow 92$ changes in $N$. The isotopes $_{62}$Sm and $_{64}$Gd with $N = 90$ are about 0.55 MeV more stable than would be expected from linear extrapolation of the mass values for $N \leq 88$, and those with $N \geq 92$ are about 1.1 MeV more stable. The interpretation given these differences is that for $N = 90$ the deformed nucleus is 0.55 MeV more stable than a spherical nucleus, and for $N \geq 92$ it is 1.1 MeV more stable.[5]

No simple explanation of the onset of deformation at $N = 90$ has been advanced. I have found that a simple explanation is provided by the close-packed-spheron theory of nuclear structure.

The close-packed-spheron theory[6] incorporates some of the features of the shell model, the alpha-particle model, and the liquid-drop model. Nuclei are considered to be close-packed aggregates of spherons (helicons, tritons, and dineutrons), arranged in spherical or ellipsoidal layers, which are called the mantle, the outer core, and the inner core. The assignment of spherons, and hence nucleons, to the layers is made in a straightforward way on the basis of the assumption of constant volume per nucleon. The relation to the shell model is that subshells (given value of $l$) occurring once are assigned to the mantle, those occurring twice to the mantle and outer core, and so on.

The spheron packing equation is

$$n_t^{1/3} = n_i^{1/3} + 1.30, \quad (1)$$

in which $n_t$ is the total number of spherons and $n_i$ the number not including those of the outer layer. For $N=82$ this equation gives one spheron in the inner core (completed $K$ shell, $1s^2$), nine in the outer core (completed $M$ shell, $2s^2 1p^6 1d^{10}$), and 31 in the mantle [completed spin-orbit subsubshell, $3s^2 2p^6 2d^{10} 1f^{14} 1g^{18}$-$(1h_{11/2})^{1/2}$]. The equation places 11 spherons in the core at about $N=87$ and 12 at about $N=92$. In icosahedral packing around a single sphere there are 12 spheres in a completed second layer and 32 in the third, a total of 45. For icosahedral packing, the transition from an inner core of one spheron to one of two spherons would be expected to take place between $N=90$ and $N=92$. The effect of the shell structure (completed mantle at 31 rather than 32 spherons) may explain why the transition occurs over the range 88 to 92 rather than more sharply at 90 to 92.

Evidence about the nature of the transition is provided by the observed proton numbers of the stable highest-$Z$ isotones. A useful empirical rule is that the largest proton number for a set of stable isotones is equal to the neutron number of the mantle. For example, the set of stable isotones with $N=50$ (from ${}_{36}Kr_{50}$ to ${}_{42}Mo_{50}$) ends at $Z=42$, which is the number of neutrons in the mantle, and the set for $N=82$ ends at $Z=62$ (${}_{62}Sm_{82}$), which again is the number of neutrons in the mantle. (This rule may be interpreted as showing that a significant contribution to the structure of the normal state of a nucleus is made by those structures with all protons in the mantle; the highest-$Z$ stable isotone then has as a contributor to its normal state the structure with a pure helion mantle and a pure neutron core.) The highest-$Z$ stable isotones in the region near $N=90$ are ${}_{60}Nd_{84}$, ${}_{62}Sm_{86}$, ${}_{64}Gd_{88}$, ${}_{66}Dy_{90}$, ${}_{66}Dy_{92}$, and ${}_{68}Er_{94}$. They correspond, with this rule, to a core of 12 spherons for $N=84$ to 90 and of 13 spherons for 92 and 94.

The onset of prolate deformation is expected at 13 spherons, with two in the inner core. A stable local environment for an inner-core spheron is that in which it has ligancy 9, as in the $KM$ core for magic number 82. We may describe this core by the symbol 3-(1)-5-1; the inner-core spheron, (1), lies in the center of the shell formed by a ring of three and a ring of five spherons, plus one spheron at the apex of the pentagonal pyramid with the ring of five as its base. [A centered icosahedron can be similarly represented as 1-5-(1)-5-1.] The corresponding structure for a two-spheron inner core and an 11-spheron outer core is 3-(1)-5-(1)-3. With this structure each of the inner-core spherons with its nine neighbors (including the other inner-core spheron) can be described as a $KM$ complex, and the entire core can be described as two overlapping $KM$ complexes. A diagram of a two-dimensional analog is shown in Fig. 1.

