

the experimental fact that the fundamental AKCR invariably appears at a slightly higher magnetic field value than is consistent with the subharmonic resonances.

When the magnetic field is rotated 90° to make $\vec{J} \perp \vec{H}$, the strong oscillations of dA/dH on the high-field side of the fundamental resonance are suppressed but similar, though less clear, variations are observed on the high-field side of the second-harmonic resonance ($\omega_c > \omega/2$). The dispersion relation for this polarization is

$$k^2/k_0^2 = \epsilon_{yy} + \epsilon_{xy}^2/\epsilon_{xx} \quad (6)$$

and corresponds to waves of mixed polarization (E partly longitudinal). In the long-wavelength limit it is possible to show that three distinct modes exist, propagation commencing at the zero in the magnetoconductivity which now occurs at the second cyclotron harmonic in agreement with experiment. Detailed comparison of the theory with experiment proves difficult because of our ignorance of the relative amplitudes of excitation of the various modes, but the qualitative features of the data do appear to be reproduced by the simple model. These modes are of particular interest as they are quite analogous to the longitudinal modes which propagate near cyclotron-resonance harmonics in inhomogeneous gaseous plasma columns.⁹

Though we have not, as yet, attempted to generalize the analysis to arbitrary Fermi surfaces, it is likely that propagation of long-wavelength excitations may occur near cyclotron resonance in other metals. The "geometric

cyclotron resonance" transmission signals observed by Schultz and Latham¹⁰ in thin metal plates may, in fact, result from such magnetically induced transparency.

The high-purity potassium required for these experiments was prepared by P. H. Schmidt. We wish to thank L. W. Rupp, Jr., for able technical assistance. Discussions with Dr. S. J. Buchsbaum and Dr. C. C. Grimes have been most stimulating.

¹For a complete list of references, see the articles by S. J. Buchsbaum and by R. Bowers, in Proceedings of the Symposium on Plasma Effects in Solids, Paris, 1964 (Academic Press, Inc., New York, 1965).

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LOCALIZED IMPURITY STATES IN METALS: EVALUATION OF $(U + 4J)^*$

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It is well known that when iron-group transition metal atoms are dissolved as dilute impurities into metals, the impurity atom does or does not possess a magnetic moment depending on the properties of the host metal.¹ To explain these phenomena Anderson² has examined the effect of s - d mixing on the localized states of iron-group atoms in metals in terms of a mod-

el based on the Friedel idea³ of virtual bound states for the solute atoms in such dilute alloys. In an attempt to make a comparison of experiment with the predictions of the Anderson theory, we present here the principal results of a study of the susceptibility and specific heat of dilute alloys of nickel in beryllium. The experimental results indicate that

there is no localized moment on the Ni; however, the addition of Ni gives rise to a relatively large change in both the temperature-independent susceptibility and the linear term in the specific heat. These results are in agreement with the Anderson theory which predicts a large enhancement of the local susceptibility when the impurity state is close to the critical condition for the formation of a moment. Comparison of the susceptibility and specific-heat results leads directly to a value for the fundamental quantity $U + 4J \approx 7$ eV, where U and J are the Coulomb and exchange integrals between two d orbitals on the same atom in the metal environment. Analysis of earlier data⁴ on Cu:Ni gives a similar value for $U + 4J$.

Assuming the Anderson model, the specific heat of a dilute alloy in the nonmagnetic case can be shown to be^{5,6}

$$C = \frac{\pi^2}{3} k^2 T (\rho_H + c\rho_d) + C_{\text{lattice}},$$

where k is the Boltzmann constant, ρ_H is the host metal density of states, c is the transition atom impurity concentration, and ρ_d is the impurity d -state density,

$$\rho_d = \frac{10}{\pi} \frac{\Delta}{(E_F - E_d)^2 + \Delta^2}.$$

In the above a five-fold orbital and two-fold spin degeneracy is assumed; E_d and Δ are, respectively, the energy and width of the virtual d levels, and E_F is the Fermi energy. Although the main body of Anderson's paper is concerned with an idealized nondegenerate orbital, the entire theory is easily generalized to the more realistic degenerate case as in the appendix to the paper. The assumption of complete orbital degeneracy requires that any crystal-field splitting be less than the virtual level width and is in agreement with experiment.⁷ Again in terms of the Anderson model, the temperature-independent susceptibility may be written

$$\chi = \chi_H + c \frac{10\mu^2}{10/\rho_d - (U + 4J)} = \chi_H + c\eta\mu^2\rho_d,$$

where η is the enhancement factor, and χ_H is the host-metal susceptibility. Note that the Anderson criterion for formation of a magnetic state is equivalent to requiring that the denominator vanish, at which point the zero-temperature susceptibility diverges as expected

