havior results simply from the fact that our data are not taken exactly on the critical isotherm,[16] and propose to test this hypothesis soon by making measurements along a family of closely-spaced isotherms passing through the critical region. If significant, this curvature would lead to even larger deviations from the law of corresponding states than we have supposed.

The broken curves in the figure were calculated from the equation

$$P-P_c=\tfrac{1}{3}B\rho_c(\rho-\rho_c)^3+\tfrac{1}{4}B(\rho-\rho_c)^4 \quad (T=T_c), \qquad (6)$$

which represents the critical isotherm in the expansion recently discussed by Tisza and Chase,[1,17]

$$(\partial\mu/\partial\rho)_T=At+B(\rho-\rho_c)^2, \qquad (7)$$

where $t=T-T_c$ and μ is the chemical potential. The constant B has been chosen to fit the experimental data at $(\rho-\rho_c)=0.01$. In view of the nonanalytic character of the thermodynamic functions at the critical point, this expansion was not expected to be quantitatively correct, and, indeed, appreciable deviations are observed. Nevertheless, it correctly represents certain qualitative features of our result, namely, the separation of the two branches of the isotherm and the fact that the higher density side may be slightly steeper.

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[17]We have slightly changed the notation of reference 1 to avoid confusion with the exponent β.

INCOHERENT MICROWAVE SCATTERING FROM RESONANT PLASMA OSCILLATIONS

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We have observed microwave scattering at the "plasma line" from controlled electron-plasma oscillations generated in a laboratory plasma. Several recent investigations[1] have been concerned with the scattering of electromagnetic radiation from fluctuations in a plasma. In particular, Kroll, Ron, and Rostoker, and Berk[2] have pointed out that the level of electron density fluctuations could be enhanced beyond their thermal equilibrium value in order to increase the scattering cross section. In our experiment, the scattering cross section is greatly enhanced by exciting in the plasma large-amplitude oscillation modes, using the so-called Tonks-Dattner resonances. These are radial electron-plasma oscillation modes strongly coupled to transverse electromagnetic waves.[3] By this means the scattering at the "plasma line" becomes easily detectable.

In the experiment, the positive column of a mercury gas discharge at 0.001 Torr (8mm in diameter) was inserted across a rectangular S-band wave guide with the discharge-tube axis normal to the propagation and electric field vectors of the TE_{10} mode. Two microwave signals were beamed at the plasma. One was a high-frequency signal of frequency f_{inc} – the incident signal – to which the plasma, whose electron-plasma frequency f_p was kept below $\frac{1}{4}f_{inc}$, was normally transparent. The other was a low-frequency signal – the resonance signal – at $f_{res} \cong f_p$, which could excite a resonant mode in the plasma. The scattering of the high-frequency signal at f_{inc} from the plasma oscillations at f_{res} led to the appearance of a scattered signal at $f_{sc} = f_{inc} - f_{res}$ whenever a resonant plasma oscillation was excited in the plasma.

Figure 1 shows a typical experimental result; here $f_{inc} = 10.5$ Gc/sec, $f_{res} = 2.6$ Gc/sec, and $f_{sc} = 7.9$ Gc/sec. Trace (a) shows the reflected power at the resonance frequency 2.6 Gc/sec as a function of ω_p^2. The peaks at $\omega_p^2/\omega_{res}^2 \cong 4$, 3.2, 1.8, etc., represent the excitation of the first, second, etc., resonant modes, respectively. The scattered power at 7.9 Gc/sec, as observed by a heterodyne detector in the forward direction, is shown in trace (b). The scattered signal appears only in those ranges of ω_p^2 at which an electron-plasma oscillation is excited.

To ensure that the observed signal at 7.9 Gc/sec is due to the linear scattering process, and not to the nonlinear mixing of two simultaneously excited resonances,[4] trace (c) shows the reflected power observed when a 7.9-Gc/sec signal is beamed at the plasma. As expected, the resonances at 7.9 Gc/sec occur at much higher values of ω_p^2 than the scattered signal at 7.9 Gc/sec. It can be estimated[5] that the nonlinear mixing produces a signal more than two orders of magnitude smaller than the observed signal.