That stable nuclear structures should involve stable local structures about each of the inner-core spherons is a reasonable consequence of the mutual interdependence of structure and potential energy function and the short range of internucleonic forces.

This core, with two spherons as inner core, has a prolate deformation from spherical symmetry. On the basis of the arguments discussed above, we assign this prolate core to $Nd_{90}$, $Sm_{90}$, and $Gd_{90}$, and their heavier isotopes.[7]

A core of 13 spherons with an inner core of a single spheron (icosahedral packing) may occur for $N=90$ (45 spherons), but not for $N \geq 92$. The energy values reported by Barber et al.[4] indicate that the change to prolate deformation is about half completed at $N=90$. Possibly these nuclei show resonance between the prolate structure and the icosahedral spherical structure.[5]

It is unlikely that ${}_{66}Dy_{90}$ has 13 spherons in its core, because it would then have two pro-

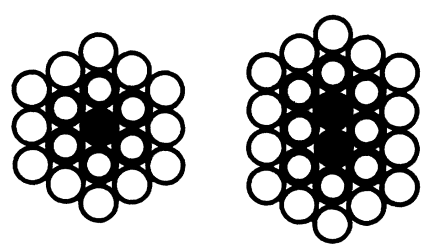

FIG. 1. Two-dimensional representation of a nearly spherical close-packed arrangement of spherons, with one in the inner core (left), and of an arrangement with prolate deformation, consequent to having two spherons in the inner core (right).

tons more than required for a pure helion mantle and pure neutron core. Hence it can be predicted that for dysprosium, prolate deformation begins at $N=92$ rather than $N=90$. The energy evidence of Barber et al. shows that the process has not begun at $Dy_{88}$ and is completed by $Dy_{96}$.

A core with two spherons as inner core may contain from 13 to 19 spherons. The 19-spheron core has the symbol 1-5-(1)-5-(1)-5-1; each of the inner-core spherons with its neighbors forms an icosahedral complex. The 19-spheron core corresponds to $N$ about 124 according to Eq. (1), and to $N=118$ according to the highest-$Z$ rule for stable isotones (${}_{80}Hg_{118}{}^{198}$). We draw the conclusion that all of the nuclei showing prolate deformation in the range $N=90$ to 120 ($A=150$ to 190) have a two-spheron inner core, and that is the geometrical properties (the shape) of this inner core that is responsible for the deformation.

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## APPLICATION OF CURRENT COMMUTATION RULES TO NONLEPTONIC DECAY OF HYPERONS*

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Recently, current commutation relations derived from the quark model[1] were successfully applied to get the axial-vector renormalization constant.[2] This success encourages us to speculate that the rules might be useful in other processes also.[3] In this paper we apply the commutation rules $[A_0^{(i)}(x), V_\mu^{(j)}(y)]_{t=0}=if_{ijk}\delta^3(x-y)A_\mu^{(k)}(x)$ and $[A_0^{(i)}(x),A_\mu^{(j)}(y)]=if_{ijk}\delta^3(x-y)V_\mu(x)$, together with other assumptions, to nonleptonic processes. These other assumptions are as follows: (I) Nonleptonic decay is described by a Hamiltonian[4]

$$H=(J_\mu J_\mu^\dagger+J_\mu^\dagger J_\mu),$$

where $J_\mu$ is the Cabibbo current.[5] (II) The commutation relation of $H$ and $A_0(x)$ is that derived from the quark model.[1] (III) $\partial_\mu A_\mu^{(k)}(x)=c\varphi^{(k)}$ ($k=1,2,3$), where $\varphi^{(k)}$ is the pion field. This is called the partial conservation of the axial-vector current (PCAC).[6] (IV) An unsubtracted dispersion relation in the momentum transfer to the baryon is valid for the nonleptonic decay vertex of the hyperons. Note that we do not assume the $|\Delta I|=\frac{1}{2}$ rule[7] nor the octet enhancement hypothesis.[8]

From assumption II we obtain

$$[\int d^3xA_0^{(k)}(x),H^{(i,j)}(0)]=if_{kil}[J_\mu^{(j)}(0)J_\mu^{(l)}(0)+J_\mu^{(l)}(0)J_\mu^{(j)}(0)]+(i\leftrightarrow j), \quad (1)$$

where

$$H^{(i,j)}=J_\mu^{(i)}(0)J_\mu^{(j)}(0)+J_\mu^{(j)}(0)J_\mu^{(i)}(0),$$