LOW-TEMPERATURE DEPENDENCE OF THE ELECTRICAL RESISTIVITY OF DEGENERATE n-TYPE GERMANIUM

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Germanium doped with more than 10^{18} donor impurities per cm^3 has electronic properties very much like those of a metal in that it has a temperature-independent carrier density in the conduction band at low temperatures. The Fermi degeneracy temperature T_D varies from 76°K for an N_D, the donor concentration, of $10^{18}/\text{cm}^3$ to 354°K for $N_D = 10^{19}/\text{cm}^3$. The Fermi surface consists of four geometrically equivalent ellipsoids of revolution centered at the centers of the hexagonal faces of the Brillouin zone of the fcc lattice. Electrical transport is determined in the main by carriers scattering from the screened ionized donor impurities which, for the samples discussed below, are equal in density to the electron density.

The temperature dependence of resistivity in heavily doped germanium has been previously investigated.[1-3] All the measurements showed a resistivity increasing with temperature at low temperatures. The results, thought at the time to be anomalous,[3-5] were not of sufficient accuracy to determine the functional dependence of resistivity on temperature and impurity concentration. We report here the first accurate measurements of the temperature dependence of the low-temperature electrical resistivity of Ge, heavily doped with either Sb or As. Figure 1 shows that the data for all three Sb-doped samples fit the expression

$$\Delta\rho \equiv \rho(T) - \rho_0 = \alpha\rho_0(T/T_D)^{3/2} \qquad (1)$$

within the experimental uncertainties. Here ρ is the resistivity, ρ_0 the resistivity at zero degrees Kelvin, and $\alpha = 0.95$ independently of the Sb concentration, as determined from the data of the figure.

The data for two As-doped samples, also shown in the figure, are fit by

$$\Delta\rho = \beta\rho_0(T/T_D)^2, \qquad (2)$$

where $\beta = 2.0$. For all samples, ρ_0 was obtained by plotting the experimental resistivity on linear graph paper versus $T^{3/2}$ for Sb doping and versus T^2 for As doping and extrapolating the resulting straight lines to 0°K.

The measurements were made in the apparatus described by Hall.[6] The carbon thermometer was the same; the calibration was rechecked at three temperatures. The data were obtained using two Leeds and Northrup potentiometers, a K-2 to measure the current, a K-3 for the voltage. The scatter in the data at the lower temperatures is due to thermal voltages of the order of 3 μV inherent in the measuring circuitry. The germanium samples have been extensively studied by Katz[3] in another connection. He obtained the carrier density from Hall-effect measurements taken at low temperatures while the samples are subjected to a large uniaxial strain. Under these conditions, the cubic symmetry is broken and the four Fermi-surface ellipsoids are no longer equivalent. Rather, one is lowered in energy sufficiently to contain all the carriers. Hall data taken under these conditions, so far as the Herring model[7] applies, is an exact measure of the carrier density for $T \ll T_D$. T_D was computed assuming the conduction band parabolic with the density-of-states mass taken as that of pure Ge.

There are two contributions to the temperature variation of the resistivity that vary as $(T/T_D)^2$. One[8] arises from the dependence on energy of the scattering of electrons by ionized impurities, the temperature dependence of the Fermi energy in the distribution function, and the temperature dependence of the screening of the ion fields by the electrons.

A second contribution to the temperature variation of resistivity is electron-electron scattering in which two electrons in different valleys scatter each other, each remaining in its original valley and conserving energy and momentum in the center-of-mass system. Because of the mass anisotropy, this process contributes appreciably to the resistivity.[9] As the electron and impurity densities are equal, the electron-electron contribution to the scattering should scale as ρ_0. Because both this electron-electron scattering and ionized-impurity scattering are screened Coulomb interactions, the matrix elements for scattering should be

