velocity ω^a is non-negative definite; (b) the null cones form two separate systems, past and future (this means that one can globally assign a time direction); (c) there is a noncompact Cauchy surface; that is, there exists a noncompact spacelike surface that intersects every timelike and null line once and once only. These conditions are satisfied by a universe of form (1) with $K=0$ or -1 and filled with normal matter. However, at S, ρ is positive by a finite amount. Therefore any perturbation of (1) which is not too large will leave ρ positive, and so there will still be a closed trapped surface and hence a physical singularity. Thus a universe that is similar on a large scale to the form (1) but has no exact local symmetries will have a singularity. Local irregularities cannot prevent it.

The presence of matter gives a unique timelike congruence (the flow lines) which may be defined as the timelike eigenvector of the Ricci tensor. Using this it is possible to replace condition (c) by the weaker conditions (d) and (e): (d) The covering space has no closed timelike lines; (e) there exists a complete connected noncompact spacelike three-surface on which the density has a positive lower bound and the scalar product of its unit normal and the velocity vector of the flow lines has an upper bound. This removes the possibility that a singularity might be avoided by there being regions of space-time for which H is not a Cauchy surface.

The author and Dr. G. F. R. Ellis are working on a proof of the occurrence of singularities in closed universes. This and an extension of the above results will be published shortly.

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DIRECT DETECTION OF TWO-PHOTON EMISSION FROM THE METASTABLE STATE OF SINGLY IONIZED HELIUM*

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In this Letter we report on the direct detection by coincidence counting techniques of the two-photon decay of the metastable $2\,^2S_{1/2}$ state of singly ionized helium. The metastability of the 2S state of hydrogenic atoms has been the subject of many theoretical studies.[1] The 2S states of H and He^+ are known to be metastable, and the lifetimes have been shown to be greater than 2.4×10^{-3} sec and 1×10^{-3} sec, respectively.[2] The present theoretical view is that an unperturbed metastable hydrogenic atom will decay by two-photon emission at a rate given by $\gamma = 8.226\ Z^6\ \mathrm{sec}^{-1}$.[3] The decay rate for all other decay modes considered so far is orders of magnitude slower than the two-photon rate.[4] Several attempts have been made to detect similar two-photon decays in excited nuclei. In no case has conclusive evidence been obtained for the existence of such transitions.[5] Extensive observations have been made of multiple-photon processes with intense laser fields.[6] Multiple-photon-induced absorption and emission processes in the rf region of the spectrum have been studied in considerable detail by molecular beams and optical pumping techniques.[7]

The present experiment was designed to detect the decay in flight of a slow (12-eV) beam of metasteble helium ions. Ionized helium was chosen in preference to hydrogen because the two-photon lifetime and the Stark quenching rate are each smaller by a factor of 64 in the helium. In addition, standard ion-beam techniques could be employed to focus and control the beam. In the case of the present beam, it is estimated that 40 000 ions/cm sec decay by two-photon emission. These photons are expected to have a continuous distribution of energy which is broadly peaked about 20 eV (600 Å). Detailed predictions of the shape of the spectrum have been made by Spitzer and Greenstein.[8] The energy sum for the pair of photons, of course, must equal the $2S-1S$ energy difference (40.1 eV). The coincidence counting rate is expected to vary as $(1+\cos^2\theta)$ if the photodetectors are not sensitive to the polarization of the photons. Here θ is the space angle between the directions of emission of the two photons.

The apparatus consists of an electron-bombardment ion source, differential pumping chambers for pressure reduction, and a detection chamber with two phototubes, one of which is movable about the beam axis. Individual photon events are processed by means of fast-coincidence circuits. Typically, the ion source is operated at a helium pressure of about 1×10^{-3} Torr, and the electron bombarding energy is 350 V. An ion beam of 5×10^{-9} A is produced, of which roughly $\frac{1}{2}\%$ is in the $2\,^2S$ state. The first differential vacuum separation chamber contains a microwave cavity through which the beam passes. This cavity may be excited at 14 kMc/sec (the Lamb-shift frequency for He^+) to quench the metastable ions. The beam is electrostatically focused into the detection chamber where the background pressure is 2×10^{-7} Torr or less. For each two-photon decay, one photon lies between 300 and 600 Å, and the complementary photon has a wavelength longer than 600 Å. In order to detect these photons, EMI 9603 windowless multipliers are used within the vacuum system. The photons are incident on the beryllium-copper photocathode. These tubes will detect all the photons from 300 to about 1200 Å with an efficiency of about 10%. Since these tubes are also sensitive to charged particles, considerable effort was made to prevent such particles from reaching the tubes. The photons enter the phototubes through a parallel-plate structure in which alternate plates are maintained at positive and negative potentials. This device effectively reduces the charged-particle background count to a small fraction of the photon count. One tube is moved with respect to the other about the beam axis within the angular range from about 67° to 292°.

