We hence have three equations [(4)-(6)] to determine two unknowns, and thus our system is overdetermined. We overcome this difficulty in the following way. We use Eq. (5) to determine the value of the coupling constant which leads to a zero for ReD; this occurs for

$$f^2 = 11.2. \tag{7}$$

With Eq. (4) this means that

$$M_w^{\ 2} = 11.2\sqrt{2}\times 10^5 M_p^{\ 2},$$

or

$$M_w \approx 1258 M_p. \tag{8}$$

[There exists, of course, the question whether the value of G chosen here is the appropriate one. If the interaction of the neutral weak current is reduced by a certain factor, then the mass of the W^0 should be higher, according to Eq. (4).]

We then use Eq. (6) as a check on the value obtained for f^2 from Eq. (5). This added constraint removes the usual flexibility of being able to adjust coupling constants and masses, which is found in most bootstrap calculations. This gives

$$\frac{3}{2}\frac{n(x)}{(d/dx)D(x)}\bigg|_{x=1} = 8.8,$$

which predicts $M_w = 1154 M_p$, and gives us added confidence in our calculation.

For bootstrapping experts we note that our solution for $n(x)$ does not differ greatly from the input term $b(x)$, and hence we can use the determinantal method to evaluate $D(x)$. We did this as a check and obtained $f^2 = 10$, which is close to the result of Eq. (7).

It is thus evident from the considerations above that we can find a self-consistent solution for a W^0 in the $\nu\bar{\nu}$ system. If this calculation is taken as a model for other processes of leptonic interactions, it turns out that the weak processes can be described by strong coupling constants, and it is only the very high mass of the intermediate vector boson that is responsible for their effective weakness.

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RENORMALIZATION OF THE STRANGENESS-CHANGING AXIAL-VECTOR COUPLING CONSTANT AND THE CABIBBO FORM OF UNIVERSALITY*

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Weisberger and Adler have independently reported consistent calculations of the renormalization of the nonstrangeness-changing axial-vector coupling constant[1] in beta decay. The calculations are based upon the equal-time commutation relations of the axial charges, and the Goldberger-Treiman relation [partially conserved axial-vector current (PCAC) hypothesis].[2] In this note the calculation of reference 1 is generalized to the strangeness-changing axial-vector coupling constant. The renormalization of the axial-vector coupling constant is related by a sum rule to off-the-mass-shell K-nucleon total cross sections and the pseudoscalar coupling constant of the strange baryons to the K meson. Although the parameters entering the calculation are not completely without uncertainty, we feel that it is worth doing the calculation to obtain some limits on this renormalization. The sum rule is very similar to that of reference 1 and the result of the calculation can be stated as follows:

$$0.73 \lesssim (1/Z_{p\Lambda})^2[1-\tfrac{1}{2}Z_{p\Lambda}^{\ 2}-\tfrac{1}{2}Z_{p\Sigma}^{\ 2}] \lesssim 0.98, \tag{1}$$

where $Z_{p\Lambda}$ and $Z_{p\Sigma}$ are the renormalization constants for the proton-lambda and proton-sigma axial-vector coupling constants. The two different limits in Eq. (1) correspond to two distinct estimates of the $g_{p\Lambda K}$ strong coupling constant. The renormalization is in the opposite direction from that in reference 1 since the K^-p total cross section is much larger than the K^+p total cross section over much of the range [see Eqs. (8) and (8′) below]. Finally, if the Cabibbo form of universality[3] is assumed and the SU(3) structure of the total hadronic weak current is taken into account, we can determine the reduced matrix elements of the currents related to weak-interaction experiments by the Cabibbo theory.

