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SINGLET-TRIPLET MIXING CORRECTION TO THE HYPERFINE STRUCTURE OF THE He^3 ATOM*

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The ratio of the hyperfine structure (hfs) of the He^3 atomic state $2\,{}^3S_1$ to the hfs of the He^{3+} ionic state $2\,{}^2S_{1/2}$ is insensitive to nuclear structure effects. Therefore, it provides a clean test of the quantum electrodynamic corrections to hfs and to the two electron system. Recently we noted[1] that a previously omitted radiative correction contributes -4 parts per million (ppm) to the atom/ion hfs ratio, reducing the difference between theory[2,3] and experiment[4] to about (6 ± 3) ppm; the uncertainty arises mainly from approximations made in the evaluation of relativistic corrections.[2] We have now found that including the mixing of the singlet and triplet $2S$ states by the hyperfine interaction leads to a further 8.9-ppm decrease in the atomic hfs, thereby bringing the calculated and measured atom/ion ratios into agreement.

The hfs interaction for S states of He^3 is given by[5]

$$\begin{aligned} H &= \lambda[\vec{s}_1\cdot\vec{I}\delta^3(\vec{r}_1)+\vec{s}_2\cdot\vec{I}\delta^3(\vec{r}_2)] \\ &= \lambda\{\tfrac{1}{2}(\vec{s}_1+\vec{s}_2)\cdot\vec{I}[\delta^3(\vec{r}_1)+\delta^3(\vec{r}_2)] \\ &\quad +\tfrac{1}{2}(\vec{s}_1-\vec{s}_2)\cdot\vec{I}[\delta^3(\vec{r}_1)-\delta^3(\vec{r}_2)]\} \\ &\equiv H_e+H_o. \end{aligned} \tag{1}$$

Here $\vec{I}$ is the nuclear spin, and

$$\lambda = 4\pi\alpha g/mM, \tag{2}$$

where gI is the He^3 magnetic moment in nuclear magnetons, m is the electron mass, and M is the proton mass; $\hbar = c = 1$. The expectation value of H_e gives the lowest order hfs.

All diagonal matrix elements of H_o vanish by symmetry. However, H_o does connect singlet and triplet states, so that in second-order perturbation theory there is an energy shift in the $2\,{}^3S_1$ state due to H_o. Neglecting all states but the $2\,{}^1S_0$, this energy shift is

$$E^{(2)} = |\langle 2\,{}^1S_0, F, M_F|H_o|2\,{}^3S_1, F, M_F\rangle|^2 \times[E(2\,{}^3S_1)-E(2\,{}^1S_0)]^{-1}. \tag{3}$$

$E^{(2)}$ vanishes if the total angular momentum $\vec{F}=\vec{I}+\vec{S}$ has the value $\frac{3}{2}$ and is negative if $F\equiv\frac{1}{2}$. Since the ratio of the hfs to the triplet-singlet separation is -35.0×10^{-6}, we can expect a significant reduction in the hfs from (3). (The $F=\frac{1}{2}$ state is above the $F=\frac{3}{2}$ state because g is negative.)

Evaluation of the spin matrix element is facilitated by noting that all other matrix elements of $(\vec{s}_1-\vec{s}_2)\cdot\vec{I}$ vanish. Closure therefore gives

$$\begin{aligned} &\langle {}^3S_1, F, M_F|(\vec{s}_1-\vec{s}_2)\cdot\vec{I}|{}^1S_0, F, M_F\rangle \\ &\times\langle {}^1S_0, F, M_F|(\vec{s}_1+\vec{s}_2)\cdot\vec{I}|{}^3S_1, F, M_F\rangle \\ &= \langle {}^3S_1, F, M_F|[(\vec{s}_1-\vec{s}_2)\cdot\vec{I}]^2|{}^3S_1, F, M_F\rangle. \end{aligned} \tag{4}$$

Using well-known Pauli spin identities gives $\frac{3}{4}$ for $F=\frac{1}{2}$ and 0 for $F=\frac{3}{2}$. The corresponding factor in the hfs squared is $9/4$. Thus the fractional change, δ, in the hfs is

$$\delta=-\tfrac{1}{3}\rho^2\times 35.0\text{ ppm}, \tag{5}$$

where ρ is the ratio of the spatial matrix elements:

$$\rho=\frac{\int\psi_{^3S}{}^*(\vec{r}_1,\vec{r}_2)\delta^3(\vec{r}_1)\psi_{^1S}(\vec{r}_1,\vec{r}_2)d^3r_1d^3r_2}{\int|\psi_{^3S}(\vec{r}_1,\vec{r}_2)|^2\delta^3(\vec{r}_1)d^3r_1d^3r_2}. \tag{6}$$

We have computed ρ using the six-term Hylleras-type variational wave functions given by Huang[6]:

$$\psi_{^3S}(\vec{r}_1,\vec{r}_2)=\tfrac{1}{2}[\varphi_{^3S}(\vec{r}_1,\vec{r}_2)-\varphi_{^3S}(\vec{r}_2,\vec{r}_1)], \tag{7}$$

$$\psi_{^1S}(\vec{r}_1,\vec{r}_2)=\tfrac{1}{2}[\varphi_{^1S}(\vec{r}_1,\vec{r}_2)+\varphi_{^1S}(\vec{r}_2,\vec{r}_1)]. \tag{8}$$

The φ's are of the form

$$\varphi(\vec{r}_1,\vec{r}_2)=Ce^{-ar_1}e^{-br_2}[c_1+c_2(r_1+r_2)+c_3(r_2-r_1)+c_4r_{12}+c_5r_{12}(r_1+r_2)+c_6r_{12}(r_2-r_1)]. \tag{9}$$

These wave functions yield the total triplet and singlet atomic binding energy to 1 and 50 parts in 10^5, respectively.[7] Using a generalization of the method due to Eckart,[5,8] we find that the corresponding error in ρ^2 should be about four parts in 10^3. We obtain therefore

$$\rho^2=0.762(1\pm 0.004) \tag{10}$$

and

$$\delta=(-8.89\pm 0.004)\text{ ppm}. \tag{11}$$

Combining this with the earlier results,[1-4] we have the new value (-3 ± 3) ppm for the difference between the theoretical and experimental atom/ion hfs ratios.

It is interesting to note that the effect on the He $2\,^3P$ fine structure of the analogous fine structure mixing of the $2\,^1P_0$ and $2\,^3P_0$ states has been known for some time.[9]

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DIRECT MEASUREMENTS OF MULTIPLE QUANTIZATION IN He II*

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The flow of the superfluid component of helium II is thought to be curl free; furthermore, around a vortex line the circulation is quantized,[1,2] i.e.,

$$\oint\vec{v}_s\cdot d\vec{s}=nh/m,\quad n=1,2,3,\text{etc.}, \tag{1}$$

where $\vec{v}_s$ is the superfluid velocity, $d\vec{s}$ is a line increment, h is Planck's constant, and m is the mass of a helium atom. While there is an accumulation of evidence for circulation values of $n=1$,[3] supporting the Onsager-Feynman theory, these experiments bear out the prediction that n can equal 2, 3, etc., as well.

The experiments reported in this Letter use a probe small enough to examine the properties of individual vortex lines. Particles of frozen H-D gas[4,5] (0.01 to 0.1 mm in diameter) are used to investigate the pressure and velocity fields near the vortex core. Because the H/D ratio is adjusted to give particles which have the same density as the helium, gravity has little influence on the particles and their motion is considered to be entirely due to the motion of the helium. While most of the particles move very slowly and smoothly through