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$p\bar{p}$ ANNIHILATION INTO TWO MESONS IN THE SPURION SCHEME OF BROKEN U(6,6) SYMMETRY

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The proton-antiproton annihilation at rest into two mesons is forbidden[1,2] in the limit of broken U(6,6) symmetry.[3] The same is true in the spurion scheme of broken U(6,6) symmetry,[4,2] provided we consider only momentum spurions in first order. Also most higher order terms vanish, but with the exception of one second-order term[5] which gives rather unsatisfactory results in comparison with the present experimental situation. On the other hand, within the scheme of broken U(6,6) symmetry, $p\bar{p}$ annihilation at rest into two mesons is forbidden in the meson-pole dominance model, even if momentum spurions are taken into account.[6] It is possible that this result is responsible for the fact that the two-meson annihilation is suppressed compared to three- and four-meson annihilations.

It is the purpose of this note to point out that the inclusion of the U(6) symmetry-breaking spurions[2,4] $(\gamma_5\otimes\gamma_5)\otimes\epsilon_1$ and $(\gamma_\mu\otimes\gamma_\mu)\otimes\epsilon_1$, which are still SU(3) singlets as indicated by the factor ϵ_1, gives rise to quite different and more reasonable predictions. It is the characteristic feature of the momentum spurions that they leave the mass terms invariant, and in this sense, preserve the U(6)-supermultiplet structure.[2] The annihilation amplitude into two mesons remains invariant under a group $U(6)_q$ with the generators $(1,\gamma_4\sigma_1,\gamma_4\sigma_2,\sigma_3)\otimes\gamma_a$, $a=0,\cdots,8$, where the relative momentum $\vec{q}$ of the final mesons is in the 3 direction.[2] On the other hand, the spurion $S_P=(\gamma_5\otimes\gamma_5)\otimes\epsilon_1$ splits the masses of singlet and octet pseudoscalar mesons if inserted into the mass term, but otherwise it preserves the degeneracy of supermultiplets.[2] Similarly, the spurion $S_V=(\gamma_\mu\otimes\gamma_\mu)\otimes\epsilon_1$ gives rise to a mass splitting between singlet and octet vector mesons, but in our model it is contracted only with the indices of baryon tensors.

Since $p\bar{p}$ annihilation at rest into two mesons is forbidden in the $U(6)_q$-invariant pole dominance model, we consider in the following two possible models:

(1) In the first case, we assume that the $U(6)_q$-invariant amplitude is supplemented by terms obtained with a U(6)-breaking spurion $(\gamma_5\otimes\gamma_5)\otimes\epsilon_1$ in arbitrary order. There remains invariance under a group $U(3)\otimes U(3)$ with generators $(1\pm\hat{\sigma}\cdot\hat{q})\otimes\lambda_a$.

(2) The $U(6)_q$-breaking [with spurions $(\gamma_5\otimes\gamma_5)\otimes\epsilon_1$ and $(\gamma_\mu\otimes\gamma_\mu)\otimes\epsilon_1$] pole terms are dominant compared to other symmetry-breaking nonpole terms. In this model, it is important to assume the meson-pole dominance, because otherwise the symmetry is broken down to SU(3). The $U(6)_q$-symmetric term will be included, since we assume it to be comparable with the $U(6)_q$-breaking pole terms.

First, we consider the model (1) in which we introduce the spurions $S_1=\not{q}\otimes\epsilon_1$ and $S_P=(\gamma_5\otimes\gamma_5)\otimes\epsilon_1$ into the $p\bar{p}$-annihilation amplitudes. Due to the requirement of charge-conjugation invariance, we find that for creation of two mesons at rest there are only three irreducible amplitudes left:

$$\lim_{\vec{p}\to 0}\{f_1\bar{B}^{ABC}(-P)B_{A'B'C}(P)(\not{q})_A{}^{A'}(\not{q})_B{}^{B'}[\bar{M}(q_1)\bar{M}(q_2)+\bar{M}(q_2)\bar{M}(q_1)]_C{}^{C'}$$
$$+f_2\bar{B}^{ABC}(-P)B_{A'B'C'}(P)(\gamma_5)_A{}^{A'}[\bar{M}(q_1)]_B{}^{B'}[\gamma_5\bar{M}(q_2)+\bar{M}(q_2)\gamma_5]_C{}^{C'}$$
$$+f_3\bar{B}^{ABC}(-P)B_{A'B'C'}(P)(\gamma_5)_A{}^{A'}(\gamma_5)_B{}^{B'}[\bar{M}(q_1)\bar{M}(q_2)+\bar{M}(q_2)\bar{M}(q_1)]_C{}^{C'}\}. \quad (1)$$

Evaluating Eq. (1) explicitly, we obtain then, for processes $p+\bar{p}\to 0^-+0^-$,

$$-f_1\frac{\vec{q}^2}{3\mu^2}(\mu+m)(\chi^+\vec{q}\cdot\vec{\sigma}\chi)\{3\,\mathrm{Tr}[\bar{N}(P_1P_2-P_2P_1)N]-3\,\mathrm{Tr}[\bar{N}N(P_1P_2-P_2P_1)]\}$$
$$+f_3\frac{1}{3\mu^2}(\mu+m)(\chi^+\vec{q}\cdot\vec{\sigma}\chi)\{5\,\mathrm{Tr}[\bar{N}(P_1P_2-P_2P_1)N]+\mathrm{Tr}[\bar{N}N(P_1P_2-P_2P_1)]\}, \quad (2)$$

and for processes $p+\bar{p}\to 0^-+1^-$,

$$-f_1\frac{\vec{q}^2}{3\mu^2}\Big\{(\mu+m)(\chi^+i\vec{q}\times\vec{\epsilon}_2\cdot\vec{\sigma}\chi)\{5\,\mathrm{Tr}[\bar{N}(P_1V_2+V_2P_1)N]+\mathrm{Tr}[\bar{N}N(P_1V_2+V_2P_1)]-2\,\mathrm{Tr}(\bar{N}N)\,\mathrm{Tr}(P_1N_2)\}$$
$$+(\chi^+\chi)D\{5\,\mathrm{Tr}[\bar{N}(P_1V_2-V_1P_1)N]+\mathrm{Tr}[\bar{N}N(P_1V_2-V_2P_1)]\}+\mathrm{terms}(1\leftrightarrow 2,q_1\leftrightarrow q_2,\epsilon_1\leftrightarrow\epsilon_2)\Big\}$$
$$+f_2\frac{(\mu+q_{10})}{3\mu^2}\;\frac{1}{2}(\chi^+i\vec{q}\times\vec{\epsilon}_2\cdot\vec{\sigma}\chi)\{\mathrm{Tr}[\bar{N}(P_1V_2+V_2P_1)N]-\mathrm{Tr}(\bar{N}P_1)\,\mathrm{Tr}(NV_2)-\mathrm{Tr}(\bar{N}V_2)\,\mathrm{Tr}(NP^1)$$
$$-4\,\mathrm{Tr}(\bar{N}V_2NP_1)+2\,\mathrm{Tr}(\bar{N}P_1NV_2)-2\,\mathrm{Tr}(\bar{N}P_1N)\,\mathrm{Tr}(V_2)\}$$
$$+f_3\frac{1}{3\mu^2}\Big\{(\mu+m)(\chi^+i\vec{q}\times\vec{\epsilon}_2\cdot\vec{\sigma}\chi)\{5\,\mathrm{Tr}[\bar{N}(P_1V_2+V_2P_1)N]+\mathrm{Tr}[\bar{N}N(P_1V_2+V_2P_1)]-2\,\mathrm{Tr}(\bar{N}N)\,\mathrm{Tr}(P_1V_2)\}$$
$$+(\chi^+\chi)D\{3\,\mathrm{Tr}[\bar{N}(P_1V_1-V_2P_1)N]-3\,\mathrm{Tr}[\bar{N}N(P_1V_2-V_2P_1)]\}+\mathrm{terms}(1\leftrightarrow 2,q_1\leftrightarrow q_2,\epsilon_1\leftrightarrow\epsilon_2)\Big\}, \quad (3)$$

