EFFECT OF THE SUHL-ABRIKOSOV RESONANCE ON THE TRANSITION TEMPERATURE OF SUPERCONDUCTORS*

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The properties of superconductors with a small concentration of randomly distributed, unpolarized paramagnetic impurities have been discussed in great detail in recent years. In all cases, the conduction-electron–impurity-spin interaction has been treated in the Born approximation (or second order). Clearly it is of great interest to see what changes occur in the classic work of Abrikosov and Gor'kov[1] when the s-d exchange interaction, of the form

$$V = -(J/N)\sum_{i=1}^{N} \vec{S}^{i}\cdot\vec{\sigma},$$

is included to all orders. In a normal metal, Suhl[2] (using dispersion theoretic techniques) and Abrikosov[3] (using better known field theoretic perturbation techniques) have recently shown that the s-d interaction may give rise to a resonance in the electronic scattering amplitude (for antiferromagnetic coupling, $J<0$). Their work was inspired by the important observation of Kondo[4] that, to third order in J, the scattering cross section seems to diverge logarithmically as the electronic energy approaches the Fermi energy ϵ_F.

In this Letter, we shall limit ourselves to a study of how the transition temperature T_c of a superconducting paramagnetic alloy is affected by the previously mentioned anomalous scattering (which is sometimes referred to as the "Kondo effect"). While the numerical calculations which we discuss are based on somewhat arbitrary values of J, ϵ_F, and T_{cp}, they do indicate the sensitive dependence of T_c on the magnitude and sign of J, ϵ_F, and the ratio T_γ/T_{cp} (T_γ is the temperature corresponding to the Suhl-Abrikosov resonance, and T_{cp} is the transition temperature of the pure superconductor). It must be emphasized that our work is somewhat tentative, since we simply assume that the additional unphysical complex poles which are found in the scattering amplitude (for $J<0$) in all current calculations[2,3,5] need not be worried about. If they are not spurious (as Nagaoka[5] has suggested), then obviously our results are not valid for temperatures $T \lesssim T_\gamma$. In any event, since this question is amenable to an experimental test, it would seem worthwhile to have theoretical predictions for $T_c \lesssim T_\gamma$ ($J<0$) on the basis of the assumption that no "phase transition" occurs. Moreover, our work may be of qualitative interest as showing how pair-breaking resonance scattering will affect T_c, quite apart from its origin.

By introducing impurity-spin Green's functions $\mathcal{G}$, in addition to the usual conduction-electron Green's function G, Abrikosov[3] has been able to make use of the powerful techniques of infinite-order perturbation theory (based on Wick's theorem and summarized by Dyson's equation). While the use of "fictitious" Fermi particles in place of impurity spin $\vec{S}$ (whose components do not commute) necessitated a change in the normalization of the original thermal averages, the extra factor caused little complication in the diagrammatic expansion if one neglects (as we shall) all interference effects due to scattering from different paramagnetic impurities. Now Abrikosov's type of discussion[6] may be easily generalized to a superconductor—in precisely the same way as when one limits oneself to the Born approximation.[1] Formally, it is very convenient to introduce a four-component electron field operator,[7] $\Psi^\dagger(x) \equiv [\psi_\uparrow^\dagger(x), \psi_\downarrow^\dagger(x), \psi_\uparrow(x), \psi_\downarrow(x)]$, since then Abrikosov's diagrams[3] are unchanged, although they now refer to 4×4 matrix equations. Of course, the calculation of the matrix Green's function $\hat{G}$ (the Gor'kov F function being the off-diagonal element in the Nambu space) is considerably more complicated than in a normal metal due to the necessity of calculating the s-d induced self-energy Σ self-consistently using the true $\hat{G}$ (this is crucial even in the Born approximation[1,7]). However, for the purpose of finding T_c, at which point the Gor'kov order parameter $\Delta \to 0$ (assuming the phase transition is second order), we only need the self-energy in the normal state.

