EXCITED STATE OF $_\Lambda Be^9$ AND THE Λ-N INTERACTION*

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A recently reported event[1] is identified as an excited state $_\Lambda Be^{9*}$ of $_\Lambda Be^9$. It is unstable with respect to the decay $_\Lambda Be^{9*} \rightarrow {}_\Lambda He^5 + He^4$ by 0.7 ± 0.5 MeV with a lifetime $\tau \gtrsim 10^{-12}$ sec. The Λ separation energy is $B_\Lambda^*({}_\Lambda Be^{9*}) = 2.5 \pm 0.5$ MeV. [The particle instability gives an upper limit of $B_\Lambda^* < 3.2 \pm 0.05$ MeV since $B_\Lambda({}_\Lambda He^5) = 3.1 \pm 0.05$ MeV and the ground-state energy of Be^8 is 0.1 MeV.] This paper reports the results of calculating B_Λ^* with a three-body model consisting of two α particles and the Λ. This model has proved very successful[2,3] for the ground state of $_\Lambda Be^9$. The excited state is taken as built on the first excited state ($J = 2$) of Be^8 by, in effect, coupling the Λ (in as s state) to the excited Be^8 core. The long lifetime τ is then readily understood as a consequence of the decay into an $l = 2$ final state of $_\Lambda He^5 + He^4$ together with the small Q value.

Bodmer and Ali[2] calculated the ground-state energy of $_\Lambda Be^9$ by use of an "equivalent" two-body method for the three-body problem. Their method gives the best s-state variational wave function of the product form,

$$\psi(r_1, r_2, r_3) = \prod_{i=1}^{3} g_i(r_i),$$

where the r_i are the interparticle separations. For the excited state, we use the generalization of this method due to Murphy and Rosati,[4] for the particular case of a total orbital angular momentum $L = 2$. Since we use central interactions, the two possible states of $_\Lambda Be^{9*}$ with $J = \frac{3}{2}$ and $\frac{5}{2}$ are degenerate in our model. Further, since $_\Lambda He^5$ is not expected to possess any low-lying excited states, we consider the orbital angular momentum to be associated only with the α particles.

The two-body (radial) Schrödinger wave function $f_{\alpha\alpha}(r_{\alpha\alpha})$, which describes the relative α-α motion[5] and which corresponds to the appropriate (three-body) product wave function in the interparticle separations (reference 4), is then determined by the Schrödinger equation[6]

$$\frac{\hbar^2}{M_\alpha} f_{\alpha\alpha}'' - \left[B^* + V_{\alpha\alpha} + \frac{6\hbar^2}{M_\alpha r_{\alpha\alpha}^2} + W_{\alpha\alpha}^{(2)}\right] f_{\alpha\alpha} = 0. \quad (1)$$

Here M_α is the α-particle mass, B^* is the total three-body binding energy ($B^* = B_\Lambda^* - 0.1$ MeV), $V_{\alpha\alpha}$ is the appropriate (i.e., $l = 2$) α-α potential, and $W_{\alpha\alpha}^{(2)}$ is the additional potential[2,4] due to the presence of the Λ. The potential $W_{\alpha\alpha}^{(2)}[g_{\alpha\Lambda}]$, through which the three-body nature of the system enters, is a functional of $g_{\alpha\Lambda}(r_{\alpha\Lambda})$ (i.e., of the relative α-Λ function), and of course also of the α-Λ interaction $V_{\alpha\Lambda}$. With our approximations, $W_{\alpha\alpha}^{(2)}$ may then be written in the form

$$W_{\alpha\alpha}^{(2)}[g_{\alpha\Lambda}] = W_{\alpha\alpha}^{(0)}[g_{\alpha\Lambda}],$$

where $W_{\alpha\alpha}^{(0)}[g_{\alpha\Lambda}]$ is just the corresponding expression[7] for the ground state ($L = 0$) of $_\Lambda Be^9$. The only essential modification of the procedure of reference 2 is then simply the presence of the $l = 2$ centrifugal barrier.