where m and μ are masses of baryons and mesons, respectively, and $D=[\frac{1}{4}\vec{q}^2-(M+q_{10})(M-q_{20})]\times\epsilon_{20}+(M+m)\vec{q}\cdot\vec{\epsilon}_2$. Concerning the processes $p+\bar{p}\to 1^-+1^-$, we shall get a more complicated expression. We remark here that annihilation amplitudes with other higher order insertions of S_1 and S_P do not give rise to independent f/d ratios in their SU(3) counterparts, and in this sense, are reducible.

From Eqs. (2) and (3), we see that the processes $p+\bar{p}\to 0^-+0^-$ proceed through the ${}^3S$ $p\bar{p}$ state, and the processes $p+\bar{p}\to 0^-+1^-$ through both ${}^1S$ and ${}^3S$ states. If we denote the annihilation matrix elements by $A(0^-0^-)$ and $A(0^-1^-)$, we obtain

$$A(\pi^+\pi^-)=3g_1+5g_3,$$
$$A(K^+K^-)=6g_1+4g_3,$$
$$A(K^0\bar{K}^0)=3g_1-g_3, \quad (4)$$

and a triangular relation[7-9]

$$A(\pi^+\pi^-)+A(K^0\bar{K}^0)-A(K^+K^-)=0 \quad (5)$$

for creation of two pseudoscalar mesons, where $g_1=-f_1(\vec{q}^2/3\mu)(\mu+m)(\chi^+\vec{q}\cdot\vec{\sigma}\chi)$ and $g_3=f_3(1/3\mu^2)\times(\mu+m)(\chi^+\vec{q}\cdot\vec{\sigma}\chi)$. The sum rule (5) is in good agreement with experimental data.[7,8,10] The sum rule (5) has been obtained here without a pole model. It also follows from SU(3) symmetry if one assumes pole dominance.

For annihilation into the pseudoscalar and vector mesons proceeding via the ^1S $p\bar{p}$ state, we find the nonvanishing matrix elements as

$$A(\pi^+\rho^-, {}^1S)=5g_1'+3g_3';$$

$$A(K^+K^{*-}, {}^1S)=4g_1'+6g_3',$$

$$A(K^0\bar{K}^{*0}, {}^1S)=-g_1'+3g_3', \quad (6)$$

and a sum rule

$$A(\pi^+\rho^-, {}^1S)+A(K^0\bar{K}^{*0}, {}^1S)-A(K^+K^{*-}, {}^1S)=0, \quad (7)$$

where $g_1'=-f_1(2\vec{q}^2/3\mu^2)(\chi^+\chi)D$ and $g_3'=f_3(2/3\mu^2)(\chi^+\chi)D$. As for annihilation via the ^3S state, we give the results in the first three columns of Table I. We note that the amplitudes f_1 and f_3 have the same f/d ratio in the present case. From the first three colums of Table I, we obtain the following relations[9]:

$$A(K^+K^{*-}, {}^3S)=-4A(K^0K^{*0}, {}^3S),$$

$$A(\eta\rho^0, {}^3S)=(5/3)A(\eta\omega, {}^3S),$$

$$A(\pi^+\rho^-, {}^3S)+A(\pi^0\omega, {}^3S)-2A(K^+K^{*-}, {}^3S)=0,$$

$$-A(\pi^+\rho^-, {}^3S)+\tfrac{3}{5}A(\eta\rho^0, {}^3S)+\tfrac{1}{2}A(K^+K^{*-}, {}^3S)=0,$$

$$A(\pi^0\varphi)=A(\eta\varphi)=0. \quad (8)$$

At present, experiments seem to indicate that the process $p+\bar{p}\to K+K^*$ proceeds at most about 35% via the ^1S state and the rest via the ^3S state[11] and that the two modes $p+\bar{p}\to K^++K^{*+}$ and $p+\bar{p}\to K^0+\bar{K}^{*0}$ are comparable.[11,12] However, the first relation in Eq. (8) does not agree with these experimental results. Thus we conclude that the model (1) is empirically not very favorable. We note that sum rules obtained in this model are the same as those obtained recently in the framework of certain SU(6)-symmetry models.[8,9]