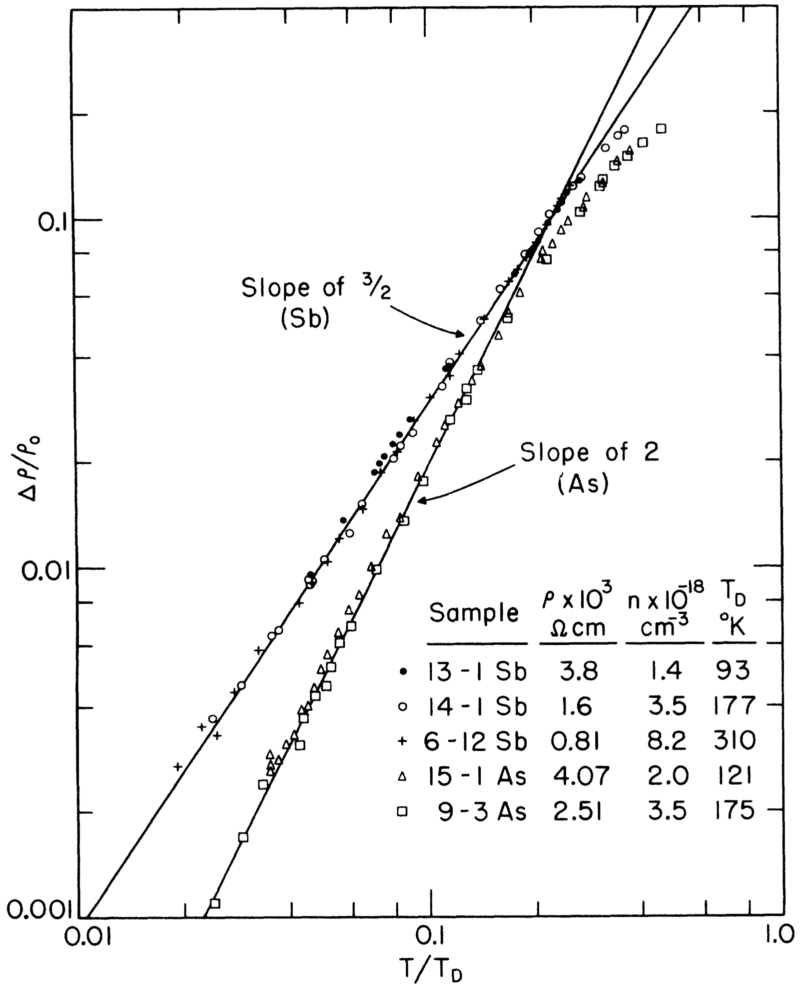

FIG. 1. Increment of resistivity normalized to the 0°K values versus temperature, in units of T_D, for several heavily doped germanium samples. The solid lines were drawn with the indicated slope to give the best fit to the data.

roughly equal; i.e., the electron-electron scattering contribution to the coefficient of $(T/T_D)^2$ should be of order unity.

The data for the As-doped samples are consistent with these mechanisms. The Sb-doped samples, however, exhibit a T dependence for which there is no explanation of which we are aware. To further clarify the experimental situation, it is clearly desirable to make similar measurements on samples with a range of compensating impurities so that the electron density can be varied independently of the density of scattering centers.

To calculate T_D from the Hall constant requires the assumption of a density-of-states mass. The fact that the curves in the figure are "universal" suggests that this mass is independent of doping; i.e., that the bands are parabolic over the range of donor concentration used.

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[8] We wish to thank J. B. Krieger for pointing out the nature of this T dependence to us.

[9] Essentially the identical mechanism has been discussed in detail for s-d scattering by W. C. Baber, Proc. Roy. Soc. (London) A158, 383 (1937). The application to multivalley semiconductors has been considered in some detail by P. J. Price (private communication); additional discussion is to be found in references 3 and 4 above. A similar T^2 dependence in very pure gallium has been reported and attributed to electron-electron scattering by M. Yaqub and J. F. Cochran, Phys. Rev. 137, A1182 (1965). A short discussion of this scattering also appears in D. Pines, Elementary Excitations in Solids (W. A. Benjamin, Inc., New York, 1963) p. 276.

SEARCH FOR INTERMEDIATE BOSONS IN PROTON-NUCLEON COLLISIONS*

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We have carried out a measurement of the yield of high-energy muons emitted at large angles to a beam of 20- and 30-BeV/c protons at the AGS. Our objective was to set an upper limit to the production and decay probability of massive unstable states decaying into muons. The most interesting candidate for such a state is the intermediate boson W, supposed to mediate weak interactions. It has been recognized[1] that one important signature for a heavy W is the large transverse momentum given to the muon in the decay process

$$W \to \mu + \nu. \qquad (1)$$

Recently, high-energy neutrino experiments have established that $m_W > 2$ BeV.[2,3] Thus, for a W of mass 2-6 BeV, the transverse momentum of the emitted muon can vary from 1 to 3 BeV/c. This is much larger than is typically found in secondary particles emerging from high-energy interactions. Furthermore, the conventional parents of muons, pions and kaons, can be largely "turned off" because of their relatively fast absorption by strong interactions in dense matter (~10 cm in tungsten) as compared to their mean free path (550 m at 10 BeV) for decay. Thus, the rate of muon counts observed at large angles, relatively easily reached by W decay, is an upper limit to $\sigma_W B$, where σ_W is the boson-production cross section for AGS protons (per nucleon), and B is the unknown partial rate for Reaction (1).

Figure 1 illustrates the experimental arrangement. The fast-extracted proton beam of the AGS was transported in a vacuum pipe up to the 82-ft steel shield of the Brookhaven National Laboratory (BNL) neutrino facility. An 18-in. block of Hevimet (90% tungsten) absorbed

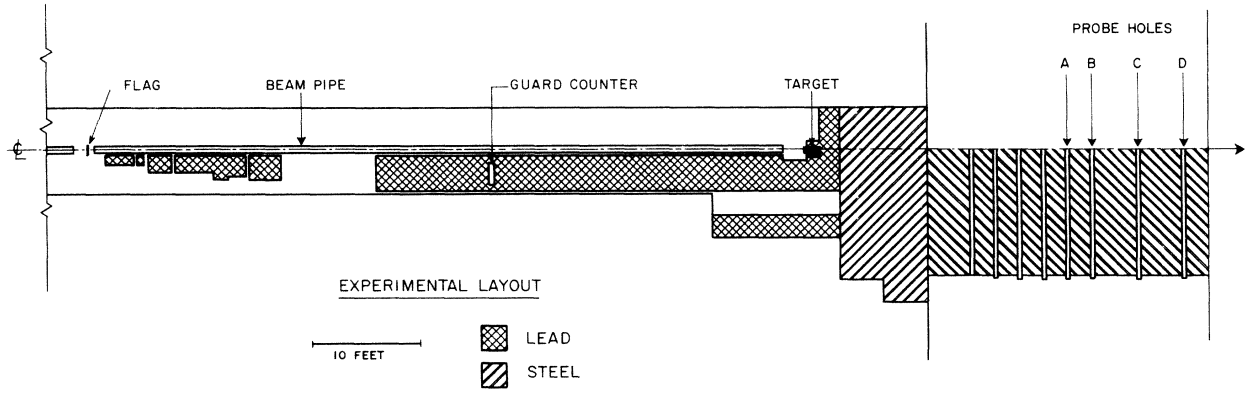

FIG. 1. Experimental apparatus.