JOSEPHSON TUNNELING AND QUANTUM MECHANICAL PHASE*

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Josephson tunneling[1] of coherent electron pairs through an insulating barrier separating two superconductors has attracted a great deal of attention. This is doubtless because this phenomenon is an especially clear example of "macroscopic quantum mechanics,"[2] which is the basis of all present ideas on superfluid behavior. As the strength of the Josephson tunneling current depends directly upon the phase change in the pair wave function across the barrier, it serves to dramatize the importance and significance of this quantum mechanical variable.

The purpose of the present note is to point out that there is a second important aspect to Josephson tunneling; because of the sensitivity of the phase to magnetic fields, Josephson tunneling is an excellent tool for the study of field penetration in superconducting films. This is already evident for thick films from the work of Rowell,[3] who confirmed the prediction of Josephson that the presence of an integral number of flux quanta in the tunneling junction would reduce the total tunneling current to zero. As the flux is proportional to the applied field times the penetration depth, Rowell's measurement give a direct determination of the penetration depth.

Not so well known is the case of films thinner than the penetration depth, such as studied by Jaklevic, Lambe, Silver, and Mercereau[4] (which we abbreviate as JLSM). The electrodynamic properties of such films are completely specified now by two parameters, instead of simply the one parameter relevant for thick films. Using Schrieffer's[5] coordinate notation for a film of thickness τ filling the region between the planes $z=0$ and $z=\tau$, we impose the boundary condition of a vanishing magnetic field at $z=\tau$. With a tangential field in the y direction at $z=0$, the vector potential $A(z)$ and the currents will have only x components. With differentiation indicated by a prime, the two electromagnetic parameters of the film are the ordinary penetration length

$$\lambda = -A(0)/A'(0), \tag{1}$$

and a different ratio which we can call the "transfer length,"

$$\lambda_T = -A(\tau)/A'(0). \tag{2}$$

Other situations involving different boundary conditions can be described in terms of λ and λ_T. For example, it is easy to show that if the fields are equal at the two surfaces, the actual negative reciprocal logarithmic derivative of the vector potential at the surface is

$$\lambda_{\text{eff}} = \lambda - \lambda_T, \tag{3}$$

and the diamagnetic susceptibility is

$$\chi = -(1/4\pi)(1 - 2\lambda_{\text{eff}}/\tau). \tag{4}$$

Equations (1) and (2) are written in a gauge such that the Gor'kov function in the current-carrying superconducting film is a constant, i.e., the mean pairing momentum vanishes. In the JLSM experiment the other film of the Josephson junction is in a field-free region. As already discussed in a similar context by the author,[6] the fact that this film is carrying no current permits us to write down immediately in terms of $A(\tau)$ an expression for the difference in phase at two points separated by the distance L:

$$\Delta\varphi = -(2eL/\hbar c)A(\tau) = (2eL\lambda_T/\hbar c)A'(0), \tag{5}$$

where e, c, and $2\pi\hbar$ are the electron charge, velocity of light, and Planck's constant, respectively. We have substituted from Eq. (2) and can now eliminate the magnetic field $A'(0)$ in terms of the film width W and the incremental current ΔJ required to increase $\Delta\varphi$ by 2π. Thus we obtain

$$\Delta J = cW\Phi_0/4\pi L\lambda_T, \tag{6}$$

where $\Phi_0 = hc/2e = 2.07$ G cm^2 is the basic flux quantum.

Equation (6) replaces the JLSM Eq. (3) which is based upon the approximation of uniform distribution of supercurrent in the film. (Our ΔJ is equal to their J_d times τW.) The improvement represented by Eq. (6) can be illustrated by a calculation of λ_T based upon London's electrodynamics. This approximation neglects the Pippard nonlocal effects and yields an expression for $A(z)$ proportional to $\cosh(\tau-z)/\lambda_L$, where λ_L is London's temperature-dependent penetration depth. Consequently, according to Eq. (2), the penetration and transfer lengths are

$$\lambda=\lambda_L \coth(\tau/\lambda_L), \tag{7}$$

$$\lambda_T=\lambda_L \operatorname{csch}(\tau/\lambda_L). \tag{8}$$

Substituted into Eqs. (3) and (4), these expressions reproduce London's result for the susceptibility of the film,

$$\chi=-\frac{1}{4\pi}[1-(\tau/2\lambda_L)^{-1}\tanh(\tau/2\lambda_L)]. \tag{9}$$