Leaving out all algebraic manipulations, we simply write down the final implicit equation for T_c in a standard form:

$$\ln\left(\frac{T_{cp}}{T_c}\right) = 2\sum_{n=0}^{\infty}\left\{\frac{1}{2n+1} - \frac{1}{2n+1+\rho_s(\omega_n)}\right\}, \qquad (1)$$

where $\rho_s(\omega_n)=[\pi\tau_s(\omega_n)T_c]^{-1}$, $\omega_n=(2n+1)\pi T_c$, and

$$\frac{1}{\tau_s(\omega)}\equiv\frac{1}{\tau_s^{\rm B}}\left|\frac{\Gamma^{(1)}(\omega)}{(J/N)}\right|^2. \tag{2}$$

We have set both Boltzmann's constant ($k_{\rm B}$) and $\hbar$ equal to unity; $\tau_s^{\rm B}$ is the usual Born approximation for the exchange scattering time, and $\Gamma^{(1)}(\omega)$ is the exchange part of the vertex function $[\Gamma(\omega)\equiv\Gamma^{(0)}(\omega)+\vec{S}\cdot\vec{\sigma}\Gamma^{(1)}(\omega)]$ discussed by Abrikosov. The normal-state Green's function $G(\vec{p},\omega)$ has a self-energy [n_i is the impurity concentration and $N(0)$ the density of states at the Fermi surface]

$$\Sigma(\vec{p},\omega)\simeq -i\,\mathrm{sgn}\omega\, n_i N(0)\left[|\Gamma^{(0)}(\omega)|^2+S(S+1)|\Gamma^{(1)}(\omega)|^2\right] \tag{3}$$

due to the exchange part of the impurity interaction, in addition to the part from direct potential scattering (in Abrikosov's analysis, nothing interesting happens when the latter is treated to all orders). Implicit in some of the approximations leading to (3) is the result $[\rho_1\equiv 2N(0)/N]$

$$\Gamma^{(1)}(\omega)\simeq(J/N)[1+\rho_1 J\ln(\epsilon_{\rm F}/|\omega|)]^{-1}, \tag{4}$$

diagrams having been evaluated using a logarithmic approximation. We note that[2,3] $\Gamma^{(1)}(\omega)$ has a resonance, for $J<0$, at $\omega_\gamma=\epsilon_{\rm F}\exp(-1/J\rho_1)$.

For a better approximation to $\Gamma^{(1)}(\omega)$ than (4), we might try the crude solution of Suhl's scattering equations,[2] which gives

$$|\Gamma^{(1)}(\omega)|^2\simeq\left(\frac{J}{N}\right)^2\frac{A^2(\omega)}{\{A^2(\omega)+[\frac{1}{2}\pi a(S+1)]^2\}[A^2(\omega)+(\frac{1}{2}\pi aS)^2]}, \tag{5}$$

where $A(\omega)=1+a\ln(\epsilon_{\rm F}/|\omega|)$, $a\equiv\rho_1 J$. A study of some of the lowest order vertex diagrams summed by Abrikosov suggests that (5) corresponds to doing the integrals one step better than the logarithmic approximation. However, as Abrikosov has emphasized, the set of vertex diagrams he leaves out are also intimately related to the broadening of the resonance given by (4). In any event, (5) is still not good enough since now the self-energy vanishes at the resonance! Recently, Suhl[8] has been able to solve his scattering equations exactly and (at least for pure exchange scattering) finds the result

$$|\Gamma^{(1)}(\omega)|^2=(J/N)^2[A^2(\omega)+(2\pi a)^2S(S+1)]^{-1}. \tag{6}$$

We shall use this in solving (1). While the Kondo effect will produce anomalies in many properties,[9] the superconducting transition temperature is particularly interesting since it is independent of $\Gamma^{(0)}(\omega)$ as well as ordinary potential scattering.[10]

The initial decrease in T_c as a function of impurity concentration n_1 is easily found to be

$$T_{cp}-T_c\simeq\left(\frac{2}{\pi\tau_s^{\rm B}}\right)\sum_{n=0}^{\infty\sim100}\left(\frac{1}{2n+1}\right)^2\left[\left(1+a\ln\frac{\epsilon_{\rm F}}{\pi(2n+1)T_{cp}}\right)^2+(2\pi a)^2S(S+1)\right]^{-1}. \tag{7}$$

The Born-approximation (BA) formula has often been fitted to the experimentally determined slope, with the general conclusion that the values of J (~0.15–0.30 eV) so found were "reasonable." However, our result for T_c is a very sensitive function of the sign and magnitude of J. One interesting feature of (7) may be seen if one fixes $|J|$, $\epsilon_{\rm F}$, and S, and considers the initial slope (dT_c/dn_i) as a function of T_{cp}. For $J<0$, the slope decreases as T_{cp} increases, while the opposite holds for $J>0$.