For the calculation of $W_{\alpha\alpha}^{(0)}$, we have considered two α-Λ potentials. These correspond to Yukawa Λ-N interactions with Yukawa ranges $\mu_{2\pi}^{-1} = 0.7$ F and $\mu_K^{-1} = 0.4$ F, appropriate to two-pion and K-meson exchange, respectively. For both these ranges, excellent agreement for $_\Lambda He^5$ is obtained by use of α-Λ potentials of the form

$$V_{\alpha\Lambda}(r) = U(e^{-\nu_1 r} - e^{-\nu_2 r}) \quad (2)$$

(which give a very good fit to the "exact" α-Λ potentials obtained by folding the spin-averaged Yukawa Λ-N interaction into the Gaussian α-particle density distribution), together with the use of an α-Λ trial function for $_\Lambda He^5$ of the form

$$g_{\alpha\Lambda}(r) = e^{-ar} + se^{-br}. \qquad (3)$$

Thus the values $U = 1040$ MeV F^3, $\nu_1 = 1.900$ F^{-1}, and $\nu_2 = 2.115$ F^{-1} for $\mu_{2\pi}$, and $U = 790$ MeV F^3, $\nu_1 = 2.150$ F^{-1}, and $\nu_2 = 2.575$ F^{-1} for μ_K reproduce the experimental value $B_\Lambda({}_\Lambda He^5) = 3.1$ MeV and also agree closely with previous calculations.[8] (U is four times the spin-averaged volume integral of the Λ-N interaction.)

With the use of the forms (2) and (3) also for the three-body problem, the potential $W_{\alpha\alpha}^{(0)}$ then becomes an algebraic function[7] of $r_{\alpha\alpha}$ and of the variational parameters a, b, and s of $g_{\alpha\Lambda}$. Numerical solution of the Schrödinger eigenvalue problem appropriate to Eq. (1) then gives B^* as a function of a, b, and s, i.e., $B^* = B^*(a, b, s)$. The required value of B^* is then the maximum of this function.

Two forms were considered for $V_{\alpha\alpha}$:

$$V_{\alpha\alpha} = \infty \text{ for } r < c,$$
$$= -V_0 \text{ for } c < r < d,$$
$$= 4e^2/r \text{ for } d < r; \qquad (\mathrm{H}N)$$

$$V_{\alpha\alpha} = V_R \exp(-\mu_R^2 r^2) - V_A \exp(-\mu_A^2 r^2) + 4e^2/r; \qquad (\mathrm{S}N)$$

corresponding to hard and soft repulsive cores, respectively. The actual potentials used are listed in Table I. Figure 1 shows the corresponding $l = 2$ phase shifts δ_2 as a function of the center-of-mass energy E of the two α particles. The experimental values[9,10] $\delta_2^{\text{expt.}}$ are also shown. The "good" potentials H1, H2, S1, and S2 all give equally adequate fits to $\delta_2^{\text{expt.}}$, although a variety of ranges and shapes are involved. H3 is the potential that Van der Spuy and Pienaar[11] found to give the best fit to $\delta_2^{\text{expt.}}$ for low energies ($E \lesssim 3$ MeV). This potential is, however, insufficiently attractive to represent $\delta_2^{\text{expt.}}$ at higher energies.[12] The fits obtained with H4 and H5 are just as reasonable as those with H3 in the resonance region, although at higher energies these potentials give even smaller values of δ_2 than does H3. For interest, the potentials H6 and S3 were also considered. These give good fits to the $l = 0$ phase shifts but cannot satisfactorily reproduce $\delta_2^{\text{expt.}}$. This is consistent with the well-known l dependence of $V_{\alpha\alpha}$ which is such that the effective repulsion for $l = 0$ is larger than for $l = 2$.[13]

The results obtained for B_Λ^* are shown in Table II. Most of these were obtained for the one-parameter trial function $g_{\alpha\Lambda}$ which is obtained from Eq. (3) by setting $s = 0$. It is seen that only a fairly small increase in B_Λ^* (by about 5.5% for μ_K and only 0.5% for $\mu_{2\pi}$) is obtained by use of the three-parameter function (for H1). The rms radius $\langle r_{\alpha\alpha}^2\rangle^{1/2}$ and rearrangement energy $E_{\alpha\alpha}$ of the Be^8 core are also shown. The energy $E_{\alpha\alpha}$ is the difference between the energy $\langle T_{\alpha\alpha} + V_{\alpha\alpha}\rangle$ of the Be^8 core with the Λ present and the $l = 2$ resonance energy which is taken as 3 MeV.