Typically, the single-photon counting rate is about 10 000/min. This result is obtained by subtracting the count obtained with the rf quenching field on from that obtained with it off and is consistent with the earlier intensity estimate. The background single-event counting rate obtained with the rf on is about 2000 counts per minute. This background arises from charged particles that are not eliminated by the parallel-plate grid system and from the excitation of the background gas.[9] The dependence of the single-photon counting rate on the electron bombarding energy is shown in Fig. 1(a). This curve shows the expected threshold at 65 eV for the formation of the $2S$ state of He^+. The dependence of the single-photon counting rate on the power used to excite the microwave quenching cavity is shown in Fig. 1(b). This curve shows the exponential dependence on rf power that is expected for

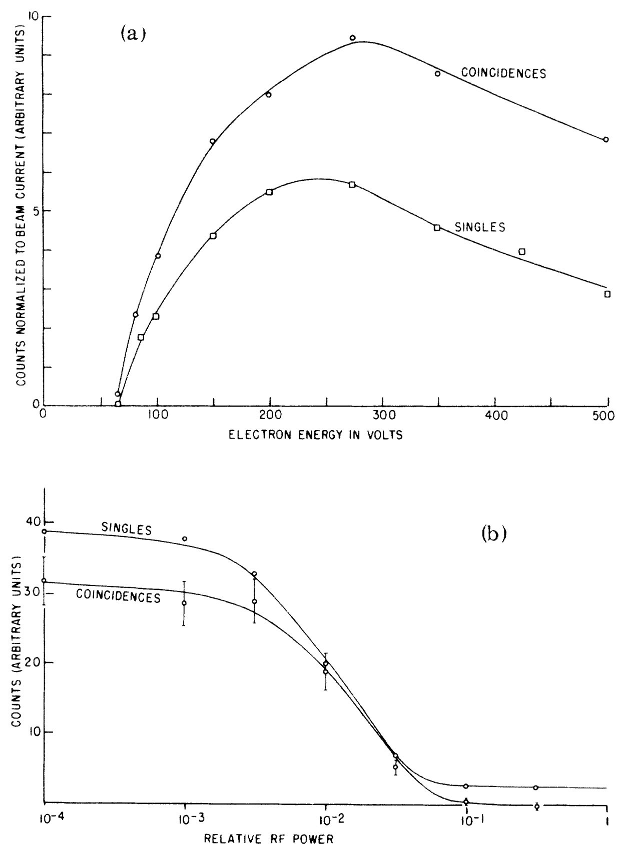

FIG. 1. (a) Dependence of the coincidence counting rate and the rf-induced singles counting rate on electron bombarding energy. (b) Dependence of the coincidence and singles counting rate on rf quenching power.

the quenching of metastable ions. The residual count at power levels between 1 and 0.1 arises from charged particles and gas excitation (see above).

Counts from the two phototubes that are in time coincidence are observed when the electron bombarding energy is greater than 65 eV and in the absence of rf quenching power. The distribution of the delay times between the pulses from the two phototubes is measured with a time-to-pulse-height converter and multichannel analyzer. A typical delay curve is shown in Fig. 2(a). The peak in this curve has a width of 18 nsec which is consistent with the known electron transit time spread in the Venetian-blind multiplier. Unfortunately, the unavailability of a fast vacuum-ultraviolet light source prevented the determination of the zero-delay position. The residual counts at long delay are consistent with the estimated random-coincidence counting rate. The total number of counts in the peak of the delay curve is taken as the "coincidence count." Typical coincidence counting rates of five per minute are observed.

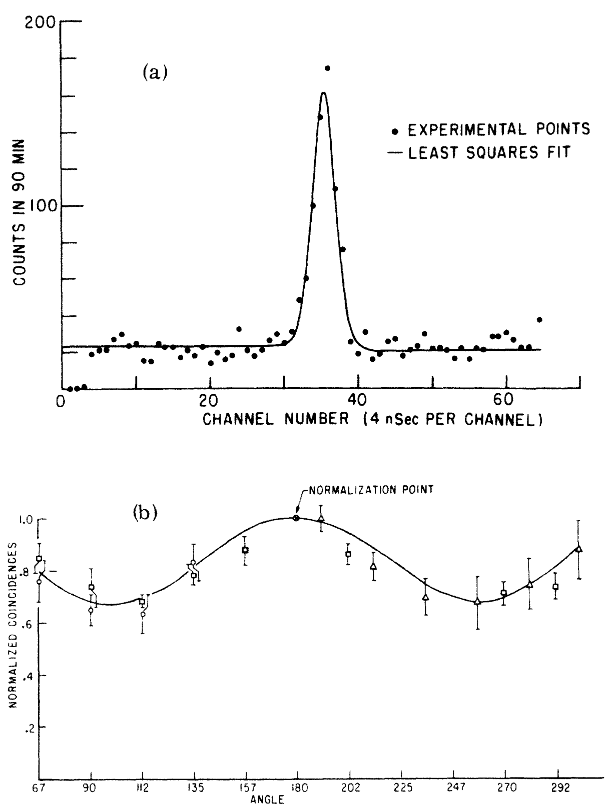

FIG. 2. (a) Typical coincidence delay curve. (b) Dependence of the coincidence counting rate on the angular separation of the photodetectors.

