pairs) are involved.

[10]It may be interesting to study Pb films having a thin normal-metal overlay.

[11]N. N. Bogoliubov, V. V. Tolmachov, and D. V. Sirkov, in Theory of Superconductivity, edited by N. N. Bogoliubov (Gordon and Breach Publishers, Inc., New York, 1962), p. 278.

[12]P. W. Anderson, Phys. Rev. 112, 1900 (1958).

[13]J. M. Blatt, Theory of Superconductivity (Academic Press, Inc., New York, 1964), p. 221.

GAPLESS SUPERCONDUCTIVITY INDUCED BY METALLIC CONTACTS*

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Numerous experiments[1] performed over the last few years show that superposition of normal and superconducting metal films can alter the superconducting properties of the films which are in contact. So it was shown that the transition temperature T_c of such a system depends on the thickness D_S and D_n of the superconducting and normal film, respectively, and theoretical investigations by de Gennes and Guyon[2] and Werthamer[3] are in good quantitative agreement with experiments. We are here primarily concerned with another sort of experiment with such contacts, as performed by Reif and Woolf,[4] in which the tunneling density of states of a superconducting film backed by a paramagnetic film is measured.

In order to get a theoretical understanding of such a situation, we first determine the tunneling density of states for a superconducting film in contact with a nonmagnetic normal film assuming that the mean free path in the metals is small compared with the coherence distance. Our calculations, which are done in the vicinity of the second-order phase transition point where the order parameter is small, show that the tunneling density of states of such a contact is similar to the one found for superconductors containing paramagnetic impurities.[5] This reveals that the essential influence of a metallic contact on superconductivity is a tendency to break electron pairs. We should like to point out that we have here, quite unexpectedly, an example of gapless superconductivity which is not caused by an interaction which breaks time-reversal symmetry. In fact, even for large order parameters where our expressions do not hold, we expect a tunneling characteristic similar to the one for superconductors with paramagnetic impurities. Interesting experiments—for example, tunneling measurements—to prove this point are suggested. Second, our calculations are extended to the case of contacts between superconducting films and paramagnetic metals. No qualitative difference arises in that case, but quantitatively paramagnetic metals exert an order of magnitude larger pair-breaking effect upon the superconducting electrons as compared with nonmagnetic normal metals.

We express the tunneling density of states $N(\vec{r},\omega)$ in terms of the Green's function $G(\vec{r}_1\vec{r}_2,\omega)$ as

$$N(\vec{r},\omega)=(1/\pi)\,\mathrm{Im}G(\vec{r},\vec{r},\omega), \tag{1}$$

where $G(\vec{r},\vec{r}',\omega)$ formally can be written up to terms of second order in the order parameter $\Delta(\vec{r})$ as

$$G=\{G_0\}_{rs}+\{G_0\Delta G_0^{-}\Delta^{+}G_0\}_{rs}.$$

Here G_0 is the Green's function for a normal metal, and $\{\cdots\}_{rs}$ indicates that an average over randomly distributed scattering centers has to be taken. This averaging process can be carried out by using a renormalization procedure developed by one of the authors (K.M.).[6] We find

$$G(\vec{r},\vec{r}',\omega)=\int\frac{d^3p}{(2\pi)^3}\frac{e^{i\vec{p}(\vec{r}-\vec{r}')}}{\tilde{\omega}-\xi_{\vec{p}}}-\int\frac{d^3p}{(2\pi)^3}d^3r_1r_2\frac{e^{i\vec{p}(\vec{r}-\vec{r}_1)}}{\tilde{\omega}-\xi_{\vec{p}}}$$

$$\times\eta_{\vec{q}_1}\Delta(\vec{r}_1)\frac{e^{i(\vec{p}+\vec{q}_1)(\vec{r}_1-\vec{r}_2)}}{\tilde{\omega}+\xi_{\vec{p}+\vec{q}_1}}\eta_{\vec{q}_2}\Delta^{+}(\vec{r}_2)\frac{e^{i(\vec{p}+\vec{q}_1-\vec{q}_2)(\vec{r}_2-\vec{r}')}}{\tilde{\omega}-\xi_{\vec{p}+\vec{q}_1-\vec{q}_2}}, \tag{2}$$

where $\xi_p = p^2/2m - \mu$ and μ is the chemical potential. Furthermore, we have set

$$\tilde{\omega} = \omega\left(1 + \frac{i}{2|\omega|\tau}\right), \quad \eta_{\vec{q}_{1,2}} = \left\{1 - \frac{i}{2|\tilde{\omega}|\tau}\left(1 - \frac{\tau_{tr}\tau v_F^2 q_{1,2}^2}{3}\right)\right\}^{-1}, \quad \vec{q}_{1,2} = -i\frac{\partial}{\partial \vec{r}_{1,2}}; \tag{3}$$

