INFLUENCE OF ZERO SOUND ON THE THERMODYNAMICS OF He^3

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Recent experimental data on the low-temperature specific heat of liquid He^3 do not follow a linear law as expected from the Landau theory,[1] but rather seem to obey a law of the form $AT\log(B/T)$.[2] Among the proposed interpretations[2] was the suggestion "that the self-energy due to the exchange of some collective excitation has an unexpectedly singular form." Such singularities are well known in quantum electrodynamics (infrared divergence),[3] and in the theory of the electron-phonon interaction.[4] In both cases, the essential cause of the singularity is the possibility for the electron to emit and absorb some soft bosons (with finite group velocity); the strength of the singularity is related to the behavior of the emission (or absorption) amplitude at small wave number. In this Letter, we briefly discuss the possibility of a finite coupling between He^3 atoms and collective excitations of the zero-sound type (at zero wave number), show that this leads to an infrared type of singularity for the quasiparticle self-energy, and note that this singularity has physically observable consequences, among which are a nonlinear temperature dependence of the specific heat, in agreement with experiment, and a possible lowering of the predicted critical temperature for superfluid He^3.

Let us assume for the moment that there exists (for energies ω less than a maximum value Ω) a boson, with propagator

$$D(k,\omega)\equiv(\omega^2-c^2k^2)^{-1}, \tag{1}$$

coupled to the fermions with a constant effective coupling Γ. Thermodynamic quantities at low temperatures are related to the fermion Green's function

$$G(k,\omega)\equiv[\omega-\epsilon_k-\Sigma(k,\omega)]^{-1}, \tag{2}$$

for ω small and k near the Fermi momentum k_F [in which case $\epsilon_k\sim v(k-k_F)$, with $v\lesssim c$]. In this region, the most singular contribution to Σ is a term (I), represented diagrammatically by Fig. 1.

If, in (I), G is replaced by the bare propagator

$$G^{(0)}\equiv(\omega-\epsilon_k)^{-1}, \tag{3}$$

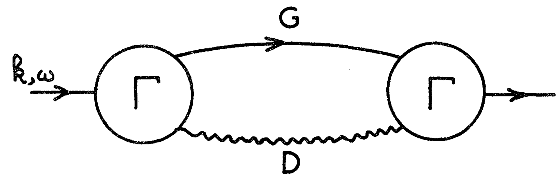

FIG. 1. Main contribution to $\Sigma(k,\omega)$, relation (I).

elementary quadratures give the result (for $|\epsilon_k|<\Omega v/c$, $|\omega|<\Omega$)

$$\begin{aligned}\mathrm{Im}\Sigma(k,\omega+i0)&=-i\pi\alpha\tfrac{1}{2}\left|\omega+\frac{c}{v}\epsilon_k\right| &&\text{for } \left|\frac{\omega}{\epsilon_k}\right|>\frac{c}{v},\\ &=-i\pi\alpha\frac{c}{c-v}|\omega-\epsilon_k| &&\text{for } \frac{c}{v}>\frac{\omega}{\epsilon_k}>1,\\ &=0 &&\text{for } 1>\frac{\omega}{\epsilon_k}>-\frac{c}{v},\end{aligned} \tag{4}$$

where

$$\alpha\equiv\Gamma^2/4\pi^2c^2(c+v), \tag{5}$$

and, consequently,

$$\mathrm{Re}\Sigma(k,\omega)\sim-\alpha\omega\ln[B/(|\epsilon_k|+|\omega|)] \tag{6}$$

for $|\epsilon_k|\ll\Omega, |\omega|\ll\Omega$ (B is a constant of order Ω). It follows that, in this approximation, G has a pole on the real axis, corresponding to a stable quasiparticle of energy

$$E_k=\epsilon_k+\Sigma(k,E_k)\sim\epsilon_k[\alpha\ln(B/|\epsilon_k|)]^{-1}. \tag{7}$$

Since both the quasiparticle velocity and the residue (E_k/ϵ_k) of G at the pole vanish for $k=k_F$, the quasiparticles do not exist right at the Fermi surface.

