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FURTHER DISCUSSION OF PARTICLE-MIXTURE THEORIES OF $K^0 \to 2\pi$ DECAY

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Fitch, Roth, Russ, and Vernon[1] have recently reported an important additional result on the apparent CP invariance violation in K_2^0 decays.[2-4] We discuss below the implications of their result, with particular reference to particle-mixture theories of long-lived $K^0 \to 2\pi$ decays.[5]

As described more fully in reference 5, the experimental results cited in references 2-4 (which we call the LLD effect) could be explained within the context of CP invariance either by postulating the existence of a spinless particle state S in the neighborhood of the K^0 mass, which mixes with $K_+ = (K^0 + \bar{K}^0)/\sqrt{2}$ and gives rise to the long-lived $\pi^+\pi^-$ component, or by ascribing the LLD effect to the action of a scalar cosmological field which, by causing K^0 and $\bar{K}^0$ to have slightly different energies, acts as a regenerating medium. It is reported[1] that there is maximal constructive interference between the regeneration amplitude which is essentially purely imaginary,[6] due to a material regenerator, and the LLD amplitude; therefore, the LLD amplitude must be purely imaginary[6] too. A classical long-range field would provide a real energy difference between K^0 and $\bar{K}^0$ and therefore give rise to a real[6] LLD amplitude; consequently, the observed interference effect provides strong evidence against the cosmological hypothesis.[7] Thus the particle-mixture theory remains as the sole survivor for explaining the LLD effect within the framework of CP invariance.

The result of reference 1 further restricts the parameters of the particle-mixture theory. Whereas on the basis of the experiments reported in references 2-4, one knew that the mass of the state Ψ_L must equal the K^0 mass to the accuracy of the mass determination in those experiments, i.e., of the order of 1 MeV, the observed interference requires that this equality hold to within the width γ_1 of the short-lived component,[8] viz., $|m_2 - m_L| \leq \gamma_1 \sim 10^{-5}$ eV.

According to the particle-mixture theory, which is devised solely to save CP invariance, the $CP = -1$ component of neutral kaons, $K_2 = K_- = (K^0 - \bar{K}^0)/\sqrt{2}$, cannot contribute to 2π decays. The occurrence of interference must be understood as follows. Due to the different interactions of K^0 and $\bar{K}^0$ with matter, the state K_2 will no longer be characterized by simple exponential decay within the regenerating medium. Using the notation of reference 5 and retaining quantities only to lowest order in θ and δ (see below), the states which undergo pure exponential decay are

$$|\Psi_s'\rangle = |\Psi_s\rangle - \delta|K_2\rangle, \quad |K_2'\rangle = |K_2\rangle + \delta|\Psi_s\rangle, \tag{1}$$

and $|\Psi_L\rangle$, which we assume to be negligibly affected[5] by the regenerator. A neutral kaon beam within the medium is described by the state vector

$$\begin{aligned}|\Psi(t)\rangle = a|\Psi_s'\rangle &\exp[-(\gamma_1'/2 + im_1')t] \\ + b|K_2'\rangle &\exp[-(\gamma_2'/2 + im_2')t] \\ + c|\Psi_L\rangle &\exp[-(\gamma_L/2 + im_L)t],\end{aligned} \tag{2}$$

where a, b, and c are constants and

$$m_1'-i\gamma_1'/2=m_1-i\gamma_1/2 -\sigma[(m_1-m_2)-i(\gamma_1+\gamma_2)/2],$$
$$m_2'-i\gamma_2'/2=m_2-i\gamma_2/2 -\sigma[(m_1-m_2)-i(\gamma_1+\gamma_2)/2], \quad (3)$$

and we have introduced the dimensionless parameters

$$\delta=(\pi N/m_K)(f-\bar{f})[(m_1-m_2)-i(\gamma_1-\gamma_2)/2]^{-1},$$
$$\sigma=(\pi N/m_K)(f+\bar{f})[(m_1-m_2)-i(\gamma_1-\gamma_2)/2]^{-1}. \quad (4)$$

