MEASUREMENT OF 0.1-keV NUCLEAR-STATE WIDTHS IN THE CONTINUUM BY FLUCTUATION AVERAGING*

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Measurements of Ericson fluctuations[1] in the cross section have enabled determinations of the average widths Γ of nuclear states when the states are broadly overlapping.[2] However, these measurements require the energy spread (resolution ρ) in the bombardments to be less than Γ, and so practical considerations have limited measurements to the $\Gamma \geqslant 5$ keV of medium-light nuclei at excitation energies above 15 MeV. We report here a method of fluctuation averaging that allows the measurement of widths that are orders-of-magnitude smaller. In these first measurements, 0.1-keV widths of the compound nucleus Zr^{90} have been measured between 17- and 21-MeV excitation energies.

These widths were determined from measurements of the amount that the fluctuations in the cross section are reduced by fluctuation averaging, which results from the resolution significantly exceeding the average width. To develop the effects of fluctuation averaging quantitatively, we first consider other effects that decrease the fluctuation. For conditions of good resolution, $\rho < \Gamma$, the fluctuation in terms of the autocorrelation function $R \equiv (\langle\sigma^2\rangle - \langle\sigma\rangle^2)/\langle\sigma\rangle^2$, which is simply the normalized variance of the cross section σ for the over-all sample, is

$$R = (1-y^2)/N_{\text{eff}}. \tag{1}$$

Here, the fluctuation damping factor N_{eff} is the effective number of m states, and y is the fraction of direct reactions.

The reaction $Y^{89}(p, \alpha_0)Sr^{86}$ was used in the present measurements of differential cross sections $\sigma(\theta)$ at angles $\theta = 51°$ and $129°$ to provide a low, well-established[2,3] $N_{\text{eff}} = 2$ from the $\frac{1}{2}$ spins of Y^{89} and the proton. Energies used were near or below the Coulomb barrier of the exit alpha particles, and the fraction y of direct reactions was then either negligible or a small quantity determined by other means. If the resolution condition $\rho < \Gamma$ of Eq. (1) were achievable with present technology, large fluctuations resulting in $R = 0.5$ from Eq. (1) would be observed for $y = 0$. Actual resolution conditions result in the autocorrelation function R being further reduced by fluctuation averaging. However, the statistical knowledge[2,4,5] of the biases and uncertainties in autocorrelation functions now enables analyses of fluctuations from realizable experiments that are more than an order of magnitude smaller than $R = 0.5$.

The decrease in fluctuations from fluctuation averaging results from each cross-section measurement being an average over a cross section that fluctuates within the energy resolution of each bombardment. To determine the width Γ from fluctuation averaging requires a statistical knowledge of how rapidly in terms of Γ the cross section changes with energy. This can be expressed in terms of the effective number[2,4,5] n of independent energies contributing to a single measurement. For a rectangular resolution function,

$$n \approx (\rho/\pi\Gamma) + 1. \tag{2}$$

With each of these n independent energies decreasing the fluctuations through the averaging process, the autocorrelation is further reduced[5] by the factor n^{-1} to

$$R \approx \frac{(1-y^2)}{(\rho/\pi\Gamma + 1)N_{\text{eff}}}. \tag{3}$$

The width Γ can now be determined from Eq. (3) by a measurement of the autocorrelation R for conditions where N_{eff}, y, and ρ are known.

The excitation functions shown in Fig. 1 resulted from resolutions which were typically $\rho = 6.0$ keV, resulting largely from the 215-μg/cm^2 target thickness of Y^{89}. In the combined resolution, the small energy spread of protons from the Los Alamos FN tandem Van de Graaff had minor importance. Batteries of transmission-type, semiconductor counters were used to detect alpha particles to the ground state of Sr^{86}. An actual value of the autocorrelation from these measurements, after bias corrections, was $R = 0.026$, which for $N_{\text{eff}} = 2$ and $y = 0$ in Eq. (3) results from a fluctuation averaging over $n \approx (1-y^2)/N_{\text{eff}}R = 19$ independent energies within each bombardment span ρ of