For high impurity concentrations, (1) must be solved by numerical or graphical means. We note that in contrast to the BA case, $T_c=0$ is not a solution of (1). However, T_c does decrease to a very small value for large concentrations. Some results are plotted in Figs. 1 and 2. The curves in Fig. 1 indicate the general behavior when viewed as functions of (T_γ/T_{cp}). The summation was cut off at $n\simeq10^3$, although the upper limit is really $n_D\simeq(\omega_D/2\pi T_c)$, with ω_D the maximum phonon energy. Negligible error was introduced by this pro-

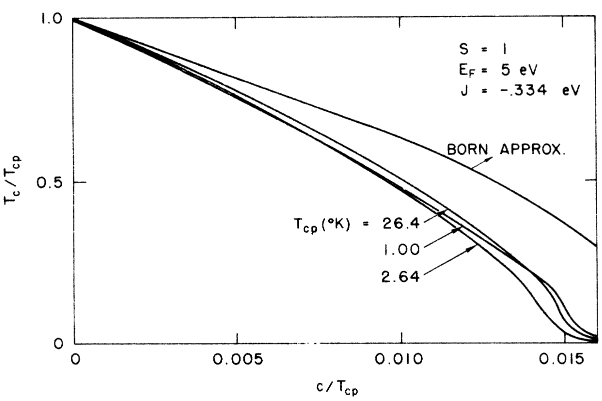

FIG. 1. Transition temperature T_c versus the impurity concentration (c, in per cent) for a fixed value of J (<0) and varying values of T_{cp}. We have normalized both scales to the pure superconductor's transition temperature T_{cp} (in °K), with the result that the Born approximation[1] is a universal curve. The temperature corresponding to the Suhl-Abrikosov resonance energy is $T_\gamma = 2.64$°K.

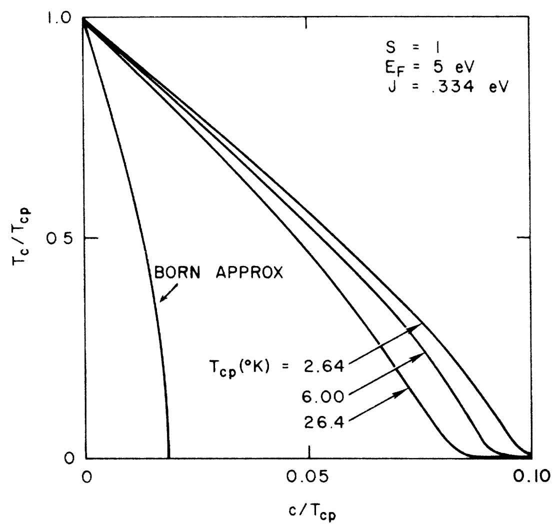

FIG. 2. Transition temperature T_c versus the impurity concentration (c) for a fixed value of J (>0). See Fig. 1 for further details.

cedure since a cutoff lower than 10^3 was found to affect T_c only when the latter was small. In the high-concentration region, the largest deviation from the BA result for T_c occurs when $T_{cp} \simeq T_\gamma$ ($J < 0$).

In a certain sense, the effect of the Suhl-Abrikosov resonance on T_c is not as drastic as one might expect, especially when T_c is close to T_γ. This is due to the fact that the determination of T_c [see (1)] involves a summation over the electronic energies $\omega_n = (2n+1)\pi T_c$. We might note here that for small concentrations, the most interesting changes in the superconducting characteristics of the Born-approximation theory[1] will probably occur just below T_c. As the temperature decreases, the energy gap $\omega_0(T)$ will increase, and the Suhl-Abrikosov resonance should take on added importance due to the density-of-states peaking. However, if the temperature becomes low enough (such that $\omega_0 \gtrsim \omega_\gamma$), the resonance will play a negligible role.

We might remark that to third order in J (the "Kondo" approximation), Liu's result[11] for T_c agrees with (1). This seems to be somewhat fortuitous, since there is no obvious reason why Liu's use of Dyson's equation should make any sense.[12]

At sufficiently low temperatures, there is the possibility of ferromagnetic ordering in superconductors with paramagnetic impurities. In principle, it is straightforward to generalize Gor'kov and Rusinov's work[13] on the phase boundaries (which was based on the Born approximation), making use of Abrikosov's recent discussion[14] for a normal metal. However, one does require the G and F functions for the paramagnetic phase in order to calculate the effective value of constant exchange field. We therefore defer any discussion to a future publication, where we hope to deal with G and F for small values of Δ.

Finally, we might make a general comment on the reduction in T_c due to paramagnetic impurity scattering. In the Born or "classical" approximation, one explains this result by noting that the impurity spins are physically equivalent to a random distribution of magnetic fields.[15] The latter depair strongly because they effectively destroy the single-particle state's degeneracy under time reversal. In contrast, the higher order effects result from the quantum mechanical nature of the impurity spin (as well as of the electrons), and thus the relevance of Anderson's theorem seems more obscure when these effects are considered. However, in a certain sense, one may still regard the electrons as being scattered by random magnetic fields, except that these fields now contain a self-induced component.

