considered isotropic, and the neutron exit channels were assumed to dominate in the total width Γ_J for compound-nucleus decay. The level densities D_J^{-1} of both the compound and final nucleus were determined from the statistical model with parameters from Gilbert and Cameron.[6] The width Γ_J was determined from the sum $2\pi\Gamma_J/D_J=\sum_f T_{lf}^n$ over the neutron transmission coefficients[7] T_{lf}^n to the final states f. Only allowed combinations of orbital angular momenta l, nucleon spin, and nuclear spins were included in the sums. The alpha-particle width was calculated from $2\pi\Gamma_J^\alpha/D_J=T_J^\alpha$, and optical-model determinations[7] were used for the capture cross sections σ_J^c.

The calculated width in the upper part of Fig. 2 was obtained by $\Gamma^{-1}=\sum_J\Gamma_J^{-1}\sigma_J/\sum_J\sigma_J$, which approximately represents the resultant width observed from fluctuations. When consideration is taken of the approximate nature of the statistical model, the omission of charged-particle exit channels, and the neglected corrections[8] for large transmission coefficients in the Γ_J/D_J determination, the agreement between measured and calculated widths could be somewhat different.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]T. Ericson, Phys. Letters 4, 258 (1963); and D. M. Brink and R. O. Stephen, Phys. Letters 5, 77 (1963).

[2]G. Dearnaley, W. R. Gibbs, R. B. Leachman, and P. C. Rogers, Phys. Rev. 139, B1170 (1965). This reference cites earlier measurements.

[3]J. Bondorf and R. B. Leachman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 34, No. 10 (1965).

[4]W. R. Gibbs, Phys. Rev. 139, 1185 (1965).

[5]W. R. Gibbs, Los Alamos Scientific Laboratory Report No. LA 3266, 1965 (unpublished; available from Clearing House for Federal Scientific and Technical Information, National Bureau of Standards, U. S. Department of Commerce, Springfield, Virginia).

[6]A. Gilbert and A. G. W. Cameron, Can. J. Phys. 43, 1446 (1965).

[7]Transmission coefficients are from J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley & Sons, Inc., New York, 1952), p. 361, for neutrons and from optical-model calculations with $V=-50$ MeV, $W=-5$ MeV, and $a=0.5$ F for protons ($R=5.5$ F) and alpha particles ($R=7.23$ F).

[8]P. A. Moldauer, Phys. Rev. 129, 754 (1963); and T. J. Krieger and C. E. Porter, J. Math. Phys. 4, 1272 (1963).

OFF-SHELL EQUATIONS FOR TWO-PARTICLE SCATTERING*

K. L. Kowalski

Department of Physics, Case Institute of Technology, Cleveland, Ohio

(Received 15 October 1965)

Quite recently a representation for the two-particle partial-wave (potential) scattering amplitudes, both off and on the energy shell, was described in terms of a solution of a manifestly nonsingular integral equation.[1] Besides exhibiting desirable qualities with respect to unitarity and threshold behavior in connection with the on-shell amplitudes, it was also pointed out[1] that certain features of this representation suggest a separable approximation to the off-shell amplitudes which permits a simplification of the Faddeev equations.[2] In the present note we first wish to establish the equivalence of the results of reference 1 to those of a previous, apparently different, analysis.[3] This, we feel, may make manifest the mathematical implications of this representation. Then using the methods of reference 3 we will determine an explicit general form for the nonseparable part of the off-shell amplitude which should be useful in estimating the magnitude of this term. Some comments on off-shell unitarity[4] are also made.

In order to include the equations employed by Lovelace,[4] it will be convenient to consider first the integral equation

$$K(p,k)=V(p,k)+\frac{2}{\pi}\mathrm{P}\int_0^\infty dq\frac{q^2}{k^2-q^2}V(p,q)K(q,k), \tag{1}$$

which is satisfied by the partial-wave amplitudes, $K(p,k)$, of the K matrix.[3,5] Equation (1) can be reduced to a Fredholm form by setting $p=k$ in (1), multiplying this by $\tau(p,k)=V(p,k)/V(k,k)$,[6] and subtracting the resultant expression from (1) to obtain