We also note in passing that when Eq. (7) is substituted into Eq. (12) of reference 6, the Josephson penetration depth is considerably reduced for films thinner than λ_L. The self-limiting effect is therefore enhanced. (As the λ of reference 6 is the mean of the penetration lengths, only one of the two films of the Josephson junction need be thin for this enhancement to occur.) Note further that $\lambda>\tau$ in this case.[7]

Returning now to the JLSM experiment, we follow JLSM in assuming a temperature dependence of the Gorter-Casimir type,

$$\lambda_L(T)=\lambda_L(0)(1-t^4)^{-1/2}, \tag{10}$$

where the best fit to the JLSM data[8] shown in Fig. 1 is achieved with the value $\lambda_L(0)=783$ Å and is shown as a solid line (t is the reduced temperature). The dashed line results from the approximation

$$\lambda\approx\lambda_T\approx\lambda_L{}^2/\tau \tag{11}$$

to Eq. (7) and corresponds to the JLSM assumption of uniform current distribution. Comparison of the two curves makes evident the desirability of taking into account the nonuniformity of the current distribution in films as thick as that used by JLSM.

As the thickness of the film is further increased, the London treatment which yielded Eq. (8) can be expected to become increasingly inaccurate. According to nonlocal anomalous skin-depth theory for a pure metal with a long electron mean free path, λ_T should pass through zero and exhibit a sign reversal. This sign reversal has been found in a radiofrequency experiment to fall between 3.4 and 2.88°K for a tin film 18 700 Å thick.[9] This will result in a striking temperature dependence in the JLSM experiment. The expected behavior has been sketched in roughly (curve labeled "REVERSAL") in Fig. 1 for a rather smaller thickness such that the transfer length passes through zero at 2.4°K (which requires a sufficiently long mean free path). Although only the absolute value of ΔJ has been drawn, the sign could be determined by admitting a small amount of magnetic flux between the two superconducting films.

In conclusion, we wish to point out that the discussion given by JLSM of their experiment employs the term "drift current," which does not have a meaning independent of the gauge used. In the present treatment the phase of the electron pairs in the current-carrying film is constant throughout the film. The current

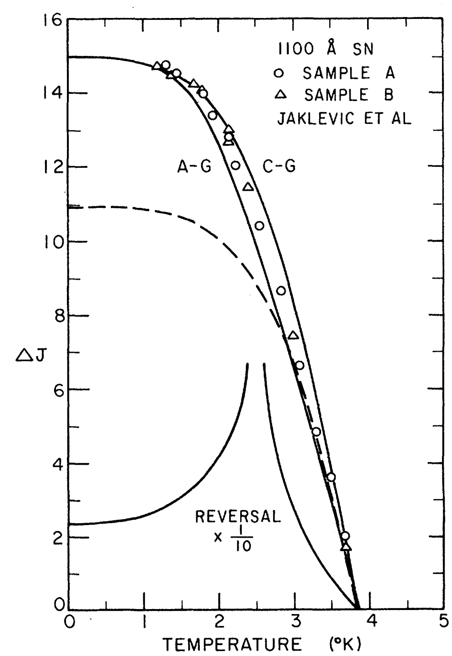

FIG. 1. Superconducting current versus temperature. ΔJ is the current needed to produce a phase difference of 2π for points in the tin film $L=8$ mm apart. The width and thickness of the film are $W=3$ mm and $\tau=1100$ Å, respectively. The data are from JLSM (reference 4) and are fitted by the present theory (solid line), which takes into account the nonuniform distribution of the superconducting current across the thickness of the film, for both the Abrikosov-Gor'kov (A-G) and the Casimir-Gorter (C-G) temperature dependences. The approximate theory of uniform distribution yields the dashed curve, for the same value of zero-temperature London penetration depth [$\lambda_L(0)=783$ Å]. The curve labeled "REVERSAL" illustrates roughly the predicted singularity and sign reversal for a thicker film of sufficiently long mean free path.

