at maximum compression, while the other symbols have the meaning and numerical values given in reference 2.

Noting that

$$\int_0^{2\pi} \cos^8 t dt = 1.72,$$

we find for the dissipation the value $8.38\times10^{18} \times[\xi_{av}]^8$ erg g^{-1} sec^{-1}, or $1.01\times10^{52}[\xi_{av}]^8$ erg sec^{-1} for a neutron star of mass 1.2×10^{33} g. As a consequence of the high exponential, this dissipation is very large in comparison with the energy lost in the exponential decay, $2.3 \times10^{45}[\xi_{av}]^2$ erg sec^{-1}, until ξ_{av} is reduced to $(2.3\times10^{45}/1.01\times10^{52})^{1/6}=0.078$; after that it becomes quite negligible. This means that during the first few days after the outburst, the reaction just described will reduce the vibrational energy to the value $\frac{1}{2}(\Gamma-\frac{4}{3})\Omega\times0.078^2 =10.1\times10^{49}$ erg; subsequently, the mechanism responsible for the exponential decay will dominate. The agreement between the theoretical value 10.1×10^{49} erg and the value 4×10^{49} erg deduced above from observation could be improved by taking into account the temperature dependence of the dissipation of vibrational energy be beta reactions.

The probable mechanism by which the vibrational energy is converted into visible light will be described in this Letter only very briefly. An outgoing shock wave accompanying each vibration[8,9] will transfer energy to the surface of the star. Here, the energy will be dissipated and emitted in the form of thermal radiation consisting of hard x rays. These x rays will be absorbed by the expanding envelope of the supernova and re-emitted in the visible region. The ratio of the period of vibration of the neutron star (which is less than 10^{-3} sec) to the half-value time $t_{1/2}$ of the luminosity is about 10^{-10}, therefore the observed exponential decay would be accounted for by assuming that the coefficient of reflection at the surface of the star differs from unity by about 10^{-10}.

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LORENTZ-COVARIANT GRAVITATIONAL ENERGY-MOMENTUM LINKAGES

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This Letter presents a gravitational energy-momentum expression with the transformation properties of a Lorentz free vector in asymptotically flat but radiative spaces. This expression and its transformation properties apply to finite regions as well as to the entire space.

In the Bondi–van der Burg–Metzner[1] formulation of the characteristic initial-value problem, certain of the field equations need only be applied on a world tube. For world tubes of topology $S^2\times E^1$, we have cast these equations in the form of conservation conditions

$$K_\xi(\Sigma_2)-K_\xi(\Sigma_1)=\int_\Gamma F_\xi{}^\alpha dS_\alpha, \qquad (1)$$

where

$$K_\xi(\Sigma)=\oint_\Sigma \xi^{[\alpha;\beta]}dS_{\alpha\beta}, \qquad (2)$$

$$F_\xi{}^\alpha \equiv \xi^{[\alpha;\beta]}{}_{;\beta}$$
$$=\xi^{(\alpha;\beta)}{}_{;\beta}-\xi^\beta{}_{;\beta}{}^\alpha+\tfrac{1}{2}\xi^\alpha R+\kappa\xi^\beta T_\beta{}^\alpha. \qquad (3)$$

Here Σ_1 and Σ_2 represent spacelike slices of the world tube with topologies S^2, Γ is the portion of the world tube bounded by Σ_1 and Σ_2, $T_\beta{}^\alpha$ is the energy-momentum tensor of the coupled fields, R is the Ricci scalar, and ξ^μ

is an arbitrary vector field. Brackets and parentheses denote antisymmetrization and symmetrization, respectively. Equation (1) is the integral form of a covariant conservation law previously considered by Komar.[2] In order to interpret the scalar functional $K_\xi(\Sigma)$ physically, we associate the vector field ξ^μ with the descriptor of an infinitesimal coordinate transformation. The ability to identify a particular descriptor with, say, a time translation would then correlate the corresponding functional with energy. To avoid the specification of any preferred spacelike three-volume in which we could say the energy resides, we will interpret this functional as the energy linkage through Σ. Linkage[3] here is meant in its topological sense as the four-dimensional analog of the three-dimensional concept of a trajectory passing through a closed loop. The energy linkage thus represents the total energy passing in time through Σ. For an arbitrary descriptor field we will refer to the corresponding functional as the ξ linkage.