The calculation is based upon the following assumptions:

(1) The hadronic current responsible for the leptonic decays has the following form:

$$J_\lambda = G_V^{\,0} V^{I+} + G_V^{\,0S} V_\lambda^{K+} + G_A^{\,0} A_\lambda^{I+} + G_A^{\,0S} A_\lambda^{K+}, \tag{2}$$

where $G_V^{\,0}$, $G_V^{\,0S}$, $G_A^{\,0}$, and $G_A^{\,0S}$ are the unrenormalized strangeness-maintaining ($\Delta S = 0, \Delta I = \frac{1}{2}$) vector coupling constant, strangeness-changing ($\Delta S = 1, \Delta I = \frac{1}{2}$) vector coupling constant, strangeness-maintaining axial-vector coupling constant, and strangeness-changing axial-vector coupling constant, respectively. In Eq. (2), I_+ and K_+ denote the currents that transform like $\frac{1}{2}(\lambda_1 + i\lambda_2)$ and $\frac{1}{2}(\lambda_4 + i\lambda_5)$ under SU(3), respectively. Here λ_a are the generators of SU(3).

(2) The space integrals of the time components (charges) of the vector and axial-vector currents generate at equal times the algebra of SU(3)⊗SU(3). This point has been emphasized by Gell-Mann repeatedly.[4] We also assume that the algebra is conserved even though the currents themselves are not conserved. The charges are defined by

$$F^a(t) = \int d^3x\, V_4^{\,a}(\vec{x}, t)$$

and

$$F_5^{\,a}(t) = \int d^3x A_4^{\,a}(\vec{x}, t). \tag{3}$$

One of the commutators that we have assumed is

$$[F_5^{\,K+}(t), F_5^{\,K-}(t)] = F^Q + F^Y. \tag{4}$$

The quantities on the right-hand side of Eq. (4) are the charge and hypercharge which are conserved by the strong and electromagnetic interactions and are hence independent of time.[5]

(3) Next we assume the partial conservation of the axial-vector current[2,4,6] generalized to the strangeness-changing current:

$$\partial_\mu A^{\mu a} = (M_K^{\,2}/2f_K)\varphi^K. \tag{5}$$

In Eq. (5), $a = 4, 5, 6, 7$ of SU(3), M_K is the K mass, φ^K is the renormalized K-meson field, and f_K is a parameter which is related to the $K \to \mu + \nu$ decay rate.

The matrix elements of the currents in Eq. (2) give us immediately the renormalization constants, for example,

$$\langle B'(P')|A_\lambda|B(P)\rangle = \left(\frac{M_B M_{B'}}{E'(\vec{P})E(\vec{P})}\right)^{1/2} \frac{G_A^{\,B'B}}{G_A^{\,0}} \bar{U}_{B'}(\vec{P}')\gamma_\lambda\gamma_5 U_B(\vec{P}). \tag{6}$$

The renormalized coupling constant is defined by Eq. (6). Here the states $|B(\vec{P})\rangle$ and $|B'(\vec{P})\rangle$ are physical one-baryon states with masses M_B and $M_{B'}$. The constant on the right-hand side of Eq. (5), which can be evaluated by standard methods,[4,6] is given by

$$i\frac{M_B + M_{B'}}{[2g_{BB'K}F_{BB'K}(0)]^{1/2}} = \frac{1}{2f_K}. \tag{7}$$

In Eq. (7), the Goldberger-Treiman relation, $g_{BB'K}$ is the renormalized pseudoscalar $K \to B + B'$ coupling constant, and $F_{BB'K}(0)$ is the invariant form factor evaluated at zero K mass $[F_{BB'K}(M_K^{\,2}) = 1]$.