We now come to the discussion of the model (2). In this case, we assume that owing to the meson-pole dominance, the $U(6)_q$-breaking pole terms are dominant compared to other nonpole terms except for the $U(6)_q$-invariant term f_1 in Eq. (1). Among the second-order spurions,[2,4]

$$S_2=(f_p\gamma_5\otimes\gamma_5+f_a\gamma_\mu\gamma_5\otimes\gamma_\mu\gamma_5 +f_v\gamma_\mu\otimes\gamma_\mu+f_t\sigma_{\mu v}\otimes\sigma_{\mu v})\otimes\epsilon_1, \quad (9)$$

we find that in substituting them into annihilation amplitudes, the spurion terms $(\gamma_\mu\gamma_5\otimes\gamma_\mu\gamma_5)\otimes\epsilon_1$ and $(\sigma_{\mu v}\otimes\sigma_{\mu v})\otimes\epsilon_1$ are reducible to $(\gamma_5\otimes\gamma_5)\otimes\epsilon_1$ and $(\gamma_\mu\otimes\gamma_\mu)\otimes\epsilon_1$, respectively. Hence, we have three independent C-invariant terms

Table I. The annihilation amplitudes[a] for $p+\bar{p}\to 0^-+1^-$ via the ^3S $p\bar{p}$ state.

$p+\bar{p}\to$	f_1	$A(0^-1^-, {}^3S)f_2$	f_3	f_4
$\pi^++\rho^-$	$3g_1''$	g_2''	$3g_3''$	$15g_4''$
$\pi^0+\varphi$	0	0	0	0
$\pi^0+\omega$	$5g_1''$	$-g_2''$	$5g_3''$	$13g_4''$
$\pi+\varphi$	0	0	0	0
$\eta+\omega$	$(3/\sqrt{3})g_1''$	$(3/\sqrt{3})g_2''$	$(3/\sqrt{3})g_3''$	$(15/\sqrt{3})g_4''$
$\eta+\rho^0$	$(5/\sqrt{3})g_1''$	$(5/\sqrt{3})g_2''$	$(5/\sqrt{3})g_3''$	$(13/\sqrt{3})g_4''$
K^++K^{*-}	$4g_1''$	0	$4g_3''$	$14g_4''$
$K^0+\bar{K}^{*0}$	$-g_1''$	0	$-g_3''$	g_4''

[a]In the entries,

$$g_1''=-f_1(2q^2/3\mu^2)(\mu+m)(\chi^+i\vec{q}\times\vec{\epsilon}_2\cdot\vec{\sigma}\chi),$$

$$g_2''=f_2(1/6\mu^2)(\mu+q_{10})(\chi^+i\vec{q}\times\vec{\epsilon}_2\cdot\vec{\sigma}\chi)$$

$$g_3''=f_3(2/3\mu^2)(\mu+m)(\chi^+i\vec{q}\times\vec{\epsilon}_2\cdot\vec{\sigma}\chi),$$

and $g_4''=f_4(1/3\mu^2)(\mu+m)(\chi^+i\vec{q}\times\vec{\epsilon}_2\cdot\vec{\sigma}\chi)$.