Figure 2(a) shows the dependence of the scattered signal on the resonance-signal power level, for a constant value (5 mW/cm^2) of the incident signal. Figure 2(b) shows the dependence of the scattered signal on the incident power level, for a constant value (5 mW/cm^2) of the resonance signal. Both of these relationships are linear, as expected. Measurements were performed at incident frequencies ranging from 8 to 12 Gc/sec and resonance frequencies between 2 and 4 Gc/sec, with results substantially similar to those shown in Figs. 1 and 2.

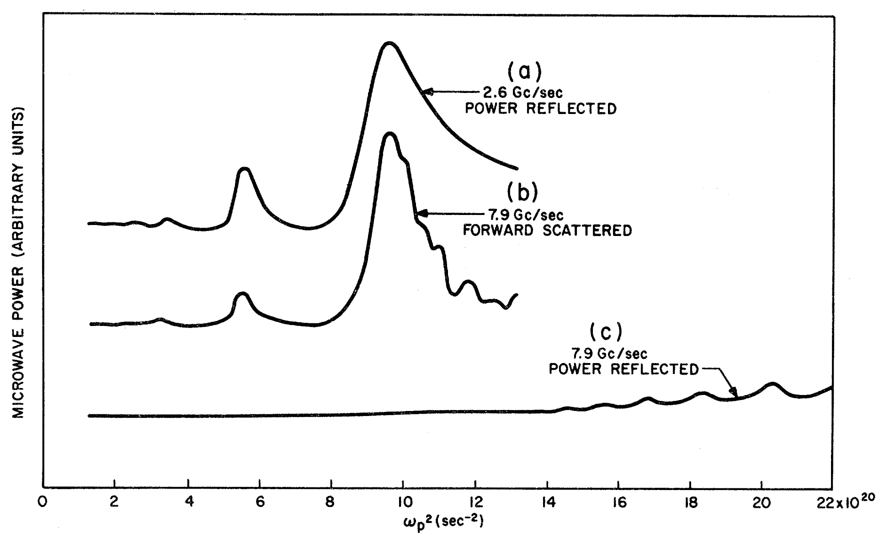

FIG. 1. Recorder traces of (a) reflected power at resonance frequency, (b) forward-scattered power, and (c) reflected power at scattered frequency as function of plasma frequency squared. Zero level of traces is offset vertically. Incident frequency is 10.5 Gc/sec.

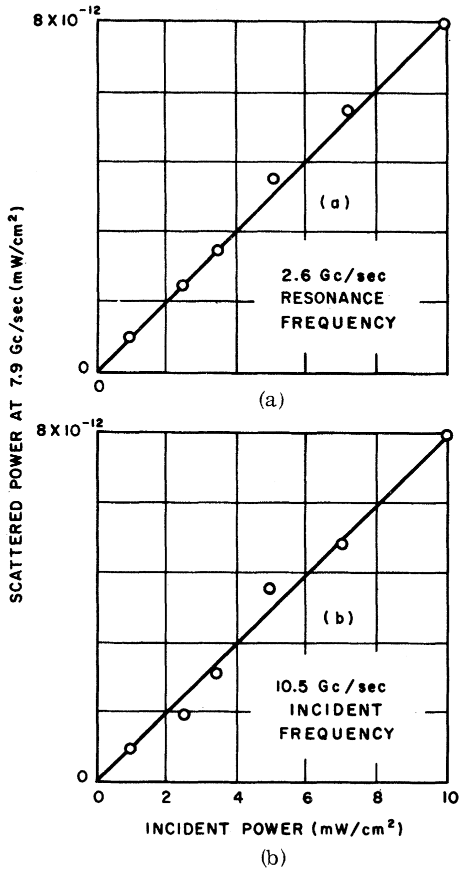

FIG. 2. Dependence of scattered power on (a) resonance power, and (b) incident power, for second resonant mode.

