[9]For ω and k in the helicon regime of interest here, Doppler-shifted cyclotron resonance effects are negligible.

[10]This dispersion relation is the same as that used in references 1-3. It should be noted that the self-magnetic field of the dc current has not been included. It may not be possible to neglect this field compared to the applied magnetic field without simultaneously endangering the one-dimensional assumption. This requires further study.

[11]A. Bers and J. D. Mills, M.I.T. Research Laboratory of Electronics and Project MAC Report (to be published).

[12]R. J. Briggs and A. Bers, M.I.T. Research Laboratory of Electronics Quarterly Progress Report No. 67, 15 October 1962 (unpublished), pp. 35-43.

[13]C. K. Birdsall, G. R. Brewer, and A. V. Haeff, Proc. IRE 41, 865 (1953).

[14]In the absence of collisions, and for gaseous plasmas, an absolute instability in this regime was pointed out by R. J. Briggs and A. Bers, in Proceedings of the Fourth Symposium on Engineering Aspects of Magnetohydrodynamics, University of California, Berkeley, California, April 1963 (unpublished), pp. 23-30.

CHARGE DISTRIBUTIONS OF Ca^{40} AND Ca^{44} FROM 250-MeV ELECTRON SCATTERING*†

R. Hofstadter,‡ G. K. Nöldeke,§ K. J. van Oostrum,‖ L. R. Suelzle, and M. R. Yearian

High-Energy Physics Laboratory and Department of Physics, Stanford University, Stanford, California

and

B. C. Clark and R. Herman

Research Laboratories, General Motors Corporation, Warren, Michigan

and

D. G. Ravenhall

University of Illinois, Urbana, Illinois

(Received 18 October 1965)

In this paper we report new information on differences between the charge distributions of Ca^{40} and of Ca^{44}, obtained by means of elastic scattering of 250-MeV electrons.[1] As is the case with single isotopes, more detail can be detected with this method than with muonic or optical isotope shifts. An interesting result was obtained recently, by the muonic-atom method, that the equivalent radius $R=[5\langle r^2\rangle/3]^{1/2}$ increases by only about 0.8%, compared with the $A^{1/3}$ prediction of 3.2%.[2,3] We agree with this result and find additionally that this 0.8% increase of R arises from a 2.2% increase in the half-radius of the charge distribution, together with a 1.6% decrease in the skin thickness.

We recall that the $A^{1/3}$ dependence of the nuclear half-radius, c, deduced from electron scattering, came from experiments on seven nuclei, Ca to Bi, widely spread through the periodic table.[4] It is thus measured only as a gross property for large variations in A. Either for simplicity, or to utilize the fact that an $A^{1/3}$ variation of c with A, coupled with a constant surface thickness, produces a central nucleon density roughly independent of A, the detailed $A^{1/3}$ dependence of c has perhaps acquired a more solid status than present experiment warrants.[5,6] What the muonic x-ray measurements obtained, and what our new electron scattering results probe in greater detail, is the modulation, possibly due to nuclear shell structure, of the gross $A^{1/3}$ dependence of the nuclear size.

The basic idea, suggested some time ago,[7] is to measure experimentally and analyze theoretically not the differential cross sections of the separate isotopes, but the isotopic difference in these quantities or, equivalently, the ratio of the cross sections. Most of the systematic experimental errors associated with an absolute cross-section measurement are eliminated by taking the ratio of the cross sections. Theoretically, the connection with the isotopic ratio of charge densities is also made more directly.

We have carried out this comparative experiment on Ca^{40} and Ca^{44} by alternating the targets and measuring the cross sections for both isotopes under the same experimental conditions. The basic method has been described previously.[8] An improvement in technique has

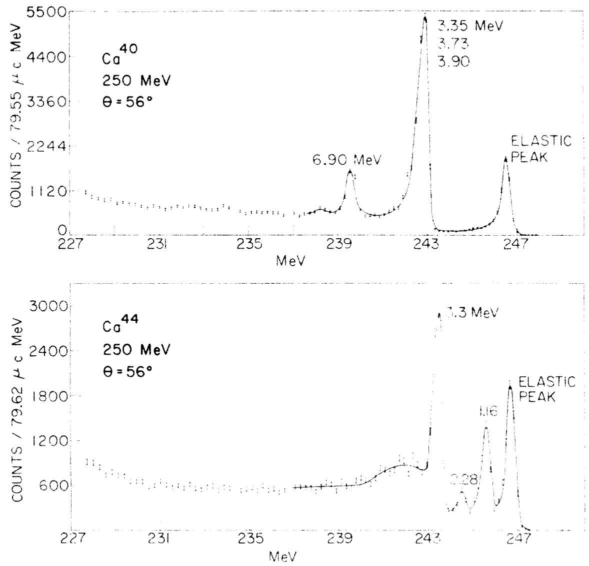

FIG. 1. Magnetically analyzed spectra of electrons scattered at 250 MeV from Ca^{40} and Ca^{44}. The 100-channel ladder was used to obtain the data. In all cases studied the elastic peaks were well separated from the inelastic structure.

