tional change, δ, in the hfs is

$$\delta = -\tfrac{1}{3}\rho^2 \times 35.0 \text{ ppm}, \tag{5}$$

where ρ is the ratio of the spatial matrix elements:

$$\rho = \frac{\int \psi_{^3S}{}^*(\vec{r}_1, \vec{r}_2)\delta^3(\vec{r}_1)\psi_{^1S}(\vec{r}_1, \vec{r}_2)d^3r_1d^3r_2}{\int |\psi_{^3S}(\vec{r}_1, \vec{r}_2)|^2\delta^3(\vec{r}_1)d^3r_1d^3r_2}. \tag{6}$$

We have computed ρ using the six-term Hylleras-type variational wave functions given by Huang[6]:

$$\psi_{^3S}(\vec{r}_1, \vec{r}_2) = \tfrac{1}{2}[\varphi_{^3S}(\vec{r}_1, \vec{r}_2) - \varphi_{^3S}(\vec{r}_2, \vec{r}_1)], \tag{7}$$

$$\psi_{^1S}(\vec{r}_1, \vec{r}_2) = \tfrac{1}{2}[\varphi_{^1S}(\vec{r}_1, \vec{r}_2) + \varphi_{^1S}(\vec{r}_2, \vec{r}_1)]. \tag{8}$$

The φ's are of the form

$$\varphi(\vec{r}_1, \vec{r}_2) = Ce^{-ar_1}e^{-br_2}[c_1 + c_2(r_1 + r_2) + c_3(r_2 - r_1) + c_4r_{12} + c_5r_{12}(r_1 + r_2) + c_6r_{12}(r_2 - r_1)]. \tag{9}$$

These wave functions yield the total triplet and singlet atomic binding energy to 1 and 50 parts in 10^5, respectively.[7] Using a generalization of the method due to Eckart,[5,8] we find that the corresponding error in ρ^2 should be about four parts in 10^3. We obtain therefore

$$\rho^2 = 0.762(1 \pm 0.004) \tag{10}$$

and

$$\delta = (-8.89 \pm 0.004) \text{ ppm}. \tag{11}$$

Combining this with the earlier results,[1-4] we have the new value (-3 ± 3) ppm for the difference between the theoretical and experimental atom/ion hfs ratios.

It is interesting to note that the effect on the He $2\,^3P$ fine structure of the analogous fine structure mixing of the $2\,^1P_0$ and $2\,^3P_0$ states has been known for some time.[9]

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DIRECT MEASUREMENTS OF MULTIPLE QUANTIZATION IN He II*

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The flow of the superfluid component of helium II is thought to be curl free; furthermore, around a vortex line the circulation is quantized,[1,2] i.e.,

$$\oint \vec{v}_s \cdot d\vec{s} = nh/m, \quad n = 1, 2, 3, \text{etc.}, \tag{1}$$

where $\vec{v}_s$ is the superfluid velocity, $d\vec{s}$ is a line increment, h is Planck's constant, and m is the mass of a helium atom. While there is an accumulation of evidence for circulation values of $n = 1$,[3] supporting the Onsager-Feynman theory, these experiments bear out the prediction that n can equal 2, 3, etc., as well.

The experiments reported in this Letter use a probe small enough to examine the properties of individual vortex lines. Particles of frozen H-D gas[4,5] (0.01 to 0.1 mm in diameter) are used to investigate the pressure and velocity fields near the vortex core. Because the H/D ratio is adjusted to give particles which have the same density as the helium, gravity has little influence on the particles and their motion is considered to be entirely due to the motion of the helium. While most of the particles move very slowly and smoothly through