To check the reasonableness of these interpretations we first need a means of geometrically selecting descriptor fields. One natural way is to utilize the symmetries or asymptotic symmetries of the space. Consider the case of a global symmetry which provides a Killing vector field satisfying

$$\xi^{(\mu;\nu)}=0. \tag{4}$$

In this case, the ξ linkage is a constant functional of Σ in empty space regions. As an example, consider the charged Kerr metric,[4] which describes a rotating ring of charge e, mass m, and spin ma:

$$ds^2=(1+W)du^2+2dudr-a\sin^2\theta(Wdu+dr)d\varphi$$
$$-R^2d\theta^2-\sin^2\theta(r^2+a^2+ag_{03})d\varphi^2,$$

where

$$R^2=r^2+a^2\cos^2\theta,\quad W=(e^2-2mr)/R^2.$$

This metric possesses a timelike translational symmetry and a rotational symmetry. The corresponding linkages are

$$E(r)=m-\frac{e^2}{2r}-\frac{e^2(r^2+a^2)}{2ar^2}\tan^{-1}\left(\frac{a}{r}\right), \tag{5a}$$

$$L(r)=ma+\frac{e^2}{4ar}(r^2-a^2)-\frac{e^2(r^2+a^2)^2}{4a^2r^2}\tan^{-1}\left(\frac{a}{r}\right), \tag{5b}$$

where $E(r)$ is the energy linkage and $L(r)$ the angular-momentum linkage through a sphere of radius r linked by the ring. The leading terms in Eqs. (5a) and (5b) describe the total linkages through a sphere at infinity. The remaining terms describe flux contributions due to the Maxwell stress tensor. For a closed two-surface not linked by the rotating ring, only the electromagnetic field terms contribute to the linkages. Newman and Janis[5] have interpreted the case $e=0$ in a similar way.

Although global symmetries do not exist in general, for certain asymptotically flat space-times there exist asymptotic symmetries,[6,7] which describe the isometries of future null infinity $\mathscr{I}^+$ and past null infinity $\mathscr{I}^-$. Here we will concentrate on the properties of $\mathscr{I}^+$. Using the conformal techniques of Penrose,[8] we treat $\mathscr{I}^+$ as a regular hypersurface with topology $S^2\times E^1$. Coordinates x^α and descriptor fields $\eta^\alpha(x)$ on $\mathscr{I}^+$ then take on a precise meaning.

The Bondi-Metzner-Sachs asymptotic symmetry group[6,7] (BMS group) is an infinite-dimensional Lie group which contains a unique four-dimensional normal translation subgroup and an infinite-dimensional normal supertranslation subgroup whose factor group is isomorphic to the orthochronous homogeneous Lorentz group. Let $\eta_Q{}^\alpha(x)$ $(Q=0,1,2,\cdots)$ be a basis for BMS descriptors defined on $\mathscr{I}^+$.

There is no geometrically intrinsic way of propagating descriptors on $\mathscr{I}^+$ throughout space time. A two-surface Σ, however, does determine a geometrical prescription for uniquely propagating descriptors from $\mathscr{I}^+$ to Σ. Each point on Σ geometrically determines two null directions which are orthogonal to the local two-space. The entirety of outward null directions on Σ defines a null hypersurface which, for simple topologies, emanates out from Σ and intersects $\mathscr{I}^+$ in a two-space Σ^+ (see Fig. 1). Let k^μ denote a vector field normal to this null hypersurface; then the propagation law

$$\xi^{(\mu;\nu)}k_\nu=\tfrac{1}{2}\xi^\rho{}_{;\rho}k^\mu \tag{6}$$

uniquely determines ξ^μ on the null hypersurface in terms of its value at Σ^+. There is an intimate connection between this propagation law and the Bondi coordinate conditions.[1] Descriptors of global isometries automatically satisfy Eq. (6) everywhere.