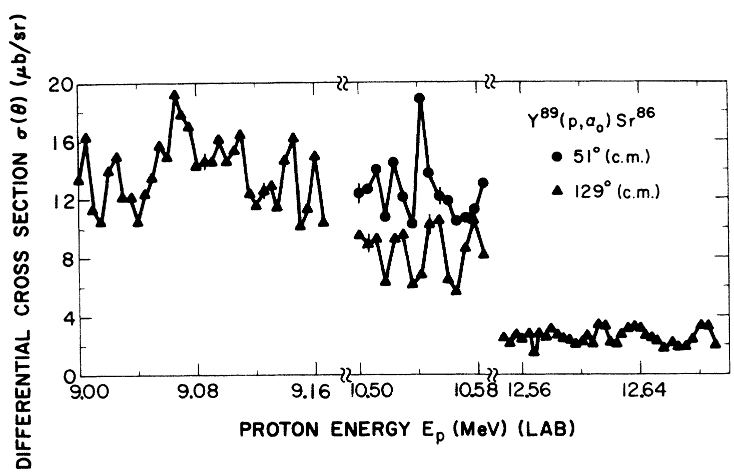

FIG. 1. Fluctuations in the $Y^{89}(p,\alpha_0)Sr^{86}$ cross section. Measurements were in 4-, 5-, or 6-keV energy steps with energy resolutions ρ typically equal to 6.0 keV.

energy. Use of $n\approx 19$ and the known $\rho = 6.0$ keV in Eq. (2) results in an average width of $\Gamma\approx\rho/(n-1)\pi = 0.11$ keV. (In the actual analyses, a small correction was made for the nonrectangular resolution functions encountered.)

The measured widths in the upper part of Fig. 2 involved appreciable fractions y of direct reactions only for $\theta = 51°$ at 10.6-MeV proton energy ($y = 0.37$) and $\theta = 129°$ at 12.6 MeV ($y = 0.34$). These were determined from the lower part of Fig. 2 by $y = 1-[\sigma(\theta)/\langle\sigma(\theta)\rangle]$, where $\sigma(\theta)$ is the calculated compound-nucleus cross section normalized at 9-MeV proton energy to the measured cross section $\langle\sigma(\theta)\rangle$, in which the fluctuations have been further reduced by the use of a large resolution (15 to 210 keV). Two arguments assure that the measured cross sections in the region of the peak at the 9-MeV normalization energy are dominantly from compound-nucleus reactions. First, the cross section exhibits fore-aft symmetry at the angles measured (and in detailed angular distributions), and second, compound-nucleus reactions are expected to produce a peak in this region whereas direct reactions are not.

The compound-nucleus cross section for each proton energy was calculated by the Hauser-Feshbach method,

$$\sigma(\theta)\approx(4\pi)^{-1}\sum_J \sigma_J^{\,c}\,\Gamma_J^{\,\alpha}/\Gamma_J, \qquad (4)$$

where J is the compound-nucleus spin, $\sigma_J^{\,c}$ is the capture cross section, and $\Gamma_J^{\,\alpha}$ is the width for alpha decay to the ground state. In the present calculation the angular distribution was