flows entirely because of the London term proportional to the vector potential—the "drift current" referred to by JLSM is absent in this gauge. But the other film of the Josephson junction has a "drift" term of just the right size to cancel the London term (in order to produce zero current), and this is where the relative phase difference of the two films arises. Use of gauge-noninvariant terminology is probably not dangerous, but it would seem wise to ascribe physical significance only to gauge-invariant relationships such as Eq. (6). "Phase is phase," and the origin of the phase difference measured by JLSM is really no different from that in their earlier experiments.[10] The main point of the present paper is that the quantum mechanical phase question arising in the JLSM experiment is answered completely by Eq. (6). This equation expresses the gauge-invariant connection between the current ΔJ which produces a phase shift of 2π and the new gauge-invariant parameter λ_T. It is necessary to supplement the conventional penetration length with this new parameter, the "transfer length," in order to have a complete description of the electrodynamics of a metallic film.

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[2]F. London, Superfluids (Dover Publications, New York, 1961), 2nd ed., Vol I.

[3]J. Rowell, Phys. Rev. Letters 11, 200 (1963).

[4]R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercereau, in Proceedings of the International Conference on Low-Temperature Physics, Columbus, Ohio, 1964 (to be published).

[5]J. R. Schrieffer, Phys. Rev. 106, 47 (1957).

[6]R. A. Ferrell and R. E. Prange, Phys. Rev. Letters 10, 479 (1963), Eq. (9).

[7]This is a consequence of the well-known fact that for a film it is not actually the enclosed flux, but the fluxoid which is important. For a recent discussion as well as references to earlier work, see P. Fulde and R. A. Ferrell, Phys. Rev. 131, 2457 (1963).

[8]I am grateful to Dr. Mercereau and his co-workers for permission to exhibit their results in advance of publication. Dr. Peter Fulde has pointed out to me that if the local approximation were justified, it would be more appropriate to assume the Abrikosov-Gor'kov temperature dependence

$$\left[\frac{\lambda_L(0)}{\lambda_L(T)}\right]^2=\frac{\Delta(T)}{\Delta(0)}\tanh\frac{\Delta(T)}{2kT},$$

where k is Boltmann's constant and $\Delta(T)$ is the BCS energy gap. This dependence is shown in Fig. 1 by the curve marked "A-G," where again we have chosen $\lambda_L(0)=783$ Å.

[9]K. E. Drangeid and R. Sommerhalder, Phys. Rev. Letters 8, 467 (1962). See also K. E. Drangeid et al., IBM J. Res. Develop. 8, 13 (1964).

[10]R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercereau, Phys. Rev. Letters 12, 159, 274 (1964). The new element of the experiment of reference 4 is the different geometrical orientation of the two films, whose relative phase gradient is measured.

PHOTON ABSORPTION CROSS SECTION OF SPHERICAL NUCLEI*

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The dynamic collective theory[1-4] which treats the coupling of dipole oscillations and surface vibrations was originally formulated for deformed nuclei. It has been extended in a systematic fashion to spherical nuclei. There the Hamiltonian, H, in the laboratory system has the form

$$H=-\tfrac{1}{2}\sqrt{3}\{(1/B_1)[\vec{\pi}^{[1]}\times\vec{\pi}^{[1]}]^{[0]}+C_1[\vec{\alpha}^{[1]}\times\vec{\alpha}^{[1]}]^{[0]}\}+\tfrac{1}{2}\sqrt{5}\{(1/B_2)[\vec{\pi}^{[2]}\times\vec{\pi}^{[2]}]^{[0]}+C_2[\vec{\alpha}^{[2]}\times\vec{\alpha}^{[2]}]^{[0]}\}+K_1[\vec{\alpha}^{[2]}\times\vec{\alpha}^{[1]}\times\vec{\alpha}^{[1]}]^{[0]}$$
$$+K_{20}[\vec{\alpha}^{[2]}\times\vec{\alpha}^{[2]}]^{[0]}[\vec{\alpha}^{[1]}\times\vec{\alpha}^{[1]}]^{[0]}+K_{22}[[\vec{\alpha}^{[2]}\times\vec{\alpha}^{[2]}]^{[2]}\times[\vec{\alpha}^{[1]}\times\vec{\alpha}^{[1]}]^{[2]}]^{[0]}+\cdots. \quad (1)$$