For both $\mu_{2\pi}$ and μ_K the value of B_Λ^* depends on the α-α interaction. In particular, the re-

Table I. Parameters of the α-α potentials.

Hard-repulsive-core potentials (HN)			
$V_{\alpha\alpha}$	R (F)	d (F)	V_0 (MeV)
H1	3.5	1.8	17.12
H2	4.0	1.0	8.83
H3	4.0	1.8	10.5
H4	4.5	1.8	6.87
H5	4.0	2.5	16.76
H6	4.0	1.7	7.2

Soft-repulsive-core potentials (SN)				
$V_{\alpha\alpha}$	μ_A (F^{-1})	V_A (MeV)	μ_R (F^{-1})	V_R (MeV)
S1	0.5	150	0.8	640
S2	0.475	130	0.7	320
S3	0.42	150	0.55	325

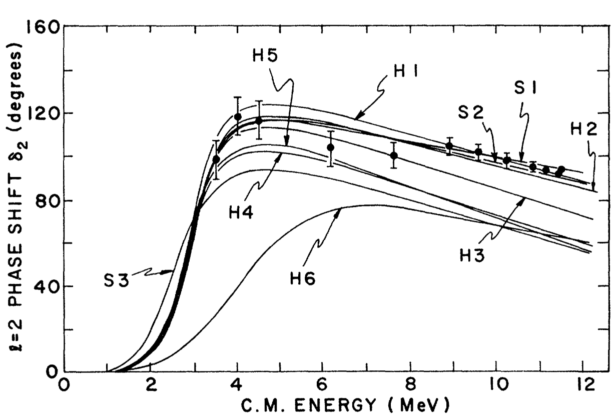

FIG. 1. The $l = 2$, α-α phase shifts as a function of the c.m. energy for the potentials of Table I. The experimental values are shown with error bars.

Table II. Results for ${}_{\Lambda}Be^{9*}$. The results are for the one-parameter function $g_{\alpha\Lambda}$ [Eq. (3) with $s=0$], except for $H1$, for which the results for the three-parameter function are also given. The potentials $V_{\alpha\alpha}$ that are marked with an asterisk are those that give a satisfactory fit to $\delta_2^{\text{expt.}}$.

	Λ-N interaction with range $\mu_{2\pi}^{-1}$				Λ-N interaction with range μ_K^{-1}			
$V_{\alpha\alpha}$	$B_\Lambda{}^*$ (MeV)	$E_{\alpha\alpha}$ (MeV)	$\langle r_{\alpha\alpha}{}^2\rangle^{1/2}$ (F)	a (F^{-1})	$B_\Lambda{}^*$ (MeV)	$E_{\alpha\alpha}$ (MeV)	$\langle r_{\alpha\alpha}{}^2\rangle^{1/2}$ (F)	a (F^{-1})
H1*	3.80	0.70	3.55	0.45	3.00	0.72	3.55	0.45
H1*	3.82	0.67	3.57	$a=0.375$	3.16	0.58	3.62	$a=0.35$
				$b=0.73$				$b=1.0$
				$s=0.8$				$s=1.0$
H2*	3.76	1.12	3.52	0.45	3.02	1.24	3.49	0.48
H3	2.80	0.72	3.91	0.425	1.97	0.69	3.94	0.43
H4	2.03	0.62	4.27	0.415	1.20	0.63	4.30	0.425
H5	1.87	0.37	4.40	0.40	1.05	0.31	4.45	0.405
H6	1.10	1.90	4.25	0.425	0.3	1.96	4.31	0.425
S1*	3.80	1.18	3.46	0.45	3.05	1.25	3.42	0.475
S2*	3.72	1.20	3.50	0.45	2.97	1.30	3.47	0.475
S3	1.70	0.73	4.43	0.42				

quirement that $V_{\alpha\alpha}$ merely reproduces the $l=2$ resonance energy is not sufficient to determine $B_\Lambda{}^*$ uniquely. Clearly, the fit to $\delta_2^{\text{expt.}}$ at higher energies is important. However, it is very satisfactory that the "good" potentials H1, H2, S1, and S2 (which all provide equally good fits also at these higher energies) all give very nearly the same value of $B_\Lambda{}^*$. All this is entirely similar to the situation for the ground state.[2] The $l=0$ potentials are seen to give considerably smaller values of $B_\Lambda{}^*$ than the $l=2$ potentials (especially the "good" ones), and also to give rather large values of $\langle r_{\alpha\alpha}{}^2\rangle^{1/2}$. This is consistent with the fact that the repulsive part of $V_{\alpha\alpha}$ is stronger for $l=0$ than for $l=2$.