m_K is the mean K^0 mass, N is the number density of nuclei in the regenerator, and f and $\bar{f}$ are the forward-scattering amplitudes for K^0 and $\bar{K}^0$, respectively. From (1) and (2), we see that $|K_2\rangle$ should be re-expressed as

$$|K_2\rangle=|K_2'\rangle+\delta|\Psi_S'\rangle \quad (5)$$

when the beam enters the medium. The Ψ_S' component dies away rapidly, leaving the long-lived K_2' component which, to our accuracy, has the same amplitude as the incident K_2. However, whereas K_2 cannot decay to 2π, the amplitude for K_2' to decay to $\pi^+\pi^-$ is, according to (1), δA_C, where A_C is the amplitude for $\Psi_S\to\pi^++\pi^-$. The interference reported in reference 1 is then, according to the particle-mixture theory, between this amplitude and the $\pi^+\pi^-$-decay amplitude a_c from the other long-lived state Ψ_L. The two amplitudes are coherent because both Ψ_L and K_2' arise from the same K^0 production events in the target in which the K^0 beam originates.[9] After traversal of a thickness L of regenerator sufficient to let the Ψ_S' component decay to a negligible level, the intensity of $\pi^+\pi^-$ decays would be

$$I_a \propto |\theta a_c+\delta A_c e^{i\varphi}|^2, \quad (6)$$

with

$$(P_K/m_K)\varphi=[(m_2-m_L)-i(\gamma_2-\gamma_L)/2]L_0 +[(m_2'-m_L)-i(\gamma_2'-\gamma_L)/2]L \quad (7)$$

where P_K is the kaon momentum and L_0 is the distance from the regenerator to the target in which the kaon beam originates. In keeping with our assumption that Ψ_L is negligibly affected by the regenerator, we take its mass and lifetime to be the same as in vacuum. We see from (7) that the relative phase factor $e^{i\varphi}$ oscillates with a period of $(P_K/m_K)2\pi/(m_2'-m_L)$ in L, and consequently, all interference effects disappear when we average the intensity I_d over an interval Δl unless $|m_2'-m_L|<(2\pi/\Delta l)\times(P_K/m_K)$. Under the conditions of reference 1, $\sigma\ll1$, so that $m_2'\cong m_2$, and the region over which interference is observed is of the order of the decay length for the amplitude of the short-lived component or greater, hence we conclude that $|m_2-m_L|\leq\gamma_1$, which is the conclusion stated at the beginning of this section.

The result of reference 1 also confirms our assessment, made in reference 5, that direct production of S particles must be weak compared to neutral kaon production. Maximum interference between the regeneration amplitude and the LLD amplitude can occur only if the two are fully coherent, which is possible only if the incoherent contribution to the LLD effect is negligible in comparison to the coherent one.

Equations (6) and (7) suggest another simple test of the particle-mixture theory. The relative phase of the two terms in (6) depends on the distance from the target to the regenerator.[10] By varying this distance, the relative phase could be varied; consequently, the magnitude of the interference effect would vary also. The measurement of the intensity I_d as a function of L_0 over an interval ΔL_0 would provide a measurement of m_L to an accuracy of $\sim1/\Delta L_0$. Only if $(m_L-i\gamma_L/2)=(m_2-i\gamma_2/2)$ would the interference effect be independent of the displacement of the regenerator, which is, of course, the result required if the LLD arises from $K_2\to2\pi$.

Possible tests for the particle-mixture theory therefore are (i) measurement of the lifetime associated with the LLD effect,[11] (ii) measurement of the interaction length for the state responsible for the LLD effect,[5,12] and (iii) measurement of the mass m_L. The discussion of section 3 suggests that a precision of $\gamma_1/10$ is quite feasible.