$$K(p,k)=\tau(p,k)K(k,k)+\int_0^\infty dq\,\Lambda(p,q)K(q,k), \tag{2}$$

with

$$\Lambda(p,q)=(2/\pi)[V(p,q)-\tau(p,k)V(k,q)](k^2-q^2)^{-1}.$$

The solution of the Fredholm equation (2) can be expressed as

$$K(p,k)=f(p,k)K(k,k), \tag{3}$$

where $f(p,k)$ satisfies (2) with an inhomogeneous term $\tau(p,k)$. If the expression (3) is inserted under the integral sign in (1) and then p is set equal to k in the over-all expression, one obtains an explicit form for $K(k,k)$ in terms of $f(p,k)$. Also, since $(q-k)\Lambda(p,q)=0$ for $q=k$, it is clear that the same analysis holds with $(k^2-q^2)^{-1}$ replaced by $(k^2-q^2+i\epsilon)^{-1}$ in Eq. (1), and so we have for the T matrix

$$T(p,k)=f(p,k)T(k,k), \tag{4}$$

where again the on-shell amplitude, $T(k,k)$, is expressed in terms of $f(p,k)$.[1,3] If we recognize that the partial wave, $a_k(p)$, of the scattered wave in momentum space is $T(p,k)(k^2-p^2+i\epsilon)^{-1}$, then we recover with Eq. (4) the formulation of reference 1.

Instead of following reference 1 in considering the complete off-shell amplitude, $T_k(p,p')$, which satisfies

$$T_k(p,p')=V(p,p')+\frac{2}{\pi}\int_0^\infty dq\frac{q^2}{k^2-q^2+i\epsilon}V(p,q)T_k(q,p'), \tag{5}$$

we again perform a Fredholm reduction on (5) to obtain[7]

$$T_k(p,p')=\tau(p,k)T_k(p',k)+[V(p,p')-\tau(p,k)V(k,p')]+\int_0^\infty dq\,\Lambda(p,q)T_k(q,p').$$

It is clear then that with the aid of (4) we obtain

$$T_k(p,p')=f(p,k)T(k,k)f(p',k)+[f(p,p')-f(p,k)\tau(p',k)]V(k,k), \tag{6}$$

where $f(p,p')$ satisfies (2) with an inhomogeneous term $V(p,p')/V(k,k)$. The nonseparable term on the right side of (6) can be reduced to a concise form if we note that the resolvent kernel, $\mathcal{R}_k(p,q)$, corresponding to $\Lambda(p,q)$, satisfies

$$\begin{aligned}\mathcal{R}_k(p,q)&=\Lambda(p,q)+\int_0^\infty dp'\,\Lambda(p,p')\mathcal{R}_k(p',q)\\&=\Lambda(p,q)+\int_0^\infty dp'\,\mathcal{R}_k(p,p')\Lambda(p',p).\end{aligned}$$

One easily finds then that

$$T_k(p,p')=f(p,k)T(k,k)f(p',k)+(\tfrac{1}{2}\pi)(k^2-p'^2)\mathcal{R}_k(p,p'). \tag{7}$$

We note that

$$(k^2-p'^2)\mathcal{R}_k(p,p')=(k^2-p^2)\mathcal{R}_k(p',p),$$

and that $\mathcal{R}_k(k,p)=0$ for all p, which shows explicitly that the nonseparable term vanishes when either p or p' is equal to k and also explains the apparent lack of symmetry of this term. Finally, we remark that the representation (7) is valid whether or not the interaction permits bound states, in contrast to the treatment of reference 1.

Since f and $\mathcal{R}_k$ are real, it is easily shown that the amplitude given by (7) satisfies the off-shell unitarity relation[4] for any approximation to f which satisfies $f(k,k)=1$ and for any (real) $\mathcal{R}_k$ in the nonseparable term. In particular, if we neglect the nonseparable term, i.e., we take $\mathcal{R}_k=0$, then we are still left with a unitary amplitude even off the energy shell.