The rearrangement energy $E_{\alpha\alpha}\approx 1.0^{+0.15}_{-0.4}$ MeV and rms radius $\langle r_{\alpha\alpha}{}^2\rangle^{1/2}=3.54\pm0.06$ F depend somewhat on the details of $V_{\alpha\alpha}$ and rather little on μ (for a given $V_{\alpha\alpha}$). The value of $E_{\alpha\alpha}$ is somewhat larger than for the ground state (0.7 ± 0.1 MeV), and $\langle r_{\alpha\alpha}{}^2\rangle^{1/2}$ is somewhat smaller than for the ground state (3.75 ± 0.05 F); this may be understood in terms of the l dependence of $V_{\alpha\alpha}$.

The calculated value of $B_\Lambda{}^*$ depends appreciably, although not dramatically, on the range μ^{-1} and is significantly smaller for the shorter range μ_K^{-1} than for $\mu_{2\pi}^{-1}$. The values are $B_\Lambda{}^*=3.79\pm0.15$ MeV for $\mu_{2\pi}$ and 3.17 ± 0.15 MeV for μ_K. [These are an average for the "good" potentials for the three-parameter function, Eq. (3), based on the three-parameter results for H1.] Here, the errors are due to the uncertainty in $V_{\alpha\Lambda}$ [due to the uncertainty in $B_\Lambda({}_\Lambda He^5)$ and in the α-particle size] and due to the uncertainty in $V_{\alpha\alpha}$. In reference 2 the difference between B_Λ for $\mu_{2\pi}$ and μ_K was found to be quite small. However, in the present work we have used an improved α-Λ potential for μ_K (the one used for $\mu_{2\pi}$ in reference 2 is very good and effectively the same as used here). With this we do find a difference also for B_Λ, although this difference is, proportionally, considerably smaller than for $B_\Lambda{}^*$; the values are $B_\Lambda(\mu_{2\pi})=6.5$ MeV and $B_\Lambda(\mu_K)=6.1$ MeV. Both these values can be considered as consistent with the experimental value of 6.5 ± 0.15 MeV, since the calculated values are lower limits (by $\lesssim 0.2$ MeV) and errors of about 0.15 MeV can arise from uncertainties in $V_{\alpha\alpha}$ and in $V_{\alpha\Lambda}$ (for a given μ).

The values of $B_\Lambda{}^*$ are lower limits because of the variational basis of our calculation. However, we expect this to give results which are accurate to within about 0.2 MeV (reference 3). Furthermore, if the energy of ${}_\Lambda Be^{9*}$ is above the threshold for ${}_\Lambda Be^{9*}\to{}_\Lambda He^5+He^4$, but not by too much, then the correspondingly small width for this two-body decay implies that a three-body bound-state calculation for ${}_\Lambda Be^{9*}$ should give a very good approximation for ${}_\Lambda Be^{9*}$ also in this case. Thus ${}_\Lambda Be^{9*}$ is predicted to be particle stable by not less than about 0.7 MeV for $\mu_{2\pi}$, in definite disagreement with experiment. For μ_K, ${}_\Lambda Be^{9*}$ could just be particle stable and $B_\Lambda{}^*$ just consistent with the experimental value (for the extreme values of the errors). However, a range even somewhat shorter than μ_K^{-1} is clearly indicated by our results.

One is reluctant to believe that K-meson exchange is the dominant process that gives rise to the attractive part of the Λ-N interaction.[14] However, either uncorrelated two-pion exchange or the exchange of a $T=0, S=0$ dipion boson (σ meson) with mass $M \gtrsim M_K$ (as is consistent with, in particular, the nucleon-nucleon interaction[15]) corresponds, in fact, to quite short ranges for the attractive part. Thus with a hard core of radius 0.42 F, the two-pion-exchange potentials of de Swart and Iddings[16] give an intrinsic range $b=1.5$ F which is the same as for a potential with the same hard core and with an attractive Yukawa part of range $\mu^{-1} = 0.23$ F (corresponding to an effective exchanged mass of 6 M_π).[17] A boson with mass $M \gtrsim M_K$ corresponds to $\mu^{-1} \lesssim 0.4$ F. We believe that our results for B_Λ^* are the first rather definite confirmation of such a short range for the attractive part of the Λ-N interaction. An important consequence of such a short range is that a comparison[18] of the hypernuclear results with the low-energy scattering Λ-p data then indicates the existence of a repulsive core in the Λ-N interaction. Thus, such a comparison rules out purely attractive interactions with $b \lesssim 1$ F ($\mu^{-1} \lesssim 0.5$ F) but permits interactions with a hard core (of radius of the order of 0.4 F) and a short attractive tail.