In comparing the results with theory, use was made of the differential scattering cross section for plane waves[6]:

$$\frac{d^2\sigma}{d\Omega d\omega}=r_0^{\,2}|\hat{E}_{\rm inc}-\hat{K}_{\rm sc}(\hat{K}_{\rm sc}\cdot\hat{E}_{\rm inc})|^2 \times \lim_{T\to\infty}\frac{|n(\overline{K},\omega)|^2}{T},$$

where $r_0^{\,2}$ is the classical electron radius, and $\hat{E}_{\rm inc}$ and $\hat{K}_{\rm sc}$ are unit vectors in the direction of the incident field and the scattered wave propagation, respectively. Since the incident and scattered signals propagate as wave-guide modes in the same plane, $\hat{K}_{\rm sc}\cdot\hat{E}_{\rm inc}=0$, and the second factor on the right-hand side above reduces to unity. Here $\omega=\omega_{\rm inc}-\omega_{\rm sc}=\omega_{\rm res}$, $\overline{K}=\overline{K}_{\rm inc}-\overline{K}_{\rm sc}$, and $n(\overline{K},\omega)$ is the spectral density (Fourier transform) of $n_e(\overline{r},t)$, the oscillating electron density in a resonant mode. The latter has the form $Af(r)\cos\theta\,\exp(i\omega_{\rm res}t)$, where A is an amplitude factor proportional to the resonance-signal strength, $f(r)$ represents the radial variation of the oscillating electron density amplitude, and $\cos\theta$ denotes the angular dipolar character of the modes. Thus $n(\overline{K},\omega)$ reduces to

$$n(\overline{K},\omega)=4\pi^2Al\int_{r=0}^{a}f(r)J_1(Kr)rdr\,\delta(\omega-\omega_{\rm res}),$$

where a and l are the radius and length of the tube, J_1 is the Bessel function of the first kind, and δ is the Dirac delta function.[7]

Using the function $f(r)$ calculated by Nickel, Parker, and Gould[3] for the second resonance in mercury, we have computed $n(\overline{K},\omega)$ for the conditions corresponding to Figs. 1 and 2. The appropriate $\overline{K}$ was calculated by decomposing the wave-guide modes into their equivalent plane-wave components. It should be noted that the resonance signal was in the TE_{10} mode, while the incident and scattered signal propagated as TE_{01} modes. In this way, the plane of the electron-plasma oscillations (parallel to the resonance-signal field vector) contained the incident and scattered propagation vectors, satisfying the momentum-conservation condition.

Equating the scattering cross section calculated in this way to the observed cross section, we obtain for the amplitude $A=1.2\times10^6$ cm^{-3} per mW of resonance signal. The ratio of the second-harmonic signal generated at resonance[4] to the fundamental reflected signal can be used to obtain an independent estimate of the amplitude; this yields 5×10^6 cm^{-3} per mW of resonance signal. Taking into account the accuracy attainable at very low signal levels, these results are in qualitative agreement.

In conclusion, in the present experiment the plasma is driven strongly out of equilibrium by a relatively low-frequency ($f_{\rm res}\cong f_p$) resonance signal, which generates an electron density wave whose amplitude is proportional to the strength of the resonance signal. By this means, the amplitude of the fluctuation spectrum $n(\overline{K},\omega)$ can be made as large as required simply by increasing the resonance power level, subject to the provision that the properties of the plasma oscillations themselves not be affected (e.g., via thermal or nonlinear effects). Because plasma resonance provides effective coupling between transverse and longitudinal waves, resonance field strengths of 0.1 V/cm suffice to bring the scattering cross section up to levels at which the scattering is easily detectable by heterodyne techniques.

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[7]A factor of order unity, involving the ratio of the incident and scattered frequencies to the wave-guide cut-off frequency, has been omitted here.