τ and τ_{tr} are the collision time and the transport collision time, respectively. Setting the above expression for G into Eq. (1) we obtain

$$N(\vec{r}, \omega) = N(0)\,\mathrm{Re}\left\{1 - \tfrac{1}{2}\prod_{j=1}^{2}\frac{1}{(i\omega - Q_j)}\Delta(\vec{r}_1)\Delta^+(\vec{r}_2)\right\}\Bigg|_{\vec{r}_1 = \vec{r}_2 = \vec{r}}, \tag{4}$$

where

$$Q_j = -\tau_{tr} v_F^2 \nabla_j^2/6. \tag{5}$$

For simplicity we assume in the following that the electronic density as well as the product $v_F^2\tau$ is equal in both metals brought into contact. The extension to the more general case is straightforward though algebraically more complicated. If we have two films of thickness D_s and D_n, with transition temperatures T_{cs}, T_{cn} $(T_{cn}/T_{cs} \ll 1)$ and electron interaction parameters V_s and V_n occupying the regions $0 < x \leq D_s$ and $-D_n \leq x < 0$, respectively, then for temperatures just below the transition temperature T_c of the metallic complex the order parameter $\Delta(x)$ is given according to reference 3 as

$$\Delta(x) = aV_s \frac{\cos k_s(x - D_s)}{\cos k_s D_s}, \qquad 0 < x \leq D_s,$$

$$= aV_n \frac{\cosh k_n(x + D_n)}{\cosh k_n D_n}, \quad -D_n \leq x < 0, \tag{6}$$

where a is a normalization factor. T_c, k_n, k_s are determined from

$$\chi(\xi^2 k_s^2) = \ln(T_{cs}/T_c), \tag{7a}$$

$$-\chi(-\xi^2 k_n^2) = \ln(T_c/T_{cn}), \tag{7b}$$

$$k_s \tan k_s D_s = k_n \tanh k_n D_n. \tag{7c}$$

Here $\chi(z)$ is defined by $\chi(z) = \psi(\frac{1}{2} + z/2) - \psi(\frac{1}{2})$ and $\xi^2 = v_F^2\tau_{tr}/6\pi T_c$. Inserting the above expressions for Δ into Eq. (4), we obtain for the tunneling density of states of a superconducting film backed by a nonmagnetic normal film

$$N(\vec{r}, \omega) = N(0)\left[1 + \frac{|\Delta(\vec{r})|^2}{2}\frac{\omega^2 - \alpha^2}{(\omega^2 + \alpha^2)^2}\right], \tag{8}$$

where

$$\alpha = \tau_{tr} v_F^2 k_s^2/6. \tag{9}$$

This expression is equivalent to the one obtained in gapless regions for superconductors with paramagnetic impurities or type-II superconductors with α characterizing the strength of the depairing effect.[7] We want to remark that because of the appearance of α our expansion of $N(\vec{r}, \omega)$ in powers of Δ^2 is justified for sufficiently small Δ. By analogy with the paramagnetic impurity case we expect that the expansion breaks down and the gap appears for $\Delta \gtrsim \alpha$. The right normalization factor for Δ can be obtained from the generalized Ginzburg-Landau equation which we write according to (M) as

$$\left\{\ln\frac{T}{T_c(\vec{r})} + \chi(-\xi_T^2\nabla^2)\right\}\Delta(\vec{r}) + \frac{1}{8(\pi T)^2}\sum_n (n + \tfrac{1}{2})\prod_{i=1}^{4}\left(n + \tfrac{1}{2} + \frac{Q_i}{2\pi T}\right)^{-1}\Delta(\vec{r}_1)\Delta^+(\vec{r}_2)\Delta(\vec{r}_3)\Bigg|_{\vec{r}_1 = \vec{r}_2 = \vec{r}_3 = \vec{r}} = 0, \tag{10}$$

where $\xi_T^2 = v_F^2\tau_{tr}/6\pi T$. From this equation one can derive

$$\langle|\Delta^2|\rangle_{av} = \frac{\pi^6}{7\zeta(3)}\frac{\kappa^2(T_c - T)}{2\kappa_2^2(T)\beta}\rho g^{-2}(\rho)[1 - g(\rho)], \tag{11}$$

where $\langle\cdots\rangle_{av}$ indicates a spatial average over the superconducting film and

$$\rho = \frac{\alpha}{2\pi T}, \quad g(\rho) = \sum_{n=0}^{\infty}(n + \tfrac{1}{2} + \rho)^{-2}, \quad \beta = \frac{\langle|\Delta^4|\rangle_{av}}{(\langle|\Delta^2|\rangle_{av})^{-2}}.$$