left in the present model:

$$\lim_{\vec{p}\to 0}\{f_1\bar{B}^{ABC}(-P)B_{A'B'C'}(P)(\not{q})_A{}^{A'}(\not{q})_B{}^{B'}[\bar{M}(q_1)\bar{M}(q_2)+\bar{M}(q_2)\bar{M}(q_1)]_C{}^{C'}$$

$$+f_3\bar{B}^{ABC}(-P)B_{A'B'C'}(P)(\gamma_5)_A{}^{A'}(\gamma_5)_B{}^{B'}[\bar{M}(q_1)\bar{M}(q_2)+\bar{M}(q_2)\bar{M}(q_1)]_C{}^{C'}$$

$$+f_4\bar{B}^{ABC}(-P)B_{A'B'C'}(P)(\gamma_\mu)_A{}^{A'}(\gamma_\mu)_B{}^{B'}[\bar{M}(q_1)\bar{M}(q_2)+\bar{M}(q_2)\bar{M}(q_1)]_C{}^{C'}\}. \quad (10)$$

The first two amplitudes in the above equation have been evaluated in the former case, whereas the third amplitude leads to

$$f_4\frac{1}{3\mu^2}(\mu+m)(\chi^+\vec{q}\cdot\vec{\sigma}\chi)\{13\,\mathrm{Tr}[\bar{N}(P_1P_2-P_2P_1)N]-\mathrm{Tr}[\bar{N}N(P_1P_2-P_2P_1)]\} \quad (11)$$

for creation of pseudoscalar mesons and

$$f_4\frac{1}{3\mu^2}\Big\{(\mu+m)(\chi^+i\vec{q}\times\vec{\epsilon}_2\cdot\vec{\sigma}\chi)\{13\,\mathrm{Tr}[\bar{N}(P_1V_2+V_2P_1)N]-\mathrm{Tr}[\bar{N}N(P_1V_2+V_2P_1)]+2\,\mathrm{Tr}(\bar{N}N)\,\mathrm{Tr}(P_1V_2)\}$$
$$+3(\chi^+\chi)D\{5\,\mathrm{Tr}[\bar{N}(P_1V_2-V_2P_1)N]+\mathrm{Tr}[\bar{N}N(P_1V_2-V_2P_1)]\}+\mathrm{terms}(1\leftrightarrow 2,q_1\leftrightarrow q_2,\epsilon_1\leftrightarrow\epsilon_2)\Big\} \quad (12)$$

for creation of pseudoscalar and vector mesons.

For processes $p+\bar{p}\to 0^-+0^-$ and $p+\bar{p}\to 0^-+1^-$ going via the ^1S state, we obtain the same triangular relation (5),

$$A(\pi^+\pi^-)+A(K^0\bar{K}^0)-A(K^+K^-)=0,$$

and the same sum rule (7),

$$A(\pi^+\rho^-,{}^1S)+A(K^0\bar{K}^{*0},{}^1S)-A(K^+K^{*-},{}^1S)=0.$$

Concerning processes $p+\bar{p}\to 0^-+1^-$ proceeding via the ^3S state, we give the annihilation matrix elements calculated from Eq. (12) in the fourth column of Table I. From the first, third, and fourth columns of Table I, we obtain now the following sum rules:

$$A(\pi^+\rho^-,{}^3S)-A(K^0\bar{K}^{*0},{}^3S)-A(K^+K^{*-},{}^3S)=0,$$

$$A(\pi^0\omega,{}^3S)+A(K^0\bar{K}^{*0},{}^3S)-A(K^+K^{*-},{}^3S)=0,$$

$$A(\pi^+\rho^-,{}^3S)=\sqrt{3}A(\eta\omega,{}^3S),$$

$$A(\pi^0\omega,{}^3S)=\sqrt{3}A(\eta\rho^0,{}^3S),$$

$$A(\pi^0\varphi,{}^3S)=A(\eta\varphi,{}^3S)=0. \quad (13)$$

We note that the sum rules obtained in the present model are somewhat different from those obtained in the meson-pole dominance model for SU(3) symmetry.[7] To the part of processes $p+\bar{p}\to 0^-+1^-$ via the ^3S state, the meson-pole dominance model in the SU(3) symmetry gives