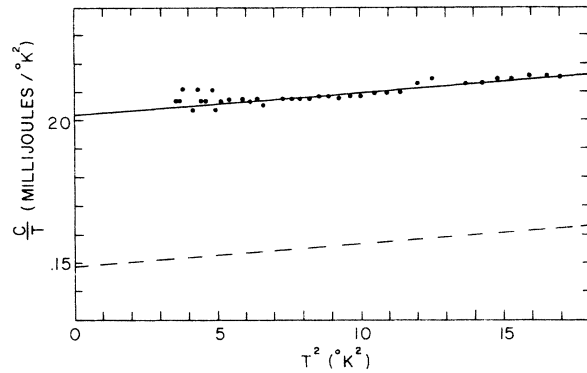


FIG. 1. Specific heat as a function of temperature for a 6.69-g sample of 2.2-at.% Be:Ni alloy. The dashed line indicates the specific heat of the same number of atoms of pure Be.

on physical grounds.

Beryllium was chosen as the host metal since it should conform to the fundamental theoretical assumption that the host-metal electron wave functions are not d -like; and because of the very small density of states at the Fermi level. Small concentrations of Ni are readily soluble in solid Be.⁸ The samples were prepared by arc melting, and were subsequently given a homogenizing anneal for 24 hours at 1050°C followed by a fast quench. Metallographic and x-ray studies showed no evidence of precipitation of any kind while chemical analysis showed the expected amounts of Ni to be present. The susceptibility was measured by the Faraday technique using a calibrated inhomogeneous field and a sensitive electronic balance. The specific-heat measurements were taken by the usual continuous heating technique in the temperature range from 1.8 to 4.2°K. Details will be published elsewhere.⁶ Except for a very small temperature dependence characteristic of pure Be,⁹ the alloy susceptibilities were independent of temperature. The specific-heat results show the expected electronic contribution (linear in T) and lattice contribution (T^3) as shown for the most concentrated alloy in Fig. 1. Examination of the figure indicates that no significant change can be detected in the very small coefficient of the T^3 term¹⁰ as compared with pure Be. The increased error at the lowest temperatures is the result of a finite uncompensated heat leak of about 0.5 erg per second. As seen from the small scatter of the points in Fig. 1, the coefficient of the linear term is obtained with an accuracy of somewhat better than 2%.

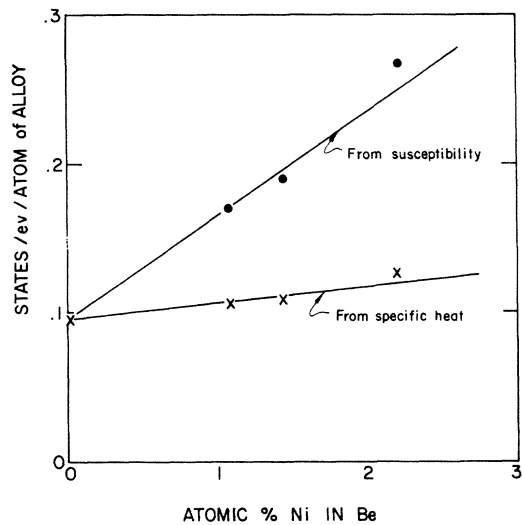


FIG. 2. True density of states as measured by specific heat and apparent density of states as measured by susceptibility for dilute alloys of Ni in Be.

In Figure 2 we show the essential experimental results. Both χ_d/μ^2 from susceptibility measurement and ρ_d as determined directly from the specific-heat data are given as a function of impurity concentration for dilute solutions of Ni in Be. A diamagnetic correction has been made to the susceptibility such as to give the true density of states for pure Be.¹⁰ The expected enhancement of the impurity contribution to the susceptibility is clearly evident. From Fig. 2, assuming the Anderson model, one finds

$$\eta^{-1} = 1 - (U + 4J)\rho_d/10 = 0.156,$$

with $\rho_d = 1.15$ states per eV per atom of Ni. Combining these results we obtain

$$U + 4J \approx 7.3 \text{ eV.}$$

The limits of error on the above value for $U + 4J$ seem to be set by the homogeneity of the samples themselves rather than by the susceptibility and specific-heat measurements as indicated by the deviations of the experimental points from the indicated straight lines. However, the above value appears to be accurate to about 10%; or in the range 6 to 8 eV. A similar study of the system Cu:Ni was made some time ago.⁴ Although it is difficult to estimate ρ_d because of lack of data at concentrations below 10% Ni in Cu, analysis of the existing data again yields a value in the same range for

$U + 4J$.

The resulting value of $U + 4J$ is to be compared with a free-atom value of more than 20 eV. However, a significant reduction in the metallic environment is not surprising. In particular, there are two obvious mechanisms by which the Coulomb and exchange integrals may be reduced in the metal; firstly, simple screening by the nearby conduction electrons, and secondly, many-body correlation effects. Although a detailed estimate is difficult, a reduction by a factor of three to the observed value due to screening alone does not seem unreasonable. On the other hand, calculations which take two-body correlation effects into account in the localized moment theory¹¹ as well as the theory of ferromagnetic metals¹² predict a somewhat greater reduction of the effective U and apparently overestimate the correlation effects.