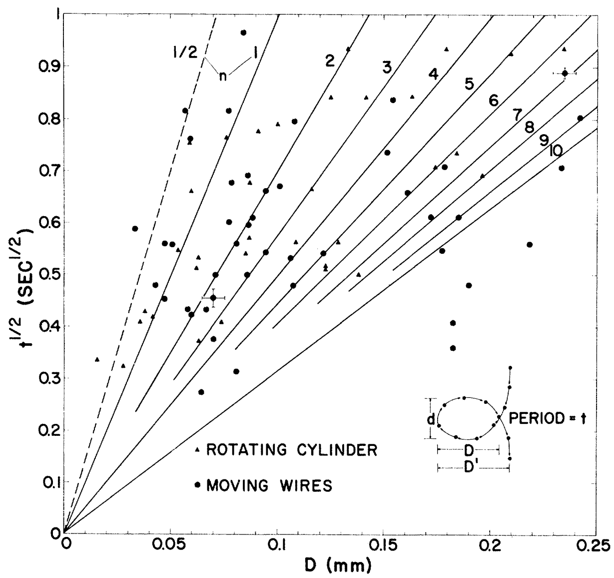

FIG. 1. Plot of the "circle" diameter (D) versus the square root of the "circle" period (t). The lines characteristic of constant circulation, n (in units of h/m) are shown.

the liquid, one in 10^3 is perturbed and executes a single loop similar to that shown schematically in Fig. 1; it then continues on along a smooth path. Rarely a particle will execute two loops around the same center. For purposes of analysis, this motion is thought of as a one-turn helix, viewed monoscopically. D, the "circle" diameter, and t, the period for apparent closure, are determined from frame-by-frame analysis of 16-mm motion pictures of the particle, generally filmed at 24 frames/second and 2:1 magnification (projected size 100:1). If this motion is indeed characteristic of the helium, we get a measure of the circulation which is given by $\pi D v_S = \pi^2 D^2/t$.

Figure 1 is a plot of $t^{1/2}$ vs D, in which circulation values of $n=\frac{1}{2}, 1, 2,$ etc., are shown by the straight lines. The errors shown are ±0.005 mm and ±0.016 sec (0.4 frame). Data for $t^{1/2}$ less than 0.3 $\sec^{1/2}$ are absent since such loops would be completed in less than two frames. Loops of $D < 0.01$ mm would not be observed because of system resolution. The data in Fig. 1 are accumulated from closed-loop particle encounters with vortices created in a rotating mesh cylinder (viewed along its axis) similar to that of reference 4. Encounters with vortices created by the 1-mm/sec sidewise passage of 7 mm×0.1 mm diam wires through the helium also appear in Fig. 1. The data of Fig. 1 show a reasonably abrupt low-vorticity cutoff along a line of constant circulation somewhere between $n=\frac{1}{2}$ and $n=1$. This absence of low vorticity values means that energy dissipation through vortex creation is not possible in an arbitrarily low-velocity helium flow. This observed cutoff, then, is necessary for superfluidity. While measurement inaccuracies give Fig. 1 somewhat of a shotgun appearance, a histogram, Fig. 2(c), averages the data showing a tendency for the circulation to group just below $n=1$, at higher integers, and possibly at $1\frac{1}{2}$. The widths of the peaks are to be expected from the errors in measurement. At temperatures 0.00 to 0.02°K below the λ point, a peculiar absence of vortices (or at least loops) was noted.

In Fig. 2(a) we have circulation measurements of closed-loop encounters with vortices associated with the counterflow of the superfluid and normal components. The distribution of data, if plotted as in Fig. 1, resembles that of the rotating cylinder. We observe that the number of encounters increases with increasing counterflow velocities and thus our microscopic studies show that vortices are indeed a mechanism for mutual friction as predicted by Vinen.[6] The counterflow is caused by a widely spaced parallel-fine-wire heater, by a plane surface heater, or by a unit composed of a heater in a closed box with a small slit. From the data in Fig. 2(a), those points are selected in which the "circle" is more open, the particle small, and any trajectory correction small.[7] The resulting data are plotted in Fig. 2(b). The strong peak near $n=\frac{1}{2}$ persists in this more restrictive analysis. Most of the data in Fig. 2 were taken between 1.4 and 2.0°K and in no case was a trend with temperature apparent. In order to improve the statistics, we plot the total of all observations meeting the aforementioned criteria in Fig. 2(d).[8] Note that the unexpected peak at $\frac{1}{2}$ originates almost entirely from the heat-current excitation.