$$A(\eta\varphi,{}^3S)\neq 0,$$

$$A(\pi^+\rho^-,{}^3S)+A(K^0\bar{K}^{*0},{}^3S)-A(K^+K^{*-},{}^3S)-\sqrt{3}A(\eta\omega,{}^3S)+\sqrt{3}A(\eta\rho^0,{}^3S)=0,$$

$$(\sqrt{3}/2)A(\eta\varphi,{}^3S)+A(K^0\bar{K}^{*0},{}^3S)+\sqrt{2}A(K^+K^{*-},{}^3S)-\sqrt{2}A(\pi^+\rho^-,{}^3S)=0,$$

$$A(\pi^0\omega,{}^3S)+A(K^0\bar{K}^{*0},{}^3S)-A(K^+K^{*-},{}^3S)=0,$$

$$A(\pi^+\rho^-,{}^3S)=\sqrt{3}A(\eta\omega,{}^3S),$$

$$A(\pi^0\omega,{}^3S)=\sqrt{3}A(\eta\rho^0,{}^3S),$$

$$A(\pi^0\varphi,{}^3S)=0. \quad (14)$$

In deriving Eq. (14) we have used the following property of $\omega\varphi$ mixing:

$$\omega=(\tfrac{1}{3})^{1/2}\omega_8+(\tfrac{2}{3})^{1/2}\omega_1$$

$$\varphi=-(\tfrac{2}{3})^{1/2}\omega_8+(\tfrac{1}{3})^{1/2}\omega_1. \quad (15)$$

From Eqs. (13) and (14) we see that the $\eta\varphi$ production amplitude can be used as a test of our model (2).

The first relation in Eq. (13) can now be compared with the present experimental data.[11-13] If we assume that the processes $p+\bar{p}\to 0^-+1^-$ proceeds predominantly via the ^3S state[12] and that a q^3 weighting factor is used to take account for phase space and the momentum dependence of the matrix element, we have then, in arbitrary units, the following values[12,13]:

$$|A(\pi^+\rho^-, {}^3S)| = 2.94 \pm 0.15,$$
$$|A(K^0\overline{K}^{*0}, {}^3S)| = 1.23 \pm 0.13,$$
$$|A(K^+K^{*-}, {}^3S)| = 1.11 \pm 0.11. \qquad (16)$$

The sum rule

$$A(\pi^+\rho^-, {}^3S) - A(K^0\overline{K}^{*0}, {}^3S) - A(K^+K^{*-}, {}^3S) = 0$$

seems well satisfied with experiment to within 20%. It thus appears that the sum rules obtained in model (2) are compatible to the present experimental data.

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EXTRA RESTRICTION ON THE FORWARD SCATTERING AMPLITUDES

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One of the most remarkable properties of the second-sheet function of the scattering amplitude, defined by the analytical continuation through the elastic cut, is that we can compute the value of the function itself in a small neighborhood of certain points from the unitarity condition alone without introducing any approximation. This is not the case for the first-sheet function, since, although in a small neighborhood of $s=m^2$ the pole term $g^2/(m^2-s)$ dominates, there still remains finite background contribution $A(s,z)-g^2/(m^2-s)$ and we have no way to compute it exactly. In this note, we restrict ourselves to the case of pion-nucleon scattering and neglect the spin of the nucleon for the reason of simplicity. Our claim is that in a small neighborhood of the point

$$s = s_+ \equiv m^2 + 2\mu^2, \qquad (1)$$

where m and μ are the masses of the nucleon and pion, respectively, the second-sheet function of the forward scattering amplitude has the form

$$A^{\mathrm{II}}(s,1) = \frac{g^2}{m^2-u(z=1)} + \frac{g^2C_1}{s_+-s} + g^2C_0' + O\left(\left|\ln\frac{s_+-s}{m^2}\right|^{-2}\right), \qquad (2)$$

where C_1 and C_0 can be expressed in terms of the μ, m, and the coupling constant $g^2/4\pi$ explicitly. [See Eqs. (29), (30), (31), and (36).] $s=s_+$ is the point where the existence of the