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[1]R. J. Piserchio, J. J. Lord, and D. Fournet Davis, Bull. Am. Phys. Soc. 10, 115 (1965).

[2]A. R. Bodmer and S. Ali, Nucl. Phys. 56, 657 (1964).

[3]Y. C. Tang and R. C. Herndon, Phys. Rev. 138, B637 (1965).

[4]J. W. Murphy and S. Rosati, Nucl. Phys. 63, 625 (1965).

[5]The function $r^{-1}f_{\alpha\alpha}(r)$ is proportional to the probability amplitude for the α-α separation (if nothing is known about the position of the Λ), whereas the function $g_{\alpha\alpha}(r)$ has no such simple probability interpretation (see reference 2).

[6]There is also a similar equation for the α-Λ wave function. The two equations are coupled through $W_{\alpha\alpha}^{(2)}[g_{\alpha\Lambda}]$ and $W_{\alpha\Lambda}^{(2)}[g_{\alpha\Lambda}, g_{\alpha\alpha}]$. We avoid having to solve the coupled equations by using a trial function for $g_{\alpha\Lambda}$ as is discussed below. The procedure and its justification is entirely analogous to that used for B_Λ in reference 2.

[7]See reference 2 for details of the definition and computation of $W_{\alpha\alpha}^{(0)}$.

[8]A. R. Bodmer and S. Sampanthar, Nucl. Phys. 31, 251 (1962); A. R. Bodmer and J. W. Murphy, Nucl. Phys. 64, 593 (1965).

[9]N. P. Heydenburg and G. M. Temmer, Phys. Rev. 104, 123 (1956); C. W. Reich, J. L. Russel, and G. C. Phillips, Phys. Rev. 104, 135 (1956); R. Nilson, W. K. Jentschke, G. R. Briggs, R. O. Kerman, and J. N. Snyder, Phys. Rev. 109, 850 (1958).

[10]C. M. Jones, G. C. Phillips, and P. D. Miller, Phys. Rev. 117, 525 (1960).

[11]E. Van der Spuy and H. J. Pienaar, Nucl. Phys. 7, 397 (1958).

[12]The cross sections at 6.15 and 7.6 MeV are found to be insensitive to δ_2 and little significance should be given to the apparent reduction in $\delta_2^{\text{expt.}}$ in this region (reference 10).

[13]P. Darriulat, G. Igo, H. G. Pugh, and H. D. Holmgren, Phys. Rev. 137, B315 (1965), have very recently obtained such l-dependent potentials from a comprehensive analysis of the phase shifts. Independently, and by a somewhat different approach for analyzing the data, S. Ali and A. R. Bodmer (unpublished) have obtained very similar potentials.

[14]See, for example, B. W. Downs and R. J. N. Phillips, Nuovo Cimento 36, 120 (1965).

[15]See, for example, R. A. Bryan and B. L. Scott, Phys. Rev. 135, B434 (1964).

[16]J. J. de Swart and C. K. Iddings, Phys. Rev. 128, 2810 (1962).

[17]Although our calculations of B_Λ^* have been made with purely attractive potentials, the presence of a hard core is not expected to have any appreciable effect on our results. The reason is that, on the one hand, the range of $V_{\alpha\Lambda}$ will be determined by the range of the attractive part of the Λ-N interaction (for a given α-particle size), while, on the other hand, $V_{\alpha\Lambda}$ will be a smoothly varying, purely attractive, potential (since it is the effective α-Λ interaction) whose strength is determined by $B_\Lambda({}_\Lambda He^5)$ for a given range of $V_{\alpha\Lambda}$.

[18]A. R. Bodmer, "Hypertriton with S' State and the Λ-N Interaction" (to be published).