The low-temperature entropy per unit volume can now be evaluated by using the expression[4]

$$S\sim-2\int\frac{d^3k}{(2\pi)^3}\int_{-\infty}^{+\infty}d\omega\frac{\omega}{T}f'(\omega)\frac{1}{\pi}\mathrm{Im}\ln G(k,\omega+i0), \tag{8}$$

where G is replaced by Eqs. (2) and (4), and where f is the Fermi factor. One finds[5]

$$S\sim\tfrac{1}{3}m^*k_F\alpha T\ln(B/T), \tag{9}$$

instead of the form $\frac{1}{3}m^*k_FT$ of the Landau theory[1]; hence, the specific heat has a similar form. The empirical law[2] is thus recovered, with $\alpha m^*/m \simeq 0.5$ and $B \simeq 20°K$.

The spin susceptibility is expected, like in the Landau theory, to tend to a constant at low temperature, because He^3 atoms are assumed to interact with spinless collective bosons.

The particle occupation number at zero temperature,

$$n(k) = -\frac{1}{\pi}\int_{-\infty}^{0} d\omega\, \mathrm{Im}G(k, \omega + i0), \tag{10}$$

has also been derived from Eqs. (2) and (4). Contrary to the Landau theory,[6] $n(k)$ is continuous at the Fermi surface, and behaves there as

$$|n(k) - n(k_F)| \sim [2\alpha \ln(B/|\epsilon_k|)]^{-1}. \tag{11}$$

One may suspect that this rounding off of the Fermi surface tends to hinder the formation of Cooper pairs at low temperature. As is well known,[7] a superfluid type of transition is expected to take place at a temperature T_c given by[4]

$$\frac{1}{\lambda} = \int d\epsilon_k T_c \sum_{n=-\infty}^{+\infty} G(k, (2n+1)i\pi T_c) \times G(-k, -(2n+1)i\pi T_c), \tag{12}$$

if, for some angular momentum, the two-particle interaction potential has an attractive part proportional to λ. In the usual theory, G has the form (3), and T_c is proportional to $\exp(-1/\lambda)$. However, the singular form (2) or (4) of G leads here to a critical temperature proportional to $\exp[-\exp(\alpha/\lambda)]$ for a weak attraction λ. A finite coupling of He^3 atoms with collective Bose excitations might therefore strongly reduce the predicted critical temperature[7] and explain the apparent absence of superfluidity for liquid He^3 above 0.0035°K.[8]

Whereas the infrared divergences of quantum electrodynamics are removed from observable quantities by the introduction of soft unobserved real photons, the situation is different here, because thermodynamical quantities are exclusively related to closed diagrams; formulas (8) and (10) were obtained by opening a single fermion line in a closed diagram, and therefore one is never led to consider diagrams with external boson lines (with vanishingly small wave number). In fact, we have not found any actual divergence, but rather an unusually strong temperature dependence of S.

The analytic structure of G obtained so far is very different from that of $G^{(0)}$, and, consequently, the inconsistent calculation described above [where G was replaced by $G^{(0)}$ in (I)] is questionable. A complete self-consistent solution is difficult, since (contrary to the case of the electron-phonon interaction) Σ depends on k as well as on ω (c/v is not a small quantity here). However, in the region where the pole $\omega = E_k$ lies, we have found in the above approximation that $\Sigma(k, \omega) \sim -\alpha\omega \ln(B/|\epsilon_k|)$. It is therefore reasonable to attack the self-consistent equation (1) by assuming a form $-\omega\varphi(\epsilon_k)$ for $\Sigma(k, \omega)$ in the region $|\omega| \ll |\epsilon_k| \ll \Omega$, and writing the integral equation for φ, which gives

$$\varphi(\epsilon_k) \sim [2\alpha(1 + v/c) \ln(B/|\epsilon_k|)]^{1/2}. \tag{13}$$

The results are not essentially different, although the singularity is weakened: ϵ_k/E_k is now equal to (13), and the low-temperature specific heat is

$$AT[\ln(B/T)]^{1/2}, \tag{14}$$

a law which, for $B \simeq 0.5°K$ and $A \simeq 1.6$ per mole, or $\alpha(1 + v/c)(m^*/m)^2 \simeq 1.2$, fits experimental data from 0.015 to 0.3°K (Fig. 2). The value

