calculated from Kasuya's formula for the most pure samples above 3.0°K and the dependence of H_0 and J_0/A_0 on carrier concentration, are under investigation.

We wish to acknowledge the technical assistance provided by Mr. C. D. Wilson. We thank Mr. M. P. Mathur for providing Ge (grown by Miss L. Roth) which he found suitable for low-temperature thermometers, and Professor H. Yearian for x-ray orientations of the samples. Support of this work by the Advanced Research Projects Agency is gratefully acknowledged.

[1]This is not quite the expectation for the very purest n-InSb due to its carrier concentration not being sufficiently degenerate; see Eq. (1) obtained from V. A. Johnson, in Progress in Semiconductors, edited by A. F. Gibson (John Wiley & Sons, Inc., New York, 1956), Vol. I, p. 63.

[2]To be reported elsewhere.

[3]L. J. Challis, J. D. N. Cheeke, and J. B. Harness, Phil. Mag. 7, 1941 (1962); M. G. Holland, Phys. Rev. 134, A471 (1964).

[4]S. M. Puri and T. H. Geballe, Phys. Rev. 136, A1767 (1964).

[5]See, for example, D. K. C. MacDonald, Thermoelectricity: An Introduction to the Principles (John Wiley & Sons, Inc., New York, 1962), p. 26.

[6]E. O. Kane, J. Phys. Chem. Solids 1, 249 (1957).

[7]C. Herring, Phys. Rev. 96, 1163 (1954).

[8]Y. Toyozawa, J. Phys. Soc. Japan 17, 986 (1962).

[9]W. Sasaki, J. Phys. Soc. Japan 20, 825 (1965).

[10]T. Kasuya, Progr. Theoret. Phys. (Kyoto) 22, 227 (1959).

[11]L. M. Roth, B. Lax, and S. Zwerdling, Phys. Rev. 114, 90 (1959).

[12]F. T. Hedgecock and D. Mathur, in Program and Abstracts of the Ninth International Conference on Low Temperature Physics, Columbus, Ohio, 1964 (unpublished), p. 51.

NEW MECHANISM FOR SUPERCONDUCTIVITY*

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(Received 16 August 1965)

It is the purpose of this note to point out a new mechanism which provides an instability against Cooper-pair formation. We find that a weakly interacting system of fermions cannot remain normal down to the absolute zero of temperature, no matter what the form of the interaction. This mechanism has nothing to do with the conventional electron-phonon attractive interaction in metals, or the long-range attractive van der Waals forces in He^3. It is present even in the case of purely repulsive forces between the particles, and is due to the sharpness of the Fermi surface for the normal system.

To understand what is involved, we first take an over-simplified view of the effect. It has long been known[1] that if a charge is placed in a metal, the screening is such that there remains a long-range oscillatory potential of the form $\cos(2k_{\mathrm{F}}r+\varphi)/r^3$ (k_{F} is the Fermi momentum). This leads to a long-range interaction between charges. Formally, the source of this long-range force is the singularity of the dielectric constant as a function of the momentum transfer $\vec{q}$, when $q=2k_{\mathrm{F}}$.[1] This singularity in the Fourier transform of the interaction gives rise to a long-ranged oscillatory force in ordinary space. All that is necessary for this effect is a sharp Fermi surface; a rounding of the Fermi surface due to (say) finite temperature or impurities will give rise to an interaction which drops off exponentially at very large distances.

It is plausible to suppose that, similarly, the effective interaction between the fermions themselves will have a long-range oscillatory part. By taking advantage of the attractive regions, Cooper pairs can form thus giving rise to superconductivity.

To investigate this possibility more systematically we consider the following model: an isotropic system of spin-$\frac{1}{2}$ fermions with weak short-range forces between them. The criterion we use for the onset of superconductivity is that the scattering amplitude for pairs of quasiparticles of equal and opposite momenta and total energy corresponding to two particles on the Fermi surface has a pole at a certain temperature T_c.[2] It can be shown that this criterion is exactly the same as one would obtain from the general theory of Cooper-pair formation,[3] in the limit as the gap approaches zero. This criterion leads to the following equation:

$$\chi(p) = -\frac{1}{\beta}\sum_{p'} K(p,p')G(p')G(-p')\chi(p'), \qquad (1)$$

where $K(p,p')$ is derived by summation over spin variables from the irreducible vertex[4] $I_{\alpha\gamma,\alpha'\gamma'}(p,-p;p',-p')$. α,γ are spin indices, p represents the momentum and (discrete) energy variables. The diagrams contributing to K up to second order are given in Fig. 1. Figure 1(a) represents the direct interaction; Fig. 1(b) represents the effect we have discussed in the general introduction, i.e., the screened direct interaction. Figures 1(c) and 1(d) are new effects, Fig. 1(c) being due to a wave-function modification of the particles and Fig. 1(d) to exchange. Equation (1) may be reduced (for small temperature) to an equation on the Fermi surface. K then becomes a function only of the angle between $\vec{k}$ and $\vec{k}'$. The solutions of Eq. (1) have an angular dependence of the form $Y_l^m(\theta,\varphi)$. For a given l, Eq. (1) can be reduced to

$$-[k_F m/(2\pi)^2]K_l \ln(\beta\epsilon_{0,l}) = 1, \qquad (2)$$

where

$$K_l = \int_0^\pi d(\cos\theta)P_l(\cos\theta)K(\cos\theta)$$

and $\epsilon_{0,l}$ is of the order of the Fermi energy. As long as K_l is negative, no matter how small, Eq. (2) has a solution for small enough temperature since $\ln(\beta\epsilon_{0,l})$ can be made arbitrarily large.

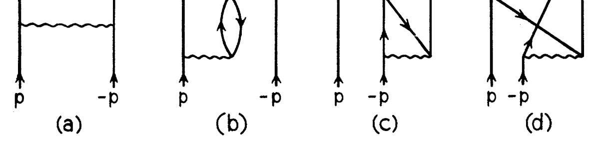

FIG. 1. Types of particle-particle interaction diagrams up to the second order which contribute to the irreducible scattering vertex.

We can in fact demonstrate that, at least for large odd values of l, K_l must become negative. The essential feature of $K(\cos\theta)$ is that its contributions from Figs. 1(b) and 1(c) are singular at $\cos\theta = -1$, and that from Fig. 1(d) at $\cos\theta = +1$. The origin of this singularity is the same as the origin of the singularity of the dielectric constant: the sharpness of the Fermi surface.[6] The singular parts of $K^{(b)}$, $K^{(c)}$, and $K^{(d)}$ are

$$K^{(b)}_{\text{sing}} = 2u^2(2k_F)Q(\cos\theta),$$

$$K^{(c)}_{\text{sing}} = -2u(2k_F)u(0)Q(\cos\theta),$$

$$K^{(d)}_{\text{sing}} = -u^2(0)Q(-\cos\theta). \qquad (3)$$

Here

$$Q(\cos\theta) = (mk_F/16\pi^2)(1+\cos\theta)\ln(1+\cos\theta), \qquad (4)$$

and $u(q)$ is the Fourier transform of the direct interaction. From these expressions we find $K_l \sim 1/l^4$. If the direct interaction is regular, its contribution to K_l [Fig. 1(a)] drops off exponentially in l, and thus for large l always becomes negligible compared to the contributions of the second-order diagrams.[7] For large l one then finds, finally, the condition

$$[(k_F m)^2/(2\pi)^4](1/l^4) \times\{u^2(0) + 2(-)^l[u(0)u(2k_F) - u^2(2k_F)]\}\ln(\beta\epsilon_0) = 1. \qquad (5)$$

It is clear that, at least for odd l, this has a solution no matter what the form of $u(k)$ is.

Of course one cannot use these formulas for real metals or for liquid He^3, since the interactions are really not weak. However, they may be used as a <u>very</u> crude estimate of the kind of transition temperatures we might expect. For He^3 we may represent the interaction by a pseudopotential

$$u(\vec{r}) = +(4\pi a/m)\delta(\vec{r}), \qquad (6)$$

where a is the diameter of the hard core of the He^3 atoms. Then (5) becomes

$$\frac{kT_c}{\epsilon_{0,l}} \sim \exp\{-[\pi^2/(k_F a)^2]l^4\} \sim \exp(-2.5l^4), \qquad (7)$$

since for He^3, $k_F a \sim 2$. For example, for $l = 2$,

$$(kT_c/\epsilon_{0,l}) \sim e^{-40} \sim 10^{-17}, \qquad (8)$$

which is utterly negligible. It should be remembered, however, that an error of a factor of 5 in the exponent (which is possible) could lead to an observable effect.