Next some details of the calculation are given. Following the methods of Weisberger and Adler[1] and Fubini and Furlan,[7] we compute the matrix elements of Eq. (4) with physical one-proton states. The contribution of the one-baryon states (Σ^0 and Λ^0 in this case) is separated from the contribution of the many-particle states, and the latter are expressed in terms of $K^\pm p$ total cross sections off the mass shell. Making use of the SU(3) structure of the current in Eq. (2), we obtain the sum rule

$$\left(\frac{1}{Z_A^{\,F}}\right)^2 = \left[1 + \frac{1}{3}\left(\frac{D}{F}\right)^2\right] + \left(\frac{M_p + M_\Lambda}{g_{p\Lambda K}}\right)^2 \frac{1}{6}\left(3 + \frac{D}{F}\right)^2 \frac{1}{4\pi} I, \tag{8}$$

where

$$I=4\pi\int_{\nu(M_\pi+M_\Lambda)}^{\nu_t}\frac{d\nu}{\nu^2}[A^-(\nu,0)-A^+(\nu,0)]+\int_{\nu_t}^{\infty}\frac{d\nu}{\nu^2}q_L(\nu)[\sigma^-(\nu)-\sigma^+(\nu)]. \qquad (8')$$

In Eq. (8′) the variable ν is related to the laboratory momentum of the incident K meson q_L by $\nu=(q_L^2+M_K^2)^{1/2}+M_K^2/2M_p$; ν_t corresponds to the threshold for elastic Kp scattering; and $\nu(M_\pi+M_\Lambda)$ corresponds to the threshold for the lowest many-particle state that has the same quantum numbers as the K^-p system (M_p = proton mass, M_Λ = lambda mass). Also, A^- is the forward-scattering absorptive part of the K^-p amplitude in the unphysical region, and $\sigma^\pm$ is the total cross section for $K^\pm p$ scattering of zero-mass K mesons.[1] We will assume without justification that the analytic continuation off the mass shell is harmless, and we can use as in reference 1 the method of Ferrari and Selleri[8] and cancel the factor of $[K_{p\Lambda K}(0)]^2$. We hope that the off-the-shell corrections amount to only a few percent as reported by Adler.[1]

From standard SU(3) analysis the quantity we are computing can be related to the physical renormalized coupling constants entering this calculation by

$$G_A^{p\Lambda}=6^{-1/2}(3+D/F)Z_A^F G_A^{0S}$$

and

$$G_A^{p\Sigma}=-2^{-1/2}(-1+D/F)Z_A^F G_A^{0S}. \qquad (9)$$

There are two uncertainties in the sum rule which make an unambiguous determination of the renormalization difficult. They are the following: the lack of knowledge of the coupling $g_{p\Lambda K}$ and the value of the absorptive part $A^-(\nu,0)$ below threshold.

Regarding the first difficulty, we can relate $g_{p\Lambda K}$ to $g_{\pi NN}$ if we assume SU(3) and a value for the D/F ratio for pseudoscalar couplings which hereafter we will denote $(D/F)_P$. This D/F ratio $(D/F)_P$ is not too well known. If we assume a value that agrees with the determination from bootstraps,[9] $(D/F)_P\approx 3$, we obtain $g_{p\Lambda K}^2\approx g_{\pi NN}^2(0.4)$, and if we assume a value of $(D/F)_{P'}$ that agrees with the determination from the weak interaction,[10] $(D/F)_P\approx 1.7$, we obtain $g_{p\Lambda K}^2\approx g_{\pi NN}^2(0.5)$. In all of the following $g_{\pi NN}^2=(4\pi)(14.6)$.

Regarding the second difficulty, the value of $A^-(\nu,0)$ is obtained from the Dalitz-Tuan analysis of K-nucleon scattering in terms of a complex scattering length[11] which fits the scattering data in the physical region. We use the values of the scattering length as obtained by Kim and by Ross and Humphrey.[12] The contribution of the unphysical region is approximately 20% of the final result. Thus uncertainties of as large as 20% in this contribution will not be too important. Since the K^-p cross section is large and climbing in the low-energy region, and the K^+p cross section small and constant, it is safe to neglect $A^+(\nu,0)$ in Eq. (8′).