κ was calculated by Gor'kov[8] and is given by

$$\kappa = \frac{3m}{2\pi^2 e\tau_{\rm tr}}\left[\frac{2\pi m}{p_0^{\,5}}7\zeta(3)\right]^{1/2}.$$

For the definition of the temperature-dependent parameter $\kappa_2(T)$ we refer to (M). The quantity β is found to be limited by $\frac{3}{2} > \beta > 1$.

In the case of a contact between a superconducting film and a paramagnetic metal we obtain a set of equations similar to Eqs. (7) with the only difference that in (7c) k_n is replaced by k_p while Eq. (7b) is modified as

$$-\chi\left(\frac{1}{\pi\tau_s T_c} - \xi^2 k_p^{\,2}\right) = \ln\frac{T_c}{T_{cp}}. \qquad (7b')$$

Here τ_s is the inverse of the spin-flip scattering rate due to the paramagnetic ions. From our calculations it is apparent that the pair-breaking effect caused by a metallic contact has its physical reason in the space dependence of the order parameter which is seen from the form of $\Delta \sim \mathrm{Re}\{e^{i\varphi}e^{ikx}\}$. Thus the situation is analogous to the current-carrying case[9] with the only difference that in our case the finite momentum k is introduced into the system by matching at the boundary while in the current case it is introduced externally.

In order to measure the depairing effect caused by the contact we note the following points:

(a) It is seen from Eq. (7a) that the strength of the pair-breaking effect, which is characterized by α, is larger the greater the decrease of the transition temperature T_c due to the metallic contact.

(b) In order to see the quantatitive difference in α for superconducting films backed with nonmagnetic and paramagnetic metal films, we note from Eq. (7c) that k_s is a monotonic function of k_n (or k_p). Since, due to Eqs. (7b), (7b′), k_n and k_p are limited to $0 < k \leq \xi^{-1}$ and $0 < k_p \leq \xi^{-1}(1 + 1/\pi T\tau_s)^{1/2}$, it follows that for $1/\pi T\tau_s$ of the order of 10, the effect of a paramagnetic metal contact is an order of magnitude larger than that of a nonmagnetic metal contact having the same dimensions.

(c) Expressions similar to Eqs. (8) and (9) are obtained for the tunneling density of states on the normal side of the contact. In that case α is of the form $\alpha = \tau_{\rm tr} v_{\rm F}^2 k_n^2/6$.

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[1]P. Hilsch, Z. Physik 167, 511 (1962). In this paper a rather complete list of the experimental work performed on that subject can be found.

[2]P. G. de Gennes and E. Guyon, Phys. Letters 3, 168 (1963); P. G. de Gennes, Rev. Mod. Phys. 36, 225 (1964).

[3]N. R. Werthamer, Phys. Rev. 132, 2440 (1963).

[4]M. A. Woolf and F. Reif, Phys. Rev. 137, A557 (1965).

[5]A. A. Abrikosov and L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 39, 1781 (1960) [translation: Soviet Phys.—JETP 12, 1243 (1961)].

[6]K. Maki, Physics 1, 21, 201 (1964). This paper will be referred to hereafter as (M).

[7]P. G. de Gennes, Phys. Condensed Matter 3, 79 (1964).

[8]L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 37, 1407 (1959) [translation: Soviet Phys.—JETP 10, 9981 (1960)].

[9]K. Maki, Progr. Theoret. Phys. (Kyoto) 29, 10 and 333 (1963).

HELICONLIKE RESONANCES IN THE INTERMEDIATE STATE*

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We have observed low-Q heliconlike resonances in the intermediate state of high-purity indium.[1] These resonances have a line shape which is similar to that observed in the normal metal; however, important differences exist between the intermediate- and normal-state resonances. In attempting to interpret our resonance results, information is obtained which is complementary to that found from other observations of the intermediate state of indium.[2] Because of the recent development of the flux-tube model of the intermediate state, it is of interest to compare our results with its predictions.[3] Some aspects of the data are consistent with this model, but there are also discrepancies which need further study.