Using Eq. (6) we can now geometrically define a ξ_Q linkage through Σ for each asymptotic

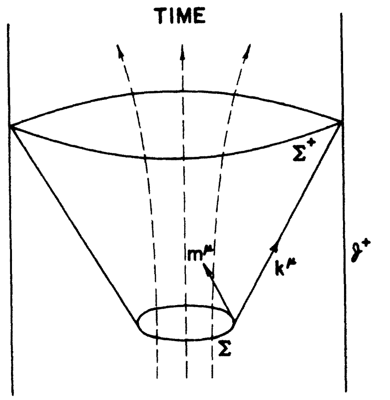

FIG. 1. In this three-dimensional drawing, Σ appears as a closed loop. The null rays k^μ generate a null hypersurface extending from Σ to Σ^+. The dotted lines depict trajectories linking Σ and Σ^+.

symmetry $\eta_Q{}^\alpha$. The total linkage through corresponding to a time translation, however, does not yield the total energy of a radiating system as defined by Bondi, van der Burg, and Metzner.[1] Instead it gives a total energy linkage which need not be monotonically smaller for infinite spheres Σ^+ at later times. We have been able to remove this shortcoming in the following way. Represent by the vectors k^μ and m^μ, respectively, the outgoing and incoming null directions which Σ picks out at each point on its surface. Normalize these vectors by

$$k^\mu m_\mu = 1. \tag{7}$$

Although Eq. (7) does not uniquely determine the extensions of k^μ and m^μ, it does completely fix bilinear products such as the bivector

$$B^{\mu\nu} = k^{[\mu}m^{\nu]}. \tag{8}$$

We now redefine the ξ linkage to be

$$L_\xi(\Sigma) = K_\xi(\Sigma) + \oint_\Sigma B^{\mu\nu} dS_{\mu\nu}. \tag{9}$$

This linkage $L_\xi(\Sigma)$ does yield the correct total energy for radiative spaces. The added bivector term has the further virtue of guaranteeing that the evaluation of $L_\xi(\Sigma)$ requires only those derivatives of ξ^μ which lie in the outgoing null hypersurface. This is crucial since the propagation law determined by Σ defines ξ^μ only on this null hypersurface. Note that for descriptors of global isometries this problem does not arise, and the added bivector term vanishes.

In order to obtain a conservation law for $L_\xi(\Sigma)$ analogous to Eq. (1), we must define the flux

$$F_\xi{}'^{\mu} \equiv (\xi^{[\mu;\nu]} + \xi^\rho{}_{;\rho} k^{[\mu}m^{\nu]})_{;\nu} \tag{10}$$

across a world tube Γ connecting Σ_1 and Σ_2. This entails defining the quantity

$$\xi^{[\mu;\nu]} + \xi^\rho{}_{;\rho} k^{[\mu}m^{\nu]}$$

along the world tube (only in-surface derivatives of this quantity are necessary to calculate the flux $F_\xi{}'$ across Γ). We can accomplish this by assigning a continuous slicing of Γ into a family of closed two-surfaces whose first member is Σ_1 and whose last member is Σ_2. Each slice then defines a descriptor field ξ^μ along its outgoing null hypersurface and a bivector $k^{[\mu}m^{\nu]}$. There are, however, an infinity of acceptable slicings. One natural choice is the slicing induced on Γ by a geodesically parallel slicing of $\mathcal{J}^+$ from $\Sigma_1{}^+$ to $\Sigma_2{}^+$ (see Fig. 2). While any choice of slicing does lead to the same total integrated flux across Γ, the lack of a unique choice presents difficulties in defining a local flux across Γ. The local flux across a null hypersurface turns out to be more fundamental. The total flux across Γ equals the sum

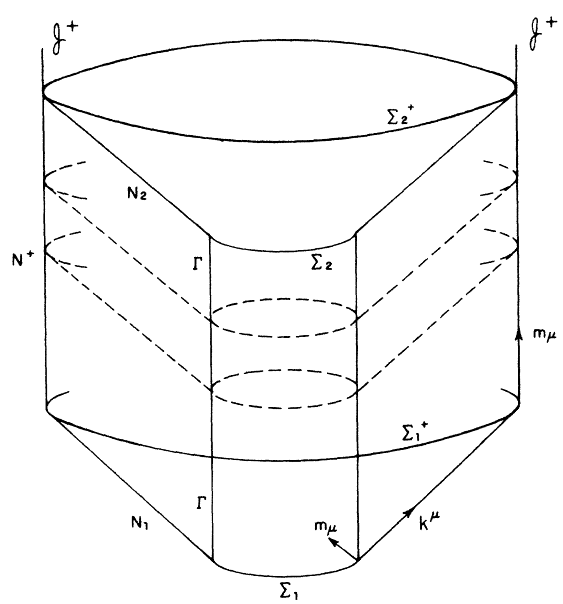

FIG. 2. The dotted lines illustrate how geodetically parallel slices on $\mathcal{J}^+$ induce a family of slices on the timelike world tube Γ. The total flux F^α across the surface of Γ is equal to the sum of the fluxes across the null surfaces N_1, N_2, and N^+.

