average fission width of the $J=0$ resonances is 403 mV, while for the $J=1$ resonances the average fission width is 41.7 mV. Thus our experiment provides direct evidence that the smaller spin is associated with the larger average fission width in accordance with the theoretical prediction for Pu^{239}.

Of our 15 spin assignments, 12 are $J=1$ and three are $J=0$. This ratio of four is consistent with the ratio of three expected from a $2J+1$ dependence for the level density.

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STATIC QUADRUPOLE MOMENT OF THE FIRST EXCITED 2^+ STATE OF ^{114}Cd†

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Recently an experimental value of the static quadrupole moment, Q_2, of the first excited (2_1^+) state of ^{114}Cd was reported by several groups of authors.[1-3] The reported values of Q_2 vary somewhat but all seem to be included in the range $Q_2 = -(0.5 \pm 0.25)$ b. This strikingly large magnitude of Q_2 made the authors of reference 1 infer that ^{114}Cd is a rotational (permanently deformed) nucleus, rather than a vibrational nucleus as has so far been believed.[4]

Indeed, $Q_2 = 0$ for a vibrational model of pure harmonic nature,[5] and this model is in contradiction with experiment. However, many known experimental data[4] (other than Q_2), such as energy levels, $B(E2)$ values, and so on, indicate that ^{114}Cd is a rather typical, though not purely harmonic, vibrational nucleus. These data are very difficult to be understood if ^{114}Cd is in fact a rotational nucleus, but are fairly satisfactorily explained by models[6,7] which assume it to be a vibrational nucleus with some anharmonicity being allowed.

The purpose of the present article is to show that it is not impossible to predict a large Q_2 value within the framework of the vibrational model. Before showing this, however, we shall list the theoretical values of Q_2 calculated by using the models so far proposed by various authors.

(i) Harmonic vibrational model[5]: This model gives $Q_2 = 0$. See above.

(ii) Shell model[8]: The dominant proton configuration will be $(g_{9/2})^{-2}$, which with[9] $e_{\text{eff}} = 0$ gives[10] $Q_2 = -0.10$ b. The magnitude is too small. Inclusion of other configurations, such as $(g_{9/2})^{-4} \times (g_{7/2})^2$, will further decrease the magnitude.

More complicated configurations and nonzero e_{eff} may be discussed more conveniently in terms of the random phase approximation [see (vii) below].

(iii) Rotational model[5]: This model gives[11] $Q_2 = -0.70$ b in agreement with experiment, but it cannot be taken seriously; see above.

(iv) Wilets-Jean model[12]: This model superposes a rotational character upon the harmonic vibrational model. Nevertheless, it predicts $Q_2 = 0$.

(v) Tamura-Komai model[6]: This model emphasizes the superposition of the rotational character upon the harmonic virbational model even more than (iv) does. It thus gives a nonvanishing value, $Q_2 = 0.003$ b, which, however, is of opposite sign and too small in magnitude.

(vi) Davidov-Filippov model[13]: From the known level structure[4] of ^{114}Cd, the parameter[13] γ_0 is fixed to be 26.75°, which leads to $Q_2 = -0.32$ b. The sign is correct and the magnitude is within the limits of the above experimental value.

(vii) Random phase approximation (RPA)[14]: Using the wave function[14] that gives the energy and collectivity (with $e_{\text{eff}} = 0.83e$) of the 2_1^+ state in agreement with experiment, we get $Q_2 = -0.077$ b. The sign is correct but the magnitude is very small.

(viii) Higher random phase approximation (HRPA)[7]: Using the wave function obtained in reference 7, we get $Q_2 = -0.082$ b, which is of slightly larger magnitude than that in (vii), but is still too small.

The above list shows that none of the vibrational models so far proposed[15] can give satisfactorily large Q_2 [except perhaps (vi), whose validity may, however, be questioned[16]]. In spite of this situation, we still feel that the problem is to be solved within the framework of the vibrational model and thus consider a very simplified model which follows.