Figure 3 demonstrates that the circulation, as measured by the particles, is in the same direction as the mesh tumbler rotation. The directions should be random if the particles were merely sensing a cylindrically symmetric pressure gradient directed toward the vortex core. This suggests that the particles are being dragged along by the superfluid component due to nonpotential flow around a particle,[9] perhaps similar to the Helmholtz helium flow investigated in reference 4. Alternatively, instead of executing solid-body rotation only, the normal fluid may be participating in the vortex circulation and carrying along the particles. Competition between super- and normal-

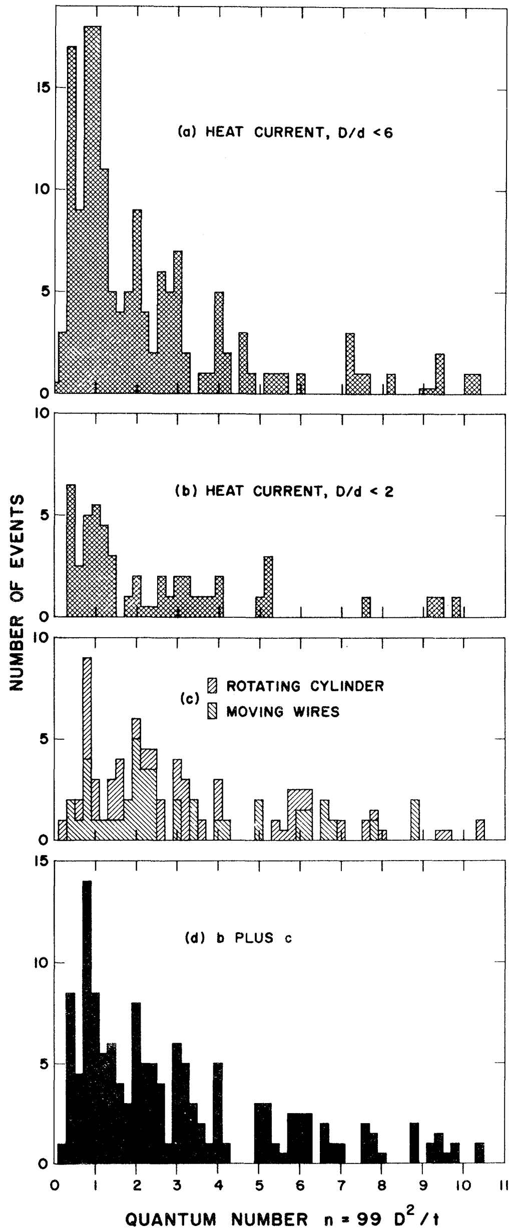

FIG. 2. Histograms of the number of events, N, versus circulation, n. Fig. 2(a) includes all events with $D/d < 6$ with vortices produced by a heat current. Data meeting more stringent criteria are corrected and plotted in Fig. 2(b) (see reference 7). Figure 2(c) shows data from vortices in a rotating mesh cylinder and from wires moving through the liquid. The interval chosen for these histograms is smaller than the measurement accuracy. Histograms made with half the interval show the same features.

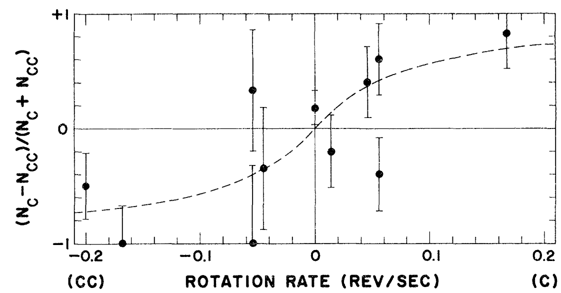

FIG. 3. Correlation between the rotation direction and "circle" direction. N_C and N_{CC} are the number of clockwise and counterclockwise "circles." The curve is calculated for 10 random vortices per mm^2 and a predicted 127 lines per mm^2 per rev/sec. With a heater, and no rotation, $(N_C - N_{CC})/(N_C + N_{CC})$ was measured as 0.07 ± 0.16.

component flow may result in the low circulation values found in the heat current. In any event, this highly sensitive method of microscopically examining helium flow clearly establishes the existence of quantized circulation[10] of $n > 1$ and, further, direct observations of vortices created in helium counterflow show that vortex generation does contribute to Gorter-Mellink mutual friction supporting Vinen's conclusions.