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[1]A. A. Abrikosov and L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 39, 1781 (1960) [translation: Soviet Phys.–JETP 12, 1243 (1961)].

[2]H. Suhl, Phys. Rev. 138, A515 (1965). For the finite temperature generalization, see H. Suhl, to be published.

[3]A. A. Abrikosov, in Proceedings of the Conference on the Many-Body Problem, Novosibirsk, USSR, March 1965 (to be published). A short summary is given in A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. 48, 990 (1965) [translation: Soviet Phys.–JETP 21, 660 (1965)]. See also A. A. Abrikosov, to be published.

[4]J. Kondo, Progr. Theoret. Phys. (Kyoto) 32, 37 (1964).

[5]Y. Nagaoka, Phys. Rev. 138, A1112 (1965); and to be published. Nagaoka decouples the equations of motion for the retarded electronic Green's function and polarization using the simplest sort of approximation which is physically reasonable.

[6]We might remark that Abrikosov computes the self-energy of the conduction electrons to all orders in J using the unperturbed value of the impurity-spin propagator $\mathcal{G}_0$, rather than solving the coupled Dyson equations for G and $\mathcal{G}$ self-consistently. In terms of Nagoka's basic idea, of course, this approximation is very bad. The formation[5] of quasibound modes at the imurity site (this being heralded by complex poles in the electronic scattering amplitude) would mean that it is crucial to find G and $\mathcal{G}$ self-consistently. Even if such collective modes do not appear, it might be that $\mathcal{G}$ is sufficiently altered from $\mathcal{G}_0$ by the s-d interaction to remove the complex poles in the G found by Abrikosov. This is under investigation.

[7]See, for example, V. Ambegaokar and A. Griffin, Phys. Rev. 137, A1151 (1965).

[8]H. Suhl, to be published. Suhl's solution has conjugate poles on the imaginary ω axis. It would seem that the only important assumption made is the restriction to single-particle intermediate states.

[9]See G. J. Van den Berg, in Progress in Low Temperature Physics, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1964), Vol. IV, p. 194.

[10]Actually, in Suhl's work,[2,8] direct potential scattering associated with a paramagnetic impurity seems to lead to some subtle effects.

[11]S. H. Liu, Phys. Rev. 137, A1209 (1965).

[12]However, if Abrikosov's technique[3] is used to compute the G and F Green's functions for a superconductor, the results to third order in J agree precisely with those found by Liu (apart from certain errors in the latter's evaluation of spin averages in second and third order). We shall discuss this elsewhere.

[13]L. P. Gor'kov and A. I. Rusinov, Zh. Eksperim. i Teor. Fiz. 46, 1363 (1964) [translation: Soviet Phys.–JETP 19, 922 (1964)].

[14]A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz.–Pisma Redakt. 1, 53 (1965) [translation: Soviet Phys.–JETP Letters 1, 33 (1965)].

[15]P. G. de Gennes and M. Tinkham, Physics 1, 107 (1964).

DIRECTIONAL ULTRASONIC NOISE AND KINK EFFECT IN BISMUTH

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In this Letter we want to report on experiments concerning the buildup of ultrasonic noise in bismuth subjected simultaneously to an electric and a magnetic field. As shown by Esaki,[1,2] in a transverse magnetic field a kink occurs in the current-voltage curve of a bismuth sample when the transverse carrier drift velocity exceeds the sound velocity.

According to the acoustoelectric explanation of the kink effect,[3,4] the strong increase of current beyond the kink point is due to an acoustoelectric current associated with acoustic noise generated by the drifting carriers. Experimental evidence of the buildup of ultrasonic noise in bismuth beyond the kink point was recently presented by Toxen and Tansal,[5] at current densities of 800 to 1000 A/cm^2. Their results, however, are rather ambiguous since the time for buildup and decay of the ultrasonic signal was more than 10 times the length of the drift-field pulse. The authors,[5] therefore, do not exclude the possibility that the effect observed might have been of thermal origin. In our experiments we observed an ultrasonic noise signal which (i) occured at much lower current densities (<50 A/cm^2), (ii) was strongly time-correlated with the drift-field pulse, and (iii) showed a marked directional effect indicating the preferential excitation of sound waves confluent with the carrier drift direction.

The present investigations were carried out at temperatures around 1.8°K using a single