Should all these measurements yield results which are indistinguishable from the parameters of the usual long-lived state K_2, we would be faced with a situation very reminiscent of the τ-θ puzzle, for which the simplest explanation would be that $\Psi_L\equiv K_2$, i.e., that CP invariance is violated. Nevertheless, we must emphasize that the observations of references 1-4 cannot by themselves demonstrate CP noninvariance, and that it is quite difficult to exorcise the particle-mixture theory by measurements on 2π decays alone. The only difference between the predictions of the CP-invariant particle-mixture theory and the hypothesis of CP noninvariance with respect to these decays is the following: Whereas the first requires that the rate of appearance of LLD events arising from a beam which consists originally of pure K^0 be exactly equal to that from a beam which is originally pure $\bar{K}^0$, the two rates need not necessarily be equal if CP invariance is abandoned. However, on the basis of TCP invariance alone[13] and the known masses and lifetimes of K_1 and K_2, we conclude that such an asymmetry cannot exceed 5.7% and may very well be much smaller. Consequently, such a direct demonstration of CP noninvariance may not be forthcoming in the near future. In the absence of such evidence, one could even argue that the postulation of a particle state S almost exactly degenerate with K^0 and a superweak[14] CP-invariant interaction (which is all that is required to provide the necessary mixing of S and K_+) may be preferable to an as yet unspecified CP-nonconserving interaction.

It is therefore a question of the utmost importance to examine processes where CP noninvariance could manifestly appear. The measurement of the charge asymmetry[15] in the leptonic decay of long-lived neutral kaons acquires a special significance in this context. If the $\Delta S = \Delta Q$ rule is valid,[16] this asymmetry is expected to be of exactly the same magnitude as the one discussed in the previous paragraph and may perhaps be easier to measure. Its value is predicted by the CP-noninvariant theory once the experiments analogous to those of references 1 and 2 have been performed for the $2\pi^0$ mode. The CP-invariant particle-mixture theory, on the other hand, forbids the existence of any charge asymmetry.

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[1]V. Fitch, R. Roth, J. Russ, and W. Vernon, Phys. Rev. Letters 15, 73 (1965).

[2]J. Christenson, J. Cronin, V. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[3]W. Galbraith et al., Phys. Rev. Letters 14, 383 (1965).

[4]X. De Bouard et al., Phys. Letters 15, 58 (1965).

[5]P. K. Kabir and R. R. Lewis, Phys. Rev. Letters 15, 306 (1965). This paper, which contains earlier references, describes the essential ingredients of all such theories. Henceforth, when we refer to the particle-mixture theory, we mean any theory which incorporates these features.

[6]Relative to the standard factor $[(m_1-m_2)+i(\gamma_1-\gamma_2)/2]$.

[7]The absorptive effect of interactions with "invisible" particles could give rise to a potential which is purely imaginary. The most likely candidate, a neutrino sea, would, however, give rise to a LLD effect which increases with kaon velocity at least as fast as $\beta^2\gamma^2$, a behavior which is clearly ruled out by the results of references 3 and 4.

[8]Conditions for observing interference were previously discussed by H. Ezawa, Y. Kim, S. Oneda, and J. Pati, Phys. Rev. Letters 14, 673 (1965).

[9]We disregard for the moment the Ψ_L which may arise from direct S production; we shall return to this point in the next paragraph.

[10]This conclusion holds whatever may be the influence of the regenerator on Ψ_L. In the event that S interacts appreciably with the regenerator, I_d would still be given by a formula of the form (6), except that a_c would then depend on the details of the interaction, and the coefficient of L in (7) would have m_L and γ_L replaced by modified constants m_L' and γ_L'.

[11]J. L. Uretsky, Phys. Letters 14, 154 (1965); K. Nishijima and H. Saffouri, Phys. Rev. Letters 14, 205 (1965).

[12]L. B. Okun and I. Ya. Pomeranchuk, Phys. Letters 16, 388 (1965), who have also pointed out that high-energy neutrino experiments already provide an upper limit to the interaction length, which rules out models in which S does not interact at all with matter.

[13]See Eqs. (28), (35), and (36) of T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. 106, 340 (1957).).

[14]In the sense of L. Wolfenstein, Phys. Rev. Letters 13, 562 (1964).

[15]Reference 13, Sec. 4(B).