of the fluxes across the three null hypersurfaces N_1, N_2, and N^+ shown in Fig. 2 ($\mathscr{I}^+$ is a null hypersurface[8]). We define m^μ along N_1 by

$$m_\mu k^\mu = 1, \quad m^\mu m_\mu = m^\mu{}_{;\nu} k^\nu = 0. \tag{11}$$

This determines m^μ up to a null rotation. The local flux across N_1 then becomes

$$F_\xi{}'^\mu \equiv \xi^{[\mu;\nu]}{}_{;\nu} + (\xi^\rho{}_{;\rho} k^\nu)_{;\nu} m^\mu \tag{12}$$

and is invariant under null rotations of m^μ because the surface element lies in the k_μ direction. We can similarly obtain geometrically defined local fluxes across N_2 and N^+. In this way the use of null hypersurfaces leads to a flux conservation law for L_ξ which does not involve an assignment of slices.

All the total linkages $L_Q(\Sigma^+)$ corresponding to asymptotic symmetries $\eta_Q{}^\alpha$ are mathematically defined finite quantities. In particular, choose a basis $\eta_a{}^\alpha(x)$ $(a = 0, 1, 2, 3)$ for the translational descriptors on $\mathscr{I}^+$ representing infinitesimal translations along four orthogonal axes, with $\eta_0{}^\alpha$ representing a timelike translation. Label the corresponding energy-momentum linkage by $P_a(\Sigma)$. In a neighborhood of $\mathscr{I}^+$, we then find as a consequence of the outgoing radiation condition[1,6]

$$P_a(\Sigma) = P_a(\Sigma^+) + O(r^{-2}), \tag{13}$$

where r is a luminosity distance along the null hypersurface from Σ to Σ^+. This equation confirms the absence of incoming fluxes of energy and momentum which could contribute to $P_a(\Sigma)$ near $\mathscr{I}^+$.

Up to now we have purposely refrained from the introduction of any special coordinate system in order to emphasize the geometrical nature of our results. There is, however, no unique choice of orthogonal basis $\eta_a{}^\mu(x)$ with which to define $P_a(\Sigma)$. The isometries of $\mathscr{I}^+$ do guarantee the existence of preferred coordinates y^α on $\mathscr{I}^+$:

$$y^0 = u, \quad y^1 = 1/r = 0, \quad y^2 = \theta, \quad y^3 = \varphi. \tag{14}$$

These coordinates are unique up to a BMS transformation. To each such coordinate system Sachs[7] has assigned a canonical basis

$$\eta_a{}^\mu(y) = [Y_a(\theta, \varphi), 0, 0, 0], \tag{15}$$

where Y_a are the spherical functions

$$Y_0 = 1, \quad Y_1 = \sin\theta \cos\varphi,$$
$$Y_2 = \sin\theta \sin\varphi, \quad Y_3 = \cos\theta. \tag{16}$$

Under a BMS transformation,

$$\bar\eta_a{}^\mu(\bar y) = \frac{\partial \bar y^\mu}{\partial y_\alpha}\eta_a{}^\alpha(y)\Big|_{\mathscr{I}^+} = L_a{}^b \eta_b{}^\mu(\bar y), \tag{17}$$

where $L_a{}^b$ is the associated orthochronous Lorentz matrix, and where $\eta_b{}^\mu(\bar y)$ is the canonical basis for the $\bar y$ coordinates:

$$\eta_b{}^\mu(\bar y) = [Y_b(\bar\theta, \bar\varphi), 0, 0, 0]. \tag{18}$$

Denote by $P_a(\Sigma)$ and $\bar P_a(\Sigma)$ the energy-momentum linkages associated with canonical bases for the y^α and $\bar y^\alpha$ coordinate systems, respectively. Then using the scalar character of the functional $P_a(\Sigma)$, the covariance of the propagation law Eq. (6), the linearity in ξ^μ of both the functional and the propagation law, and Eq. (17), we obtain

$$P_a(\Sigma) = L_a{}^b \bar P_b(\Sigma). \tag{19}$$

This transformation law states that the energy-momentum linkage behaves as a Lorentz free vector when interpreted by the preferred observers on $\mathscr{I}^+$.