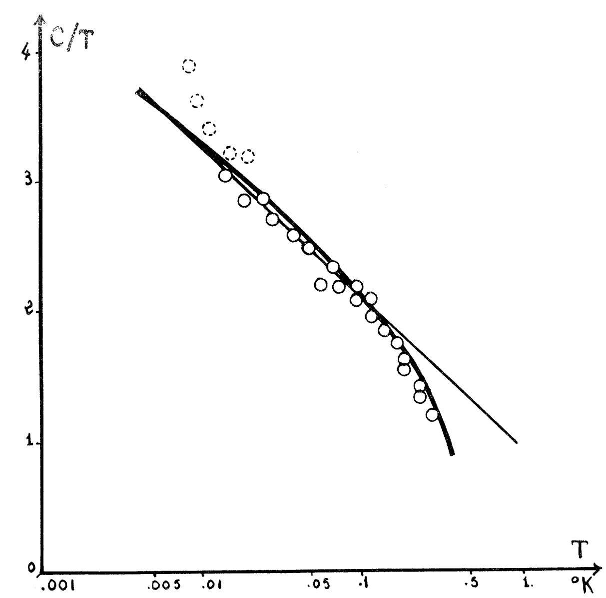

FIG. 2. Specific heat per mole of He^3 versus temperature. Experimental data are those collected in reference 1 (dashed circles represent raw data). The straight line [Eq. (9)] is the empirical law of reference 1; the curve represents Eq. (14).

0.5°K for B is reasonable, since we expected B to have the same order of magnitude as Ω.

We have still to justify the crucial assumption of a finite Γ. In order to define this quantity, we consider the two-body fermion Green's function G_2 at temperature T, corresponding to the creation of particles 1, 3 and absorption of particles 2, 4. Its Fourier transform, defined for $\omega_1-\omega_2=\omega_4-\omega_3$, $\omega_i=(2n_i+1)i\pi T$, is analytically continued in the whole three-complex-dimensional space $\omega_1-\omega_2=\omega_4-\omega_3$ with the help of a spectral representation,[9] and the uniqueness of this continuation can be proved by proper use of Carlson's theorem.[10] The singularities of G_2 then lie on the surfaces $\mathrm{Im}\omega_i=0$, $\mathrm{Im}(\omega_1-\omega_2)=0$, $\mathrm{Im}(\omega_1-\omega_4)=0$, $\mathrm{Im}(\omega_1+\omega_3)=0$. A particle-hole collective excitation (without damping) will manifest itself as a pole in the variable $\omega_1-\omega_2$ with a residue factorizing out into parts depending on (1, 2) and (3, 4). We thus separate from G_2 a term

$$G(1)G(2)\Gamma(k_1\omega_1, k_2\omega_2)D(k_1-k_2, \omega_1-\omega_2) \times\Gamma(k_3\omega_3, k_4\omega_4)G(3)G(4), \qquad (15)$$

in which D is interpreted as the propagator for the zero-sound type of excitation, and Γ as the vertex for its coupling to the fermions. Equation (15) defines Γ only on the energy shell $(\omega_1-\omega_2)^2=c^2(k_1-k_2)^2$, so that Γ is somewhat arbitrary off the shell.

The equation for Γ is obtained by isolating the contributions of the pole D in each term of the (particle-hole) Bethe-Salpeter equation for G_2. This equation (II) is represented diagrammatically by Fig. 3. The kernel of the Bethe-Salpeter equation has been separated into two parts, one corresponding to the exchange of a boson between a particle and a hole, the remainder (denoted by R) depending on the two-body interaction. In principle, one should solve Eq. (2), using the term (I) represented in Fig. 1, and the equation (II) represented by Fig. 3, self-consistently for Γ and G. We have not achieved this program, but we may justify the assumption that $\Gamma(1,2)$ is constant for 1–2 small in the following way. Let us compute the right-hand side of the equation (II), represented in Fig. 3, for 1 = 2, replacing Γ by a constant. If we take into account the singular structure of G, both terms of this expression are finite. Therefore, there does not seem to be an obvious inconsistency in our assuming a finite coupling between atoms and soft Bose excitations.

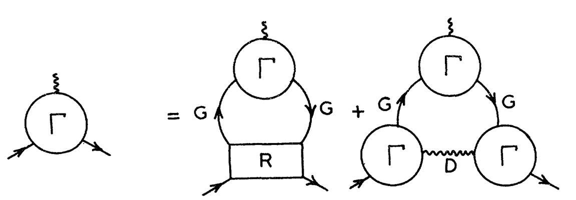

FIG. 3. Equation for Γ, relation (II).

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