In conclusion, we have found the results of the present study to be consistent with the Anderson model of localized transition atom impurity states in metals. The interpretation of the experimental results in terms of this model leads to a value of the quantity $U + 4J \approx 7 \text{ eV}$.

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PARAMAGNETIC RESONANT ABSORPTION OF MICROWAVE PHONONS IN ZERO FIELD

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We have observed an intense, magnetic-field-dependent interaction between 10-kMc/sec longitudinal acoustic waves and Dy^{3+} ions in CaF_2 . In a sample containing 0.16% Dy ions, ultrasonic absorption was maximum at or near $H_0 = 0$, and it was observed to decrease with increasing magnetic field until reaching a field-independent

value at $H_0 \approx 4000$ Oe (Fig. 1). The integrated absorption, when adjusted for concentration differences, is comparable to the stronger but narrower banded absorption reported by Shiren¹ for Fe^{2+} in MgO and its peak value, $\alpha_m = 10.5$ dB/cm, is in sharp contrast to the weak interactions, $\alpha_m \leq 0.1$ dB/cm, reported² for similar

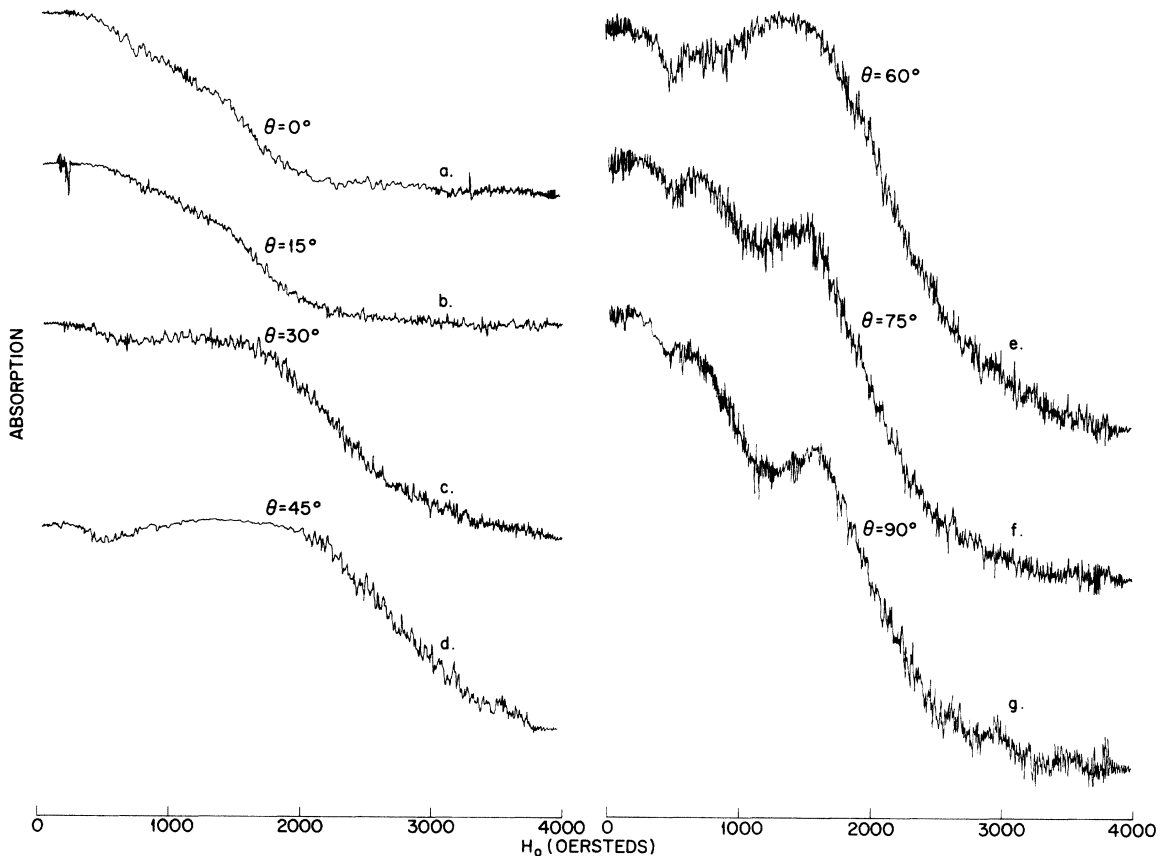


FIG. 1. Absorption of longitudinal 10-kMc/sec phonons by Dy^{3+} in CaF_2 in absence of saturation versus magnetic field. θ is the angle between H_0 and propagation direction along [100] axis. Each curve was calibrated separately, since gain conditions varied. The absorption at $H_0 = 0$ for all curves is 10.5 dB/cm.