Since the coincidence counting rate is proportional to the product of the detector efficiencies, and since these are estimated to be 10^{-3}, this counting rate agrees with the observed singles rate and the estimated two-photon decay rate. The dependence of the coincidence rate on the electron bombarding energy is shown in Fig. 1(a); this curve has the same threshold and shape as the singles excitation curve. The dependence of the coincidence rate on rf quenching power is shown in Fig. 1(b). This curve is similar to the singles quenching curve except that there is no residual signal at full rf power. This indicates that all of the coincidence counts arise from metastable ions.

The coincidence count observed at various angular positions of the movable phototube is shown in Fig. 2(b). The widths and positions of the peaks in the delay curves obtained at different angles were constant to within about one nanosecond. This variation is no greater that the expected statistical fluctuations in these quantities. The solid curve shown in Fig. 2(b), which was obtained by integrating the theoretical angular factor $(1+\cos^2\theta)$ over the area of the phototube cathodes and over the length of the beam exposed to the detector, is in agreement with the observations.[10]

The dependence of the coincidence counting rate on the angular separation of the phototubes as well as on the electron bombarding energy and rf quenching power provides very strong evidence that we have detected the two-photon decay of the metastable helium ion. Our knowledge of the detector efficiency is too poor to allow a precise determination of the two-photon decay rate. Work is in progress to measure the spectral distribution of the photons.

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KOHN ANOMALY AND COHERENT SCATTERING OF SLOW NEUTRONS IN LIQUID METALS*

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It was first pointed out by Kohn,[1] and later experimentally verified by Brockhouse et al.,[2] that the presence of a logarithmic singularity in the derivative of the static random-phase-approximation dielectric function $\epsilon(\kappa)$ at $\kappa=2k_F$, k_F being the Fermi momentum, should produce a corresponding discontinuity in the gradient of the phonon dispersion curve $\omega=\omega(\vec{q})$. The same fact also leads to the important result that the asymptotic form of the screened Coulomb potential is an oscillatory function of the form $\cos(2k_F r+\varphi)/r^3$. There is again experimental evidence[3] to support this result.

The purpose of this Letter is to show that there exists a Kohn anomaly in liquid metals which exhibits itself as a logarithmic singularity at $\kappa=2k_F$ in the gradient of the fourth moment $\langle\omega_{coh}{}^4(\kappa)\rangle$ of the usual scattering function $S_{coh}(\vec{\kappa},\omega)$. Since the width $\Delta E(\kappa)$ of the quasi-elastic scattering is related to $\langle\omega_{coh}{}^4(\kappa)\rangle$, the former also exhibits a similar anomalous behavior. The very existence of a dispersion curve in liquid metals is of somewhat doubtful validity; and even if it exists,[4] it is more like a broad band. The latter fact precludes the possibility of detecting Kohn anomaly except in an indirect way as Johnson et al.[5] have done by computing the interionic potential, using the Born-Green or the Percus-Yevick equations and experimentally determined values of the pair-distribution function, and establishing the oscillatory nature of the potential. In view of some of the approximations and possible errors in the Johnson et al. analysis leading to a long-range oscillatory $V(r)$, it would be desirable to observe the complementary aspect of the singularity in a parameter that is directly observable in wave vector space. In the following we shall establish the presence of this anomaly.

The ratio of the fourth moment to the square of the second moment of $S_{coh}(\vec{\kappa},\omega)$ for a coherent classical liquid as derived by de Gennes[6] is given by

$$\frac{\langle\omega_{coh}{}^4(\kappa)\rangle}{\langle\omega_{coh}{}^2(\kappa)\rangle^2} = 3[1+\Gamma(\kappa)]+\frac{1+\Gamma(\kappa)}{\kappa^2 k_B T}\left[\int g(r)\frac{\partial^2 V}{\partial x^2}(1-\cos\kappa x)d\vec{r}\right], \quad (1)$$

where $\Gamma(\kappa)$ is defined by

$$\Gamma(\kappa)=\int[g(r)-g_0]e^{i\vec{\kappa}\cdot\vec{r}}d\vec{r}, \quad (2)$$

and $\langle\omega_{coh}{}^2(\kappa)\rangle$ by

$$\langle\omega_{coh}{}^2(\kappa)\rangle=\kappa^2 k_B T/M[1+\Gamma(\kappa)]. \quad (3)$$