The $K^\pm p$ cross sections above threshold are well known and are given by several authors.[13] In the asymptotic region $q_L>4$ BeV/c the cross-section difference is fit by a Regge-pole model.[14] The breakdown of the final result is as follows:

$$I=[(I_<=13.04)+(I_>=29.6)+(I_A=16.80)]\text{ mb}, \qquad (10)$$

where $I_<$ is the contribution to the integral of Eq. (8′) below threshold, $I_>$ is the contribution above threshold but not asymptotic ($q_L<4$ BeV/c), and I_A is the contribution from the asymptotic region. Also, the $Y_1^*(1385)$ is put in as a bound state; the residue of this pole is related to that of the $N^*(1238)$ isobar by SU(3), and its contribution is shown to be negligible (≲1%). Because of the uncertainties in the problem, we can regard the final result as an estimate.

The facts that the quantity $\Delta\sigma=\sigma^--\sigma^+$ is always positive and large and the quantity $g_{p\Lambda K}$ is small for reasonable $(D/F)_P$ conspire to produce a large renormalization in the direction opposite to that in reference 1, and using Eq. (9) in Eqs. (8) and (8′) we obtain Eq. (1).

If we make the Cabibbo assumption for the current[15] of Eq. (2), then

$$G_V^0=G_A^0=G\cos\theta,\quad G_V^{0S}=G_A^{0S}=G\sin\theta, \qquad (11)$$

where θ is the Cabibbo angle. In the axial-vector contribution there are three quantities related to weak-interaction experiments: θ, D, and F. The latter two are the reduced matrix elements of the currents in Eq. (2) and correspond exactly to our $Z_A{}^D$ and $Z_A{}^F$.

Using SU(3) analysis the Adler-Weisberger result can be stated:

$$1.20 \approx (1+D/F)Z_A{}^F = Z_A{}^D + Z_A{}^F. \tag{12}$$

Taking the lower limit in our result Eq. (1) ($g_{P\Lambda K}{}^2 \approx 0.5 g_{\pi NN}{}^2$) and using Eqs. (9) and (12), we obtain

$$Z_A{}^F \approx 0.45 \text{ and } Z_A{}^D \approx 0.75, \tag{13}$$

which agree well with the solution A of Willis et al.[10] who report $Z_A{}^F \approx 0.436$ and $Z_A{}^D \approx 0.742$. The precise numerical values are not to be taken too seriously since the calculation is only an estimate.

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ELECTRODYNAMICS OF INTERMEDIATE BOSONS AND *CP* NONINVARIANCE

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The simplest explanation of the experimental result of Christenson, Cronin, Fitch, and Turlay[1] is that *CP* invariance is violated in weak interactions. With this in mind, we reinvestigate the charged vector-meson theory of Lee and Yang.[2,3] We shall restrict ourselves to the case $\kappa = 0$, for the reasons given by Bernstein and Lee.[4] Furthermore, we assume the validity of the principle of minimal electromagnetic interaction, in the form recently given by Lee.[5] More explicitly, we assume that the electromagnetic coupling constant appears in the combination ∂_μ only.

In the absence of electromagnetic interactions, the Lagrangian density for a free vector-meson field is

$$-\frac{1}{2}\left(\frac{\partial\varphi_\nu}{\partial x_\mu}-\frac{\partial\varphi_\mu}{\partial x_\nu}\right)^*\left(\frac{\partial\varphi_\nu}{\partial x_\mu}-\frac{\partial\varphi_\mu}{\partial x_\nu}\right)-m^2\varphi_\mu{}^*\varphi_\mu. \tag{1}$$

By the principle of minimal electromagnetic interaction, we replace $\partial/\partial x_\mu$ by ∂_μ in (1) to get the Lagrangian density

$$\mathcal{L}(e,m) = -\tfrac{1}{2}(\partial A_\mu/\partial x_\nu)(\partial A_\mu/\partial x_\nu) -\tfrac{1}{2}G_{\mu\nu}{}^*G_{\mu\nu} - m^2\varphi_\mu{}^*\varphi_\mu. \tag{2}$$

In the ξ-limiting formalism of Lee and Yang,[2] the following Lagrangian density is considered