(A) We assume that the wave functions $\psi(2_1^+)$ and $\psi(2_2^+)$ of the first and second excited 2^+ states are written as linear combinations of the one- and two-phonon harmonic vibrational states, $|1\rangle$ and $|2\rangle$: $\psi(2_1^+) = a_1|1\rangle + a_2|2\rangle$ and $\psi(2_2^+) = -a_2|1\rangle + a_1|2\rangle$, with $a_1^2 + a_2^2 = 1$. We then consider the ratio $R_1 = B(E2; 2_2^+ \to 2_1^+)/B(E2; 2_1^+ \to \text{ground})$, which with the above ψ's becomes $R_1 = 2(2a_1^2-1)^2/a_1^2$. The experimental value[4] of $R_1(=1.2)$ gives $a_1^2 = 0.86$. (With the harmonic model $a_1^2 = 1$ and $R_1 = 2$.) In this model $Q_2 = (12/5)(7\pi)^{-1/2}a_1a_2ZR_0^2\beta$ ($Z = 48$ for Cd); and with the above value of a_1^2 we get $Q_2 = \pm 0.58$ b, which is sufficiently large.[17-19]

In the light of the successful result of (A), we may now say that the failure of (vii) in giving a large Q_2 is by no means a difficulty, since (vii) is essentially a microscopic description of the model (i). On the other hand, the microscopic model (viii) corresponds to (A) and thus the failure of (viii) is disturbing. We now show, however, that this difficulty can also be resolved.

In RPA the state $|1\rangle$ was written as $B^\dagger|0\rangle = (A^\dagger + A)|0\rangle$, where $A^\dagger$ (A) stands for a linear combination of quasiparticle-pair creation (destruction) operators, while $|0\rangle$ means the ground state. With this $B^\dagger$, the state $|2\rangle$ is to be written (with appropriate vector coupling and normalization being understood) as $B^\dagger B^\dagger|0\rangle = (A^\dagger + A)(A^\dagger + A)|0\rangle$. In reference 7, however, $B^\dagger B^\dagger|0\rangle$ was approximated as $(A^\dagger A^\dagger + AA)|0\rangle$ in order to avoid some mathematically difficulty. We then recall that $B(E2; 2_2^+ \to 2_1^+)$ was predicted[7] somewhat too small, perhaps because of this approximation. It is then quite possible that the failure of (viii) to give a large Q_2 has the same origin. To show that this is in fact the case, we consider the following model.

(B) We put $|1\rangle = B^\dagger|0\rangle$ and $|2\rangle = B^\dagger B^\dagger|0\rangle$ and consider the interaction[7] H_{31} as a perturbation which mixes $|1\rangle$ and $|2\rangle$, resulting in states $\psi(2_1^+)$ and $\psi(2_2^+)$ similar to those in (A). Using this $\psi(2_1^+)$, we find that $Q_2 = -0.44$ b with $e_{\text{eff}} = 0.83e$. (If $e_{\text{eff}} = 1.2e$ as in reference 7, $Q_2 = -0.59$ b.) Now Q_2 is obtained with sufficiently large magnitude and with correct sign.

Clearly more refined calculations have to be made in order to completely establish the conclusion that the experimental Q_2 can be reproduced within the framework of the vibrational model. The results of (A) and (B), however, are encouraging and convince us that such an effort would be worthwhile.

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[10]Throughout this article we assume that the charge radius $R_0 = 1.2A^{1/3}$ fm.

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[18]Another test of the validity of this model may be made by evaluating the ratio $R_2 = B(E2;\ 2_2^+ \to \text{ground})/B(E2; 2_2^+ \to 2_1^+)$ which becomes 0.14 for $a_1^2 = 0.86$. This is about a factor of five too large compared with its experimental value[4] $R_2 = 0.03$, but still embodies the experimental fact that the crossover transition is weak. The more elaborate model will reduce R_2, without modifying R_1 too much, and so the above discrepancy may not be so serious.

[19]Recently, experiments similar to that reported in reference 2 were performed also for ^{112}Cd and ^{116}Cd, and it is found (P. H. Stelson, private communication) that $|Q_2(^{112}\text{Cd})| < |Q_2(^{114}\text{Cd})| < |Q_2(^{116}\text{Cd})|$. Our model (A) can explain this result too, since it is known[2] that $R_1(^{112}\text{Cd}) > R_1(^{114}\text{Cd}) > R_1(^{116}\text{Cd})$, and thus the factor a_1a_2 in the expression for Q_2 is increased in going from ^{112}Cd to ^{116}Cd. [Note that $B(E2; 2_1^+ \to \text{ground})$ and thus β is almost the same[4] in these three isotopes.]