been made since the earlier work by adding a 100-channel "ladder" detector in the region of the focal plane of the Stanford 72-in. spectrometer.[9] Figure 1 shows an example of the spectra of scattered electrons observed with each isotope. In this paper we shall concentrate only on the elastic peak, and for the purposes of this paper we form the quantity $D_{\rm exp} = [\sigma(40) - \sigma(44)]/[\sigma(40)+\sigma(44)]$ by inserting the appropriate elastic cross sections for the two calcium isotopes.

The targets employed were obtained through the kindness of Dr. George L. Rogosa of the U. S. Atomic Energy Commission. Actual target plates were fabricated at the Oak Ridge National Laboratory by Mr. T. H. Kobisk who is responsible for producing these targets of isotopically pure metallic Ca^{40} (99.97% purity) and Ca^{44} (98.6% purity).

Differential cross sections and muon energy levels are calculated exactly using methods described previously.[10] Nuclear recoil, quite significant in the muonic energy-level difference, is included in that problem by using the reduced mass. For the electron scattering, it is assumed that potential scattering describes the problem best in the center-of-mass system,[11] so that cross sections are obtained in that system and transformed to the laboratory system. They are folded to allow for finite angular resolution of the spectrometer.

The analysis reported here is restricted to describing the charge density of each isotope by the same functional form, i.e.,

$$\sigma(r) = \sigma_0\{\exp[(r^n - c^n)/z^n] + 1\}^{-1},$$

where $n = 1$ and $n = 2$ give what are usually called the Fermi and modified Gaussian shapes, respectively.

The values for Ca^{40} of the radius parameter c and the surface parameter z have been determined previously.[8] Taking for the values in Ca^{44} of these parameters $c + \Delta c$ and $z + \Delta z$, we calculate the theoretical quantity

$$D(\Delta c, \Delta z) = [\sigma(40) - \sigma(44)]/[\sigma(40) + \sigma(44)].$$

A least-squares fit of $D(\Delta c, \Delta z)$ is made to the measured $D_{\rm exp}$ to determine $\Delta c, \Delta z$. The goodness of fit depends mainly on Δz and Δc, and is quite insensitive to variations of c and z, the Ca^{40} parameters, of a size allowed by the earlier estimates of error.[8] A simplification useful in qualitative examination of the data and preliminary analysis is that at each angle $D(\Delta c, \Delta z)$ depends approximately linearly on Δc and Δz for small Δc and Δz. It is thus possible to separate and identify the contributions to D of changes in radius and in skin thickness. As Fig. 2 shows, Δc by itself ($\Delta z = 0$) produces a characteristic oscillation of D. With Δz alone ($\Delta c = 0$), D decreases (for $\Delta z < 0$) more or less steadily with θ. (These effects can be understood in terms of the variation of the differential cross sections with c, z.) The property of the experimental results which indicates a negative Δz is, qualitatively, the size of the minimum at 60° compared with that of the maximum at 45°. With $\Delta z = 0$ the ratio would be roughly 0.7, whereas experimentally it is about 1.0.

Numerical results for $\Delta c/c$, $\Delta z/z$, and related quantities are given in Table I, for both the Fermi and the modified Gaussian shapes. The true half-radius, $r_{0.5}$, the value of r such that $\rho(r_{0.5}) = 0.5\rho(0)$, is not the same as c in each of our shapes, because ρ_0 is not equal to $\rho(0)$. In the isotopic difference this distinction is magnified. We note the remarkable similarity in the numerical values of $\Delta r_{0.5}/r_{0.5}$ and $\Delta t/t$, where t is the 90-10% skin thickness, obtained for the two theoretical shapes. The isotopic difference in charge density, illustrated by the product $r^2[\rho(40) - \rho(44)]$ is shown in the