The validity of the approximation of setting $\mathcal{R}_k=0$ in the nonseparable term is unknown at present. However, in calculating $f(p,k)$, $\mathcal{R}_k=0$ is a fair approximation for typical potentials, and in the calculation of the scattering amplitude $\mathcal{R}_k=0$ is precisely equivalent to assuming a plane-wave trial function in the Schwinger variational expression for $T(k,k)$.[3] Some separable approximations to $\mathcal{R}_k$ even when the potential is not separable have been discussed elsewhere.[3]

We should like to thank Professor H. P. Noyes for several communications concerning these questions.

---

*This work was supported in part by the U. S. Atomic Energy Commission.

[1]H. P. Noyes, Phys. Rev. Letters 15, 538 (1965).

[2]L. I. Faddeev, Zh. Eksperim. i Teor. Fiz. 39, 1459 (1960) [translation: Soviet Phys.-JETP 12, 1014 (1961)].

[3]K. L. Kowalski and D. Feldman, J. Math. Phys. 4, 507 (1963). See also K. L. Kowalski and D. Feldman, J. Math. Phys. 2, 499 (1961); Phys. Rev. 130, 276 (1963).

[4]C. Lovelace, Phys. Rev. 135, B1225 (1964).

[5]We consider only the single-channel case for simplicity. The extension to the multichannel case is straightforward (see reference 3).

[6]We assume, of course, that the partial-wave amplitude, $V(k,k)$, of the potential is nonzero. See references 1 and 3 for a discussion of the cases when this is violated.

[7]We have assumed that $V(p,q)=V(q,p)$, which implies that $T_k(p',p)=T_k(p,p')$. Reference 3 includes a complete discussion of the unsymmetrical case.

SEARCH FOR INTERMEDIATE VECTOR BOSON PRODUCTION IN NUCLEON-NUCLEON COLLISIONS*

R. C. Lamb, R. A. Lundy, T. B. Novey, and D. D. Yovanovitch

Argonne National Laboratory, Argonne, Illinois

and

M. L. Good, R. Hartung, M. W. Peters, and A. Subramanian†

University of Wisconsin, Madison, Wisconsin

(Received 20 September 1965)

Neutrino experiments carried out at CERN[1] and BNL[2] have indicated that the mass of the intermediate vector boson, the W, must be greater than 2 BeV. This lower limit is essentially set by the low yield of high-energy neutrinos available from present accelerators. It appears, however, feasible to search for the W produced in nucleon-nucleon interactions. This process can yield W's of mass up to 3.3 BeV with the proton energy available at the Argonne zero-gradient synchrotron (ZGS). Recent theoretical calculations[3] indicate that the cross section for W production is of the order of 10^{-6} of geometric. This cross section taken with a branching ratio of $\frac{1}{3}$ to the channel $W \to \mu + \nu$ gives a flux of muons at large momentum and angle (4-6 BeV/c at ~20° lab angle) which is significantly above the unavoidable background of muons from other processes. We have conducted an experiment to detect W production in nucleon-nucleon interactions which is sensitive to W masses in the range of 2 to 3 BeV. We find no muon signal in excess of that due to π and K decay.

The experimental apparatus used is shown schematically in Fig. 1. A proton beam of 12.5-

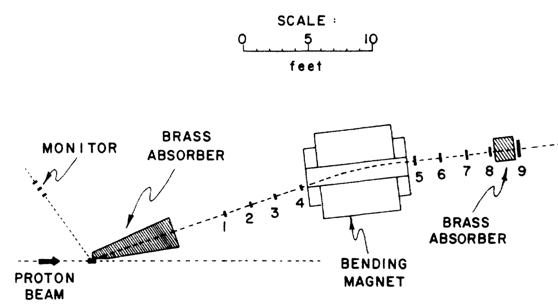

FIG. 1. Layout of experimental apparatus. Numbers 1 through 9 indicate plastic scintillator counters. Counters and the target are not to scale.

BeV/c momentum (2×10^9/pulse) was incident on a 3-in.-long uranium target. In this parasite extracted proton beam, about $\frac{1}{2}$% of the circulating beam of the ZGS is spilled during the 250-msec flat top. The beam spot, 1 cm in diameter, was continuously observed by a closed circuit TV-scintillator system. The incident proton intensity was calibrated by gold-foil activation and monitored throughout the run by a triple telescope located at 150° production angle. A magnetic spectrometer viewed the target at a lab angle of 20°. This