m yields

$$\delta\sigma(m,\pm) = \pm 4\omega(2-B')/m^2\pi^2 \qquad (10)$$

so that each pair is separated in angular frequency by $8\omega(2-B')/m^2\pi^2$. The modes having even m are not separated to the first order of approximation. As before, separate excitation of these modes enables $2-B'$ to be determined directly, and besides simplicity this method has the advantage of giving the sign of $2-B'$. Snyder has obtained values of B and B' from double-peaked resonance curves using combinations of the $m=1$ modes, and his formula for the approximate separation is in agreement with (10). Note that Lin's and Hall's systems both give the same result when $B'=2$ as expected.

It is interesting to record that this type of problem also occurs in the theory of tides. Lamb[7] has calculated the separation of the degenerate modes of surface water waves on circular and square rotating lakes of uniform depth. His result for the circular lake agrees with (8) when powers of ω/σ higher than the first are ignored and his result for the $m=1$ mode of a square lake agrees with (10). Later workers, e.g., Corkan and Doodson,[8] obtain similar results.

The author has performed experiments on the $n=1, s=2$ modes of a cylindrical resonator and finds that application of (8) to the mode separations observed leads to $B'=0.08\pm0.08$ at 1.603°K and $B'=0.2\pm0.25$ at 1.426°K which are compatible with Snyder's square resonator results using (10). Equation (8) gives consistent values of B' for the (1, 1) modes as well, but errors are larger for these modes as they are more sensitive to geometrical imperfections of the resonator. It does not seem likely that the error in measuring $2-B'$ can be reduced much below 1% using this type of method because any such improvement requires an increased resonator Q, and this is limited by the attenuation factor ωB. Since the frequency separations are an order of magnitude smaller for the modes where $n=2$, further experimental checks on (8) await improvement of the apparatus.

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OBSERVATION OF A NEW TYPE OF INSTABILITY IN A THERMAL PLASMA*

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Some years ago Bernstein et al.[1] reported an observation of a low-frequency instability in the Project Matterhorn B-3 Stellarator. The instability there described depended for its existence on electron drift. We wish to report observation of an instability which occurs without axial electron drift in a thermally ionized potassium plasma and which may possibly be related to that observed in the Stellarator. However, the rf fields are only strong where there is a radial gradient of density within the plasma.

We have been working with both a one- and two-ended potassium plasma system 10 cm long with cathodes 1 cm in diameter operating in a vapor mode. The cathodes are usually operated at a temperature of 2200°K. The temperature of the oil bath surrounding the system is usually kept at temperatures in the range 80 to 100°C to obtain vapor pressures yielding maximum densities in the range 5×10^{10} to 2×10^{11} cm^{-3}. Two movable probes within the

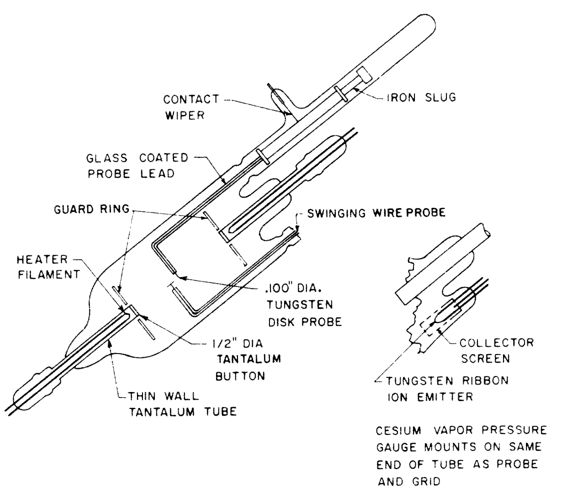

FIG. 1. A schematic of the experimental apparatus.

system shown in Fig. 1 may be used to detect rf signals or as Langmuir probes. Guard rings usually operated at floating potential are placed around each cathode. These guard rings are of great importance, for by changing their potential it is possible to change the profile of the plasma. At floating potential, the diffusion rate is virtually ambipolar. If the guard rings are missing from the system, or when present and operated near plasma potential, the diffusion rate is far greater. A typical profile of the plasma with the guard rings floating is shown in Fig. 2. With a probe at the center, positive with respect to the plasma, ion cyclotron waves are observed in the uniform plasma region if the current drawn to the probe exceeds a critical value, such as reported previously by Motley and D'Angelo.[2] When there is an excess of ions at the two cathodes and no current is drawn from either cathode, universal instabilities are observed with strong rf fields only in the region where the density gradient is large. These are similar to those observed previously in the Q device at Princeton[3,4] and predicted theoretically by others.[5,6] Farther out radially, where the density is a few percent of that in the middle and there is a weak density gradient extending to three times the cathode radius, another type of instability is observed. This we have called a "magnetoionic sound wave." It exists in both one- and two-ended systems although most coherent in the latter one. The rf oscillation observed is almost sinusoidal in the two-ended system with amplitude of approximately 0.2 V as measured on a floating probe. The instability is observed whether there is drift to the probe or not, and whether there is an excess of ions or electrons at one or both cathodes. The oscillations are present only if the intensity of the magnetic field exceeds a critical value of 420 G. A plot of the experimentally measured frequency variation with respect to magnetic field is given in Fig. 3. It should be observed that the frequency increases with magnetic field, unlike the universal instability.[3] Phase measurements indicate $m = 1$ type flutelike instability propagating in the direction of ion rotation.