[16]There are strong theoretical arguments {R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); B. L. Ioffe, Zh. Eksper. i Teor. Fiz. 42, 1411 (1962) [translation: Soviet Phys.—JETP 15, 978 (1962)]} for expecting $\Delta S = -\Delta Q$ amplitudes, if at all present, to be very much smaller than $\Delta S = \Delta Q$ amplitudes. Inclusion of $\Delta S = -\Delta Q$ amplitudes with a relative magnitude X modifies the asymmetry from the value corresponding to $X = 0$ by a factor which lies between $(1 \pm X)$. Thus, even if we admit $\Delta S = -\Delta Q$, amplitudes of the magnitudes reported by M. Baldo-Ceolin et al.,

Nuovo Cimento 38, 684 (1965) or by B. Aubert et al., Phys. Letters 17, 59 (1965), an anticipated charge asymmetry (for $X=0$) may be reduced (or enhanced!) but cannot disappear.

BOOTSTRAP CALCULATION FOR THE MASS OF THE INTERMEDIATE VECTOR BOSON

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The S-matrix methods that have been developed in recent years have usually been applied to problems involving strong interactions only.[1] The methods are based on the assumptions of unitarity and analyticity. However, it is possible that these assumptions might also hold true for the other types of reactions, and thus it is instructive to use these techniques in other fields too. In this note we apply S-matrix methods to interactions between leptons; in particular, we try to estimate the mass of the intermediate vector boson using the bootstrap technique.[2]

In our calculation we limit ourselves to the simplest type of process, namely, that of elastic neutrino-antineutrino scattering, for which only one helicity amplitude has to be considered. We ask ourselves whether a neutral intermediate vector boson W^0 can exist. If it does exist, it should appear as a pole both in the s and t channels. Following the usual bootstrap prescription, we use the pole in the t channel as the input for our system of N/D equations. We then look for a zero of $\mathrm{Re}(D)$ in the s channel, which should occur at the same value as the pole in the t channel. Moreover, at these two points the residue at the pole should have the same value. If this happens we can say that the system of equations explains the occurrence of the W^0 in a self-consistent way.

For $\nu\bar{\nu}$ elastic scattering, the only parameter which has the dimensions of mass is M_w, i.e., the mass of the W^0. It is advantageous to use the variable $x=s/M_w^2$ in writing down the dispersion relation (s, as usual, denotes the square of the center-of-mass energy in the s channel). If we define $b=4\pi sB$ and $n=4\pi sN$, then our input takes the form

$$b(x)=2f^2\{(1+1/x)^2\ln(x+1)-(\tfrac{3}{2}+1/x)\}, \tag{1}$$

while the dispersion relations for the numerator N and denominator D functions are now of the form

$$n(x)=b(x)+\frac{x}{4\pi^2}\int_0^\infty \frac{n(x')}{x'-x}\cdot\frac{1}{x'}\left[\frac{x}{x'}b(x')-b(x)\right]dx', \tag{2}$$

$$D(x)=1-\frac{x}{4\pi^2}\int_0^\infty \frac{n(x')dx'}{x'(x'-x)}. \tag{3}$$

If we describe the effective Lagrangian of the weak interaction by

$$L_i=-(G/\sqrt{2})JJ^+,$$

then our coupling constant is given by

$$f^2=GM_w^{\ 2}/\sqrt{2}. \tag{4a}$$

Assuming that the coupling constant of the scattering $\nu+\bar{\nu}\to\nu+\bar{\nu}$ is the usual G of the weak interactions, we find that

$$f^2=10^{-5}M_w^{\ 2}/M_p^{\ 2}\sqrt{2} \tag{4b}$$

(where M_p denotes the proton mass), which gives us a relation between the coupling constant and the mass of the intermediate vector boson.

As mentioned previously a single-channel bootstrap calculation provides for the self-consistent determination of two parameters, usually the mass and the coupling constant of the exchanged particle. The equation for finding the mass in our system is

$$\mathrm{Re}D(x=1)=0, \tag{5}$$

and the coupling constant can be determined from

$$f^2=\tfrac{3}{2}\left.\frac{n(x)}{(d/dx)D(x)}\right|_{x=1}. \tag{6}$$