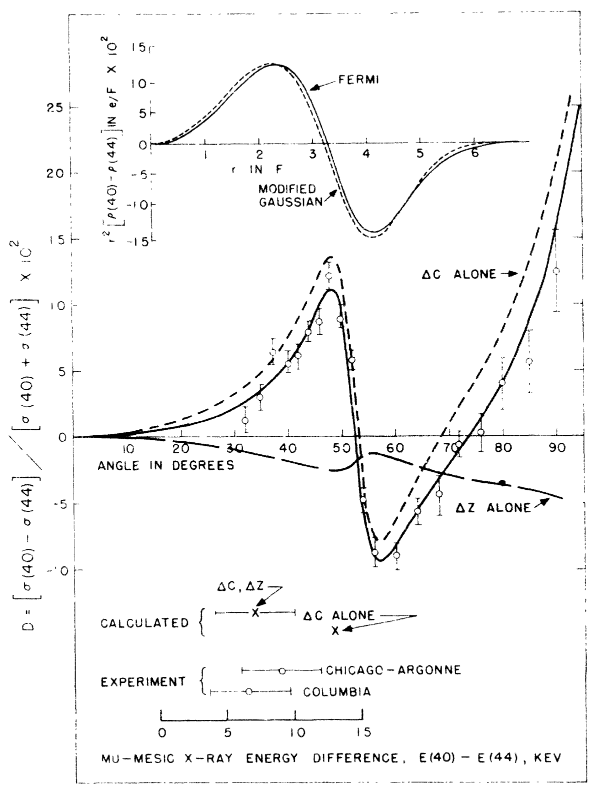

FIG. 2. Isotopic difference of electron scattering from Ca^{40} and Ca^{44} at 250 MeV. The curves are for a Fermi shape, with $c = 3.602$ F, $z = 0.576$ F as the Ca^{40} parameters, and with Ca^{44} parameters changed according to $\Delta c/c = 2.18\,\%$, $\Delta z/z = -1.66\,\%$. In the insets are the changes in charge density, times r^2, for the Fermi and modified Gaussian model, and the comparison with experiment of the Fermi shape predictions for the muonic K_α x-ray energy difference.

inset to Fig. 2. The curves for the Fermi and modified Gaussian fits are remarkably similar. This indicates to us that to some extent the method is actually measuring the isotopic difference in charge density, independent of the density assumed for Ca^{40}. More detailed investigation of this model independence is under way. As regards errors in the determination of these isotopic differences, the results for the directly determined parameters Δc and Δz for the Fermi shape are

$$\Delta c/c = (2.18 \pm 0.07)\%$$

$$\Delta z/z = (-1.66 \pm 0.82)\%.$$

Some of the error is the standard deviation due to the statistical and other uncertainties of each angular point. At the best fit, χ^2 has the

Table I. Isotopic changes in the charge distribution parameters of Ca^{44} compared with Ca^{40}. The quantities c and z occur in the definition of $\rho(r)$. Their values for Ca^{40} are taken as in reference 7, $c = 3.602$ F, $z = 0.576$ F, Fermi; $c = 3.373$ F, $z = 2.20$ F, modified Gaussian. The changes in the true half-radius $r_{0.5}$, the 90-10% surface thickness t, and the equivalent radius R, are obtained from Δc and Δz. Errors on these quantities are given in the text.

Shape	$\Delta c/c$ (%)	$\Delta z/z$ (%)	$\Delta r_{0.5}/r_{0.5}$ (%)	$\Delta t/t$ (%)	$\Delta R/R$ (%)
Fermi	2.18	−1.66	2.16	−1.58	0.78
Modified Gaussian	3.36	−0.78	2.51	−2.01	0.85

value 15.6 (18 degrees of freedom). The major part of the error in $\Delta z/z$, however, comes from the overall ±1% uncertainty in the origin of the vertical scale of $D_{\rm exp}$. The skin thickness change Δt has the same relative error as Δz, and $\Delta r_{0.5}$ the same as Δc.

A confirmation of our results is provided by the muonic x-ray energy difference, which depends essentially on the relative change in R. A graphical comparison of our prediction, from electron scattering, with the Argonne-Chicago[2] and Columbia[3] experiments is shown in an inset to Fig. 2. The decrease in surface thickness improves the agreement with those experiments.

While Ca^{40}, doubly magic, has its first strongly excited state at 3.9 MeV, Ca^{44} has a low-lying $E(2)$ state at 1.16 MeV which is excited in electron scattering. To the extent that virtual nuclear excitation contributes to the elastic scattering, this is a possible source of difference between the isotopes. The only experimental evidence adduced for such a possibility is by Peterson, Ziegler, and Clark.[12] In comparing scattering by Pb^{207} and Pb^{208}, they find an anomalous energy dependence of $\sigma(\theta)$ at electron energies of 10 MeV. The implication of this result for our problem is not at all clear, our energy being so much higher. This feature of the theoretical analysis requires further study.