We have used a collisionless pressure theory in Cartesian coordinates to predict the properties of this mode. We assume the zero-order electric field to be zero and neglect finite resistivity, and the electrostatic approximation is used by taking $\vec{E} = -\nabla\varphi_1$, where φ_1 is the perturbed potential. The electron density

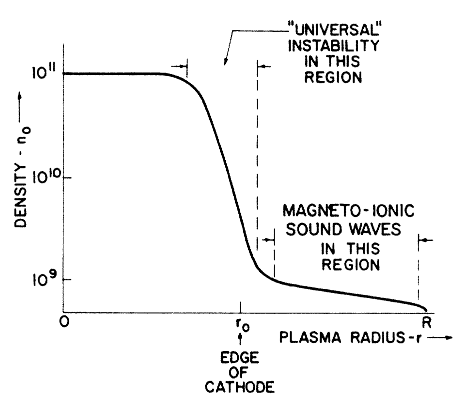

FIG. 2. A typical density profile of the potassium plasma taken midway between the cathodes.

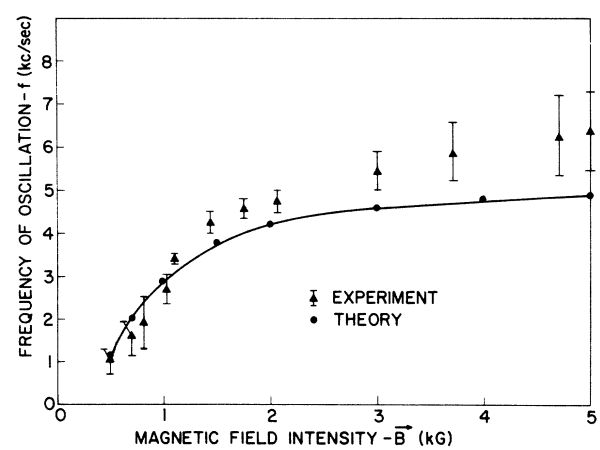

FIG. 3. Experimental and theoretical frequency variations of the magnetoionic sound wave instability with respect to frequency.

is assumed to be described by the Boltzman distribution so that to first order $e\varphi_1/kT_e = n_1/n_0$, where n_1 is the perturbed density and n_0 is the plasma density. Quasineutrality is assumed by taking $n_{1e} = n_{1i}$. For ions the equation of motion and the equation of continuity are used. The viscosity tensor giving finite Larmor radius effects[7] is neglected. With the magnetic field along the z direction and the density gradient along the x direction, perturbation theory yields for the dc ion and electron-drift velocities

$$V_{i0y} = GV_i^2/\omega_{ci} V_{e0y} = -GV_e^2/\omega_{ce}, \qquad (1)$$

where ω_{ci} (ω_{ce}) is the ion (electron) cyclotron frequency, $G = (\partial/\partial x)(\ln n_0)$, $V_i = kT_i/m_i$, and $V_e = kT_e/m_e$. We find that for frequencies well below the ion cyclotron frequency the perturbed potential φ_1 within the plasma obeys the following wave equation:

$$r_L^2 \frac{d^2\psi}{dx^2} + \left[\frac{k_z^2 V_{\text{th}}^2}{\Omega_i^2} - (1 + k_y^2 r_L^2) - \frac{Gk_y V_{\text{th}}^2}{\Omega_i \omega_{ci}} - \frac{1}{4}G^2 r_L^2\right]\psi = 0, \qquad (2)$$

where $\varphi_1 = C\psi/n_0^{1/2}$, where C is an arbitrary constant. We have defined $V_{\text{th}}^2 = k(T_e + T_i)/m_i$, $r_L^2 = V_{\text{th}}^2/\omega_{ci}^2$, $\Omega_i = \omega - k_y V_{i0y}$, and φ_1 is taken to vary as $\varphi_1(x)\exp[i(k_y y + k_z z - \omega t)]$. For simplicity it has been assumed that the gradient of density, G, is constant. As Eq. (2) is an equation with constant coefficients, we take the solution to be of the form $\psi(x) \sim \exp(ik_x x)$ and obtain the following dispersion relations:

$$\Omega_i \approx -\frac{Gk_y V_{\text{th}}^2}{2\omega_{ci}(1 + K^2 r_L^2)} \pm \frac{k_z V_{\text{th}}}{(1 + K^2 r_L^2)^{1/2}}, \qquad (3)$$

which for $T_e = T_i$ (corresponding to our experimental conditions) reduces to

$$\omega = \frac{K^2 r_L^2}{1 + K^2 r_L^2} k_y V_{i0y} \pm \frac{k_z V_{\text{th}}}{(1 + K^2 r_L^2)^{1/2}}, \qquad (4)$$

where $K^2 = k_x^2 + k_y^2 + \frac{1}{4}G^2$, it having been assumed, in contrast to universal instability theory, that $k_z V_{\text{th}} \gg k_y V_{\text{th}}^2 G/\omega_{ci}$. Since the value of G is small enough outside the edge of cathodes to satisfy this approximation, the universal instability would not be expected to be present in this region. On the other hand, for radii just less than the radius of the cathodes, G is so large that the foregoing inequality is not valid and thus the universal instability will be present.

It will be seen that this dispersion relationship yields an ion sound wave with $\omega \approx k_z V_{\text{th}}$ at large values of magnetic field, the frequency well below this at lower values of magnetic field. Assuming then that in the region where the oscillation is observed the boundary condition is such that $k_x l = \pi$, where l is the width of this low-density small-gradient region, the dispersion relation of Eq. (4) has been plotted in Fig. 3. It will be seen that there is fairly good agreement between the experimental and theoretical results, although the agreement would be much better if it were assumed that the ion temperature was approximately twice that of the cathodes. Results indicating such higher effective temperatures were also obtained previously by others.[8] It should be noted that if k_x were not assumed to be finite, the agreement would be far poorer in the low-frequency region where $\omega \approx \pm(k_z/K)\omega_{ci} + k_y V_{i0y}$. We may therefore conclude that there is some type of instability whose real frequency is governed by a dispersion relationship of the type of Eq. (3). The instability probably depends for its existence on the fact that there is a density gradient.

An examination of the theory with colllsions between electrons and ions taken into account has been undertaken. It can be shown that there is a possible resistive instability also predicted previously by others,[9] but because the density is so low, this resistive instability would be extremely weak. Consequently, we do not feel that such an explanation of the effect is satisfactory.

We would like to thank Mr. H. R. Andrews, who constructed the experimental tube.

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ABSOLUTE INSTABILITIES WITH DRIFTED HELICONS

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In this Letter we report analytic results based on an exact computer analysis of helicon-wave instabilities in a drifted electron-hole plasma. Previous analyses[1-3] have not distinguished between absolute (nonconvective) instabilities, which correspond to growth in time at every point in space or an oscillator, and convective instabilities, which correspond to spatial amplification over a range of real frequencies. It has been tacitly assumed that in order to have spatial amplification, it is sufficient for the dispersion relation to yield complex k's for some real ω provided that complex ω's also exist for some real k's; whereas, in fact, one must in addition show that no absolute instabilities exist, for otherwise oscillations will occur at the frequency and wave number of the absolute instability. In view of the considerable experimental work[4-7] in this area, it would seem most important to clarify the theoretical picture. Our results, which are based on a recently developed technique for determining the nature of instabilities,[8] show for example that in InSb with equal electron and hole concentrations, an absolute instability should appear above a threshold magnetic field, at an arbitrarily small electric field. Furthermore, from our results we develop a physical picture of the instabilities, radically different from the one given by either Bok and Nozières[1] or Misawa,[2] and show it to be correct in that it predicts the exact computer results. This picture has the same basis as the one recently and independently described by Hasegawa,[3] who failed, however, to correctly identify the nature of the instability.

We restrict our discussion here to isotropic and nondegenerate semiconductors, and use a hydrodynamic description[9] of the free carriers. The dispersion relation for transverse waves, $\exp(i\omega t-ikz)$, with k along the applied magnetic field B_0, and electrons and holes counterstreaming parallel to B_0 with velocities v_e and v_h, respectively, is[10]

$$\omega^2-c^2k^2=\frac{\omega_{pe}{}^2(\omega-kv_e)}{(\omega-kv_e-i\nu_e\pm\omega_{ce})}+\frac{\omega_{ph}{}^2(\omega+kv_h)}{(\omega+kv_h-i\nu_h\mp\omega_{ch})}, \quad (1)$$

where c is the velocity of light, ω_{pe} and ω_{ph} the electron and hole plasma frequencies, ω_{ce} and ω_{ch} the electron and hole cyclotron frequencies, and ν_e and ν_h the electron and hole collision frequencies. The stability analysis consists in first mapping from (1) the real k axis into the complex ω plane; if part of the real k axis maps into the lower half ω plane the system may exhibit some kind of instability. Any portion of the lower half of the ω plane containing a mapping of part of the real k axis is then investigated by mapping lines of constant Reω