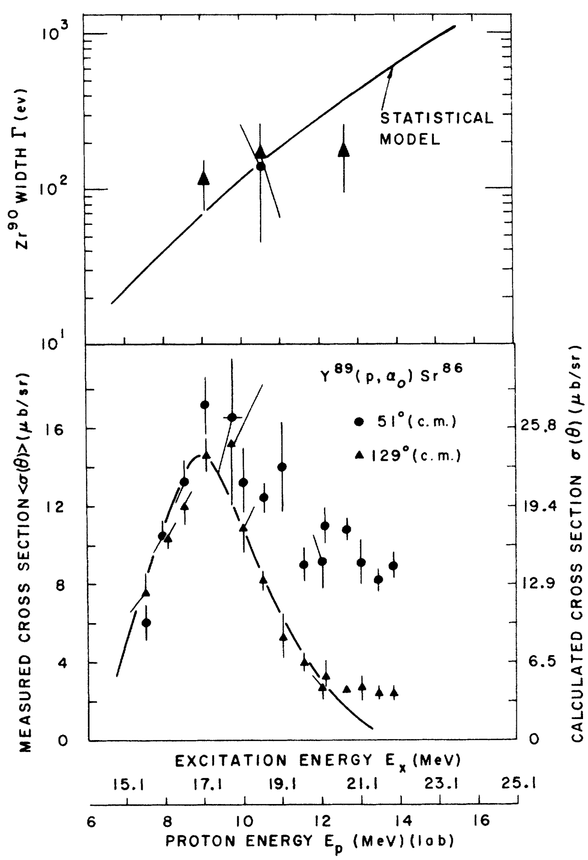

FIG. 2. Determination of Zr^{90} widths. The lower figure shows the gross excitation function measured with large energy resolutions. Uncertainties include both measurement uncertainties and cross-section fluctuations. Calculations of this excitation function and the width (upper figure) are described in the text. The measured widths in the upper figure are based on the fluctuations in Fig. 1 and on fractions of direct reactions determined from comparisons of measured and calculated excitation functions in the lower figure.

considered isotropic, and the neutron exit channels were assumed to dominate in the total width Γ_J for compound-nucleus decay. The level densities D_J^{-1} of both the compound and final nucleus were determined from the statistical model with parameters from Gilbert and Cameron.[6] The width Γ_J was determined from the sum $2\pi\Gamma_J/D_J=\sum_f T_{lf}^{\,n}$ over the neutron transmission coefficients[7] $T_{lf}^{\,n}$ to the final states f. Only allowed combinations of orbital angular momenta l, nucleon spin, and nuclear spins were included in the sums. The alpha-particle width was calculated from $2\pi\Gamma_J^{\,\alpha}/D_J=T_J^{\,\alpha}$, and optical-model determinations[7] were used for the capture cross sections $\sigma_J^{\,c}$.

The calculated width in the upper part of Fig. 2 was obtained by $\Gamma^{-1}=\sum_J\Gamma_J^{-1}\sigma_J/\sum_J\sigma_J$, which approximately represents the resultant width observed from fluctuations. When consideration is taken of the approximate nature of the statistical model, the omission of charged-particle exit channels, and the neglected corrections[8] for large transmission coefficients in the Γ_J/D_J determination, the agreement between measured and calculated widths could be somewhat different.

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OFF-SHELL EQUATIONS FOR TWO-PARTICLE SCATTERING*

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Quite recently a representation for the two-particle partial-wave (potential) scattering amplitudes, both off and on the energy shell, was described in terms of a solution of a manifestly nonsingular integral equation.[1] Besides exhibiting desirable qualities with respect to unitarity and threshold behavior in connection with the on-shell amplitudes, it was also pointed out[1] that certain features of this representation suggest a separable approximation to the off-shell amplitudes which permits a simplification of the Faddeev equations.[2] In the present note we first wish to establish the equivalence of the results of reference 1 to those of a previous, apparently different, analysis.[3] This, we feel, may make manifest the mathematical implications of this representation. Then using the methods of reference 3 we will determine an explicit general form for the nonseparable part of the off-shell amplitude which should be useful in estimating the magnitude of this term. Some comments on off-shell unitarity[4] are also made.

In order to include the equations employed by Lovelace,[4] it will be convenient to consider first the integral equation

$$K(p,k)=V(p,k)+\frac{2}{\pi}\mathrm{P}\int_0^\infty dq\frac{q^2}{k^2-q^2}V(p,q)K(q,k),\tag{1}$$

which is satisfied by the partial-wave amplitudes, $K(p,k)$, of the K matrix.[3,5] Equation (1) can be reduced to a Fredholm form by setting $p=k$ in (1), multiplying this by $\tau(p,k)=V(p,k)/V(k,k)$,[6] and subtracting the resultant expression from (1) to obtain

$$K(p,k)=\tau(p,k)K(k,k)+\int_0^\infty dq\,\Lambda(p,q)K(q,k),\tag{2}$$