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[7]On Fig. 1, 2(b), 2(c), and 2(d) the points shown are data for well levitated particles which are small compared to D, D/d was less than 2, and $D/D' \geq 0.8$. If

the particle path is a screw helix viewed off-axis, D is an underestimate of the true diameter; a closer measure of the true diameter is D' (Fig. 1). Since for $D/D' < 1$ we underestimate the diameter and the period, it is necessary to apply a correction calculated to be 10% to the circulation ($\propto D^2/t$) when $D/D' = 0.8$. This correction, made on data shown in Fig. 2(b), 2(c), and 2(d), is required for about one point in five. In all cases, D and t were measured, and a correction, if necessary, decided upon before n was calculated after which, to avoid bias, no re-evaluation was permitted.

[8]The number of events in Fig. 2(d) at $n = 1$ (not 0.8), 2, 3, 4, and 5 were analyzed with Pearson's χ^2 test. Compared to several smooth analytical curves adjusted to have an area equal to the histogram, the probability of the observed positive deviations being random fluctuations was less than 0.02.

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[10]One might consider that the $n = 2$, 3, etc. points in Fig. 2 are associated with orbits around several $n = 1$ lines of the same polarity. However, in the rotation system there are only 10 to 30 lines/mm^2, yet many "circles" 0.07 to 0.1 mm in diameter have n's of about 2 and 3 [Figs. 1 and 2(c)]. This is improbable, unless there is a clumping of lines—a higher energy, unstable arrangement.

THERMAL EXPANSION COEFFICIENT AND COMPRESSIBILITY OF SOLID He^3†

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Liquid helium has been the object of extensive research for many years, while considerably less attention has been devoted to solid helium. Particularly lacking have been direct measurements of the thermal expansion coefficient and compressibility of solid helium, although theoretical calculations[1-3] and calculations from other data[4,5] have been made. This lack of experimental data is due to the difficulty of measuring accurately the small changes in pressure resulting from changes in temperature of the solid. We report here direct observations of the thermal expansion coefficient and compressibility of solid He^3. The measurements are all in the bcc phase for molar volumes from 22.39 to 24.61 cm^3/mole and over the temperature range from 0.3°K to the melting point.

A capacitance-type strain gauge which permitted observation of changes in pressure on the solid sample of less than 10^{-3} atm was used. The details of the sample chamber and pressure measuring apparatus will be reported elsewhere.[6]

For each sample studied the solid was obtained by applying pressure at a temperature just above the melting point. The filling capillary above the sample chamber was then blocked by quickly cooling it below the melting point, usually to 0.3°K, where it was held during the course of the measurements on a given sample. Observations of pressure versus temperature, made for both decreasing and increasing temperature with up to several hours time lapse between, showed good reproducibility. This indicated that there was negligible slipping of the solid blocking the capillary and very little hysteresis in the strain gauge. For a change in temperature from 0.3°K to the melting temperature the fractional change in pressure was of the order of 1%, while the fractional change in volume due to stretching of the sample chamber was calculated to be of the order of 0.001%. Therefore, for all practical purposes, the process was at constant volume.

To obtain the compressibility, $P(T)$ was measured for various molar volumes. The molar volume for each sample was determined from the intersection of the isochore with the melting curve using the data of Mills, Grilly, and Sydoriak.[4] A plot of V vs P for fixed temperatures was then made, with the slope of these curves determining the compressibility $\beta = -V^{-1}(\partial V/\partial P)_T$. The values of β obtained in this way are shown in Fig. 1. Because of the extremely small temperature dependence of β, only the values at 0.3°K are shown. Data from other sources are shown for comparison. Our results compare reasonably well with those