OCCURRENCE OF SINGULARITIES IN OPEN UNIVERSES

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At the present time the universe is observed to be expanding. If we assume that there is no creation of matter, this indicates that the density must have been higher in the past. The question then arises, was there some time in the past when the density was infinite (i.e., was there a singularity of space-time), or did the universe contract until it reached a finite maximum density and then expand again? This was partly answered by Robertson[1] who showed that if the universe was spatially homogeneous and isotropic, its metric could be written

$$ds^2 = dt^2 - R^2(t)[dr^2/(1-Kr^2) + r^2(d\theta^2 + \sin^2\theta d\varphi^2)], \quad (1)$$

$$K = -1, \ 0, \ \text{or} \ +1.$$

Robertson showed that provided the matter has normal properties and the Einstein equations without cosmological constant held,

$$R_{ab} - \tfrac{1}{2} g_{ab} R = -T_{ab},$$

then there would be a physical singularity in any universe whose metric had the form (1). This form restricts the flow of the matter to be acceleration-, shear-, and rotation-free. It has been suggested[2] that if these restrictions were dropped there might not be a physical singularity (although there might still be a coordinate singularity). However, it has been shown that there will still be a physical singularity if the flow of the matter is acceleration- and rotation-free[3,4] or if the universe is homogeneous but not isotropic[5] (in which case there could be acceleration, shear, and rotation). All of these cases place some restriction or exact symmetry on the flow, however, and it has been claimed[6] that in the absence of such restrictions or exact symmetries there will not be a physical singularity. That is, if we have a model that is a small perturbation of one of these restricted models, then the perturbations will grow as we go back in time and will prevent the occurrence of a physical singularity. This claim has already been proved false by Penrose[7] for the case of a collapsing star. Using similar methods it will be shown that this claim is also false for a class of universe models.

Consider universes of the form (1) where $K = -1$. In these the surfaces of homogeneity H (t = const) have constant negative curvature. They may or may not be compact, but if they are compact, they will not be simply connected. However, the covering space will be noncompact. Since a singularity in the covering space implies a singularity in the original space, it will be sufficient to consider the case where the surfaces of homogeneity are noncompact.[8]

In one of the three-surfaces H with future directed unit normal V^a $(1,0,0,0)$, consider a two-sphere S (r = const, t = const). Let q^a be the outward directed unit normal to S in H. Consider the two families of past directed null geodesics with tangent vector l^a, that intersect S orthogonally. Then at S, $l^a = -V^a \pm q^a$. Introduce two unit spacelike vectors s^a and t^a orthogonal to each other and to V^a and q^a. Form the complex combinations

$$m^a = 2^{-1/2}(s^a + it^a), \quad \bar{m}^a = 2^{-1/2}(s^a - it^a).$$

Then the convergence of the null geodesics is represented by the quantity[9]

$$\rho = l_{a;b} m^a \bar{m}^b.$$

Thus at S,

$$\rho = (-V_{a;b} \pm q_{a;b}) m^a \bar{m}^b$$
$$= \frac{2\dot{R}}{R} \pm \frac{2(1-Kr^2)^{1/2}}{rR}.$$

But $3(K + \dot{R}^2)/R^2 = \mu$, where μ is the density of matter.[10] Therefore,

$$\rho = (2/R)[(\tfrac{1}{3}\mu R^2 - K)^{1/2} \mp (1-Kr^2)^{1/2}/r].$$

If $\mu > 0$ and $K = 0$ or -1, ρ can be made positive at S for both families of null geodesics by taking r large enough. Thus both families of null geodesics normal to S will be converging. Therefore, in the language of Penrose,[7] S will be a closed trapped surface. Penrose has shown that either a physical singularity must occur or space-time is incomplete if there is a closed trapped surface, and (a) the energy density $T_{ab}\omega^a\omega^b$ in the rest frame of any observer with velocity ω^a is non-negative definite; (b) the null cones form two separate systems, past and future (this means that one can globally assign a time direction); (c) there is a noncompact Cauchy surface; that is, there exists a noncompact spacelike surface that intersects every timelike and null line once and once only. These conditions are satisfied by a universe of form (1) with $K=0$ or -1 and filled with normal matter. However, at S, ρ is positive by a finite amount. Therefore any perturbation of (1) which is not too large will leave ρ positive, and so there will still be a closed trapped surface and hence a physical singularity. Thus a universe that is similar on a large scale to the form (1) but has no exact local symmetries will have a singularity. Local irregularities cannot prevent it.

The presence of matter gives a unique timelike congruence (the flow lines) which may be defined as the timelike eigenvector of the Ricci tensor. Using this it is possible to replace condition (c) by the weaker conditions (d) and (e): (d) The covering space has no closed timelike lines; (e) there exists a complete connected noncompact spacelike three-surface on which the density has a positive lower bound and the scalar product of its unit normal and the velocity vector of the flow lines has an upper bound. This removes the possibility that a singularity might be avoided by there being regions of space-time for which H is not a Cauchy surface.

The author and Dr. G. F. R. Ellis are working on a proof of the occurrence of singularities in closed universes. This and an extension of the above results will be published shortly.

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DIRECT DETECTION OF TWO-PHOTON EMISSION FROM THE METASTABLE STATE OF SINGLY IONIZED HELIUM*

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In this Letter we report on the direct detection by coincidence counting techniques of the two-photon decay of the metastable $2\,^2S_{1/2}$ state of singly ionized helium. The metastability of the 2S state of hydrogenic atoms has been the subject of many theoretical studies.[1] The 2S states of H and He$^+$ are known to be metastable, and the lifetimes have been shown to be greater than 2.4×10^{-3} sec and 1×10^{-3} sec, respectively.[2] The present theoretical view is that an unperturbed metastable hydrogenic atom will decay by two-photon emission at a rate given by $\gamma=8.226\ Z^6\ \mathrm{sec}^{-1}$.[3] The decay rate for all other decay modes considered so far is orders of magnitude slower than the two-photon rate.[4] Several attempts have been made to detect similar two-photon decays in excited nuclei. In no case has conclusive evidence been obtained for the existence of such transitions.[5] Extensive observations have been made of multiple-photon processes with intense laser fields.[6] Multiple-photon-induced absorption and emission processes in the rf region of the spectrum have been studied in considerable detail by molecular beams and optical pumping techniques.[7]

The present experiment was designed to detect the decay in flight of a slow (12-eV) beam of metasteble helium ions. Ionized helium was chosen in preference to hydrogen because the two-photon lifetime and the Stark quenching rate are each smaller by a factor of 64 in the helium. In addition, standard ion-beam techniques could be employed to focus and control the beam. In the case of the present beam,