For electrons in metals we can make estimates using a screening Coulomb potential. It is easily seen that in the high-density limit $u(0)$ dominates and becomes $u(0)=\pi^2/mk_F$; therefore,

$$(kT_c/\epsilon_0)\sim\exp[-(2l)^4]. \qquad (9)$$

Is there then any hope of observing superconductivity (or superfluidity) which is largely due to the mechanism proposed here? In He^3 it seems unlikely, since $\epsilon_{0,l}$ corresponds to about a degree, and therefore the exponent must be fairly small. For metals, on the other hand, the situation is not quite as bad, for the following reasons. First, $\epsilon_{0,l}\sim\epsilon_F\sim10^4\,°K$, which means a factor of $kT_c/\epsilon_{0,l}$ of even 10^{-7} may give an observable transition temperature. Secondly, by taking metals with different parameters and nonspherical Fermi surfaces, it may prove possible to enhance the effect appreciably. In fact, it is known that some substances[8] (for example, molybdenum, niobium, vanadium, and tantalum) show unexpectedly strong and sharp structure in their phonon spectra, which can only arise from strong long-range oscillatory forces between the ions. This, in turn, is very likely due to Friedel oscillations, which have the same origin as the effects studied here. It is interesting in this connection that some of these substances have rather high transition temperatures.[9]

One may easily see that a flattening of the Fermi surface or an abnormally high density of states can result in a considerable enhancement of phonon anomalies[10] as well as of the mechanism discussed here. A factor of 10 in the exponent does not appear out of the question.

We also mention that since the electron-phonon interaction is screened by the same kind of mechanism, it too should contribute to long-range effects.

We would like to express our appreciation to the staff of the Bell Telephone Laboratories at Murray Hill for their hospitality during the summer of 1965, when this work was carried out.

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*Work performed at Bell Telephone Laboratories, Murray Hill, New Jersey.

[1]H. Friedel, Advan. Phys. 3, 446 (1954); Nuovo Cimento Suppl. 2, 287 (1958). W. Kohn, Phys. Rev. Letters 2, 393 (1959); W. Kohn and S. H. Vosko, Phys. Rev. 119, 912 (1960); J. S. Langer and S. H. Vosko, J. Phys. Chem. Solids 12, 196 (1959).

[2]This criterion has been used by several authors. See, for example, D. J. Thouless, Ann. Phys. (N.Y.) 10, 553 (1960); L. P. Gor'kov and T. K. Melik-Barkhudarov, Zh. Eksperim. i Teor. Fiz. 40, 1452 (1961) [translation: Soviet Phys.—JETP 13, 1018 (1961)]; L. P. Gor'kov and L. P. Pitaevski, Zh. Eksperim. i Teor. Fiz. 42, 600 (1962) [translation: Soviet Phys.—JETP 15, 417 (1962)].

[3]See, for example, A. I. Larkin and A. B. Migdal, Zh. Eksperim. i Teor. Fiz. 44, 1703 (1963) [translation: Soviet Phys.—JETP 17, 1146 (1963)]. There only the $T=0$ case is treated, but the analysis for $T\neq0$ is almost identical.

[4]See A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics (Prentice-Hall, Inc., 1963), p. 289 ff.

[5]See Gor'kov and Pitaevski, reference 2; and L. P. Pitaevski, Zh. Eksperim. i Teor. Fiz. 37, 1794 (1959) [translation: Soviet Phys.—JETP 10, 1267 (1960)], for similar considerations applied to the long-ranged van der Waals forces in liquid He^3.

[6]For $kT_c \ll \mu$, it is permissible to calculate K in the limit $T=0$.

[7]Even for an r^{-6} attraction, as in He^3, the direct term contributes only $O(l^{-5})$ to K_l.

[8]A. D. B. Woods and S. H. Chen, Solid State Commun. 2, 233 (1964); A. D. B. Woods, Phys. Rev. 136, A781 (1964); Y. Nakagawa and A. D. B. Woods, Phys. Rev. Letters 11, 271 (1963); J. Peretti, I. Pelah, and W. Kley, Phys. Letters 3, 105 (1962).

[9]B. T. Matthias, T. H. Geballe, and V. B. Compton, Rev. Mod. Phys. 35, 1 (1963).

[10]A. M. Afanasev and Yu. Kagan, Zh. Eksperim. i Teor. Fiz. 43, 1456 (1962) [translation: Soviet Phys.—JETP 16, 1030 (1963)].