Assuming that our results relate directly to the nuclear ground state, the most straightforward nuclear-model calculation suggested by this work would seem to be the independent-particle shell model, with a realistic Woods-Saxon effective potential for the nucleons, such as has been applied with success to the Ca^{40}

elastic scattering by Elton, Swift, and Towner.[13] Calculations of the equivalent radii R of the calcium isotopes with this model, by Perey and Schiffer,[14] show even a decrease in R from Ca^{40} to Ca^{44}. A basic assumption for such calculations is, of course, the isotopic variation of the potential itself. Exploration of isotopic variations in elastic scattering of protons or alpha particles on Ca will yield information on this point, to the extent that the effect of the separate parameters in the optical potential can be disentangled. In this regard, we tentatively suggest an approach like ours, in which one focuses attention, experimentally and theoretically, on the difference in scattering. It may be, as with our work, that some of the possible ambiguities (in our case the actual choice made for the Ca^{40} parameters) turn out to be unimportant.

We wish to thank Dr. G. L. Rogosa of the U. S. Atomic Energy Commission and Mr. T. H. Kobisk of Oak Ridge National Laboratory for their kind help in obtaining the target materials for this experiment. The High-Energy Physics Laboratory accelerator crew has been helpful, as always, in providing running time and we are very grateful to them.

---

*Work supported in part by the U. S. Office of Naval Research, Contract No. Nonr 225(67), and by the National Science Foundation.

†Preliminary data on the calcium isotopes were presented by the present authors at the International Symposium on Electron and Photon Interactions at High Energies, Hamburg, Germany, 8-12 June 1965.

‡On sabbatical leave September 1965 to September 1966, Imperial College, London, England.

§On leave from the Institut für Kernphysik, Mainz, Germany.

‖On leave from the Instituut voor Kernphysisch Onderzoek, Amsterdam, The Netherlands.

[1]The study of the isotopes Ca^{40} and Ca^{44} was suggested to one of us (R.H.) by A. de Shalit.

[2]J. A. Bjorkland, S. Raboy, C. C. Trail, R. D. Ehrlich, and R. J. Powers, Phys. Rev. 136, B341 (1964).

[3]R. C. Cohen, S. Devons, A. D. Kanaris, and C. Nissim-Sabat, Phys. Letters 11, 70 (1964).

[4]B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev. 101, 1131 (1956).

[5]L. R. B. Elton, Nucl. Phys. 5, 173 (1957).

[6]See, for example, A. R. Bodmer, Nucl. Phys. 9, 371 (1958).

[7]B. Hahn, R. Hofstadter, and D. G. Ravenhall, Phys. Rev. 105, 1353 (1957).

[8]M. Croissiaux, R. Hofstadter, A. E. Walker, M. R. Yearian, D. G. Ravenhall, B. C. Clark, and R. Herman, Phys. Rev. 137, B865 (1965).

[9]L. R. Suelzle and M. R. Yearian, Nuclear Structure (Stanford University Press, Stanford, California, 1964), pp. 360-361.

[10]D. G. Ravenhall, R. Herman, and B. C. Clark, Phys. Rev. 136, B589 (1964).

[11]L. L. Foldy, K. W. Ford, and D. R. Yennie, Phys. Rev. 113, 1147 (1959).

[12]G. A. Peterson, J. F. Ziegler, and R. B. Clark, Phys. Letters 17, 320 (1965).

[13]L. R. B. Elton, A. D. Swift, and I. S. Towner, Rutherford High Energy Physics Laboratory Report No. NIRL/R/81, 1964 (unpublished), p. 76; and private communication.

[14]F. G. Perey and J. P. Schiffer, unpublished manuscript. We thank Dr. Schiffer for a copy of this work, and also for very interesting discussions.

SCATTERING MEASUREMENTS ON NEUTRON RESONANCES IN Pu^{239}*

G. D. Sauter† and C. D. Bowman

Lawrence Radiation Laboratory, University of California, Livermore, California

(Received 16 September 1965)

Epithermal neutron resonance scattering measurements on Pu^{239} have recently been made, using the Livermore electron linear accelerator as a pulsed neutron source for a time-of-flight experiment. The goal of this experiment was to obtain total spin values J for as many resonances of Pu^{239} as possible, and to confirm the prediction of Bohr[1] and Wheeler[2] that the total spin and average fission width are correlated. Our method, a previously reported variation[3] of the "bright line" technique,[4] eliminates the fission neutrons and capture gamma rays which complicate the analysis of an earlier scattering experiment on Pu^{239} by Fraser and Schwartz.[5] From these measurements, we have assigned J values to 15 levels of Pu^{239} up to 75.2 eV. These include seven levels (14.3, 15.5, 32.3, 35.3, 47.6, 52.6, and 75.2 eV) not determined by Fraser and Schwartz and values for three levels (14.7, 22.2, and 44.5 eV) which are in disagreement with their reported values. In addition, we have determined values of $g\Gamma_n{}^2/\Gamma$ for two other resonances (50.0 and 85.6 eV),