Similar, although more complicated, statements can be made about the transformation properties of all the asymptotic symmetry linkages $L_Q(\Sigma)$. Constructs such as the angular-momentum linkage do not have transformation properties in complete analogy with Lorentz-covariant theories due to the supertranslation freedom.

A more detailed account of these results and their connection with the works of Bondi, van der Burg, and Metzner, of Penrose, and of Sachs is being prepared.

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X-RAY SPECTRA FROM SCORPIUS (SCO-XR-1) AND THE SUN OBSERVED ABOVE THE ATMOSPHERE*

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An important question which must be answered about the recently discovered stellar x-ray sources is the nature of the emission spectrum of these sources. Several experiments[1-5] have been performed in the past few years and a number of theoretical models[6,7] have been proposed to explain the observations. To obtain more precise information about the spectrum of the x-ray source in Scorpius (SCO-XR-1),[1] a proportional counter sensitive to photons with quantum energies between 2 and 20 keV was flown on a rocket. With this detector and a special telemetry system it was possible to measure the spectrum in this energy region with high resolution.

The counter employed in this experiment was a proportional gas counter filled with a 90%-10% xenon-methane mixture at atmospheric pressure; it had a resolution of 20% full width at half-maximum, at 5.9 keV. The counter window was made of 3-mil beryllium and had a rectangular shape with an area of 8.67 cm^2. Surrounding the counter (except over the window and over one end) was $\frac{1}{8}$ in. of plastic scintillator which was viewed by an RCA 4440 photomultiplier. This scintillator functioned as an anticoincidence shield against high-energy charged particles in cosmic rays. In addition, $\frac{1}{4}$-mil aluminized Mylar was placed over the counter window to protect the scintillator from light. The proportional counter was collimated to a transmission half-angle of $\pm 10°$ in azimuth and $\pm 45°$ in elevation, and it had a geometric factor of 3.41 cm^2 sr. The calculated efficiency of the proportional counter as a function of quantum energy is shown in Fig. 1.

The counter was mounted on an Honest John-Nike-Nike rocket which was launched on 12 June 1965 from Kauai, Hawaii. The launch time was 1515 hours UT, which was 39 minutes before local sunrise. At that time, the zenith coordinates were 22 hours right ascension and +22° declination. The rocket was launched towards an azimuth of 340° and 5° from the zenith and reached an altitude of 170 km. Apogee of the flight occurred at 160.0° W and 22.5° N at 1518 UT.

An Fe^{55}, Cd^{109} source was mounted on the inside of the nose cone for continual calibration during launch. The nose cone and this source were detached from the vehicle at an altitude of 88 km. Data were taken for the next 275 sec. The payload was spin-stabilized with a spin rate of 6.0 rev/sec, and was observed to precess in a cone of half-angle 3° with a frequency of 0.11 rev/sec. Therefore, the rocket spin vector pointed at a spot on the celestial sphere which traced out a circle of radius 3°, the center of which was located at 21 hours 50 min. right ascension and 27° declination. Scorpius was scanned every revolution during this time, and the x rays from the sun were detected during the 70-sec period centered on apogee during which the sun was high enough above the rocket's horizon so that atmospheric absorption did not completely attenuate the x rays. At apogee there was approximately 6×10^{-3} g/cm^2 of atmosphere between the counter and the sun.

Signals from the proportional counter were amplified and lengthened to 1.2 msec. This was done only with pulses not accompanied by

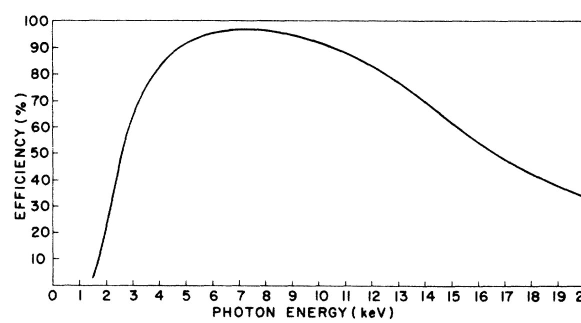

FIG. 1. Efficiency of Xe-CH_4-filled proportional counter as a function of x-ray energy.