PHYSICAL REVIEW LETTERS

VOLUME 15 | 13 SEPTEMBER 1965 | NUMBER 11

GENERALIZED GOLDSTONE THEOREM*

R. F. Streater

Department of Physics, Imperial College, London, England

(Received 16 August 1965)

The paper of Goldstone, Salam, and Weinberg[1] gave a very simple proof of an important theorem on broken symmetry: If $\varphi_1(x) \to \varphi_2(x)$ is a transformation of fields such that $\langle\varphi_2(x)\rangle_0 = n \neq 0$ and such that the transformation is generated by a conserved current, then there must be zero-mass particles in the theory with the quantum numbers of φ_1. (The proof depends on the absence of states of negative norm.[2]) In this paper we show that a significant generalization, along the same lines, can be proved; namely, if a local conserved current generates a transformation $\varphi_1(x) \to \varphi_2(x)$ between fields corresponding to particles of different mass, then there must be states in the theory with arbitrarily small mass (again in the absence of states of negative norm).

If a conserved current $j^\mu(x)$ generates a transformation of the fields under which the vacuum is not invariant, the symmetry of the theory is said to be spontaneously broken. This idea can be formulated mathematically in terms of axiomatic quantum field theory,[3] and is a very appealing explanation for the existence of groups, like SU(3), which are not exact symmetry groups. But the present result shows that any mass differences between particles, one of which is stable, must be caused by a symmetry breaking which is not of the spontaneous type. This contradicts the hopes of Glashow[4] as far as the strongly interacting particles are concerned.[5]

Let $\varphi_1(x)$ and $\varphi_2(x)$ be quantized fields (not necessarily spin 0) such that the Källén-Lehmann weight functions $\rho_1(K^2)$ and $\rho_2(K^2)$ have the form

$$\rho_1(K^2) = c_1\delta(K^2-m_1^2) + \sigma_1(K^2);$$

$$\rho_2(K^2) = c_2\delta(K^2-m_2^2) + \sigma_2(K^2),$$

where $m_1 \neq m_2$, $c_1 \neq 0$, and $\sigma_1(K^2) = 0$ if $K^2 < M_1^2$, $\sigma_2(K^2) = 0$ if $K^2 < M_2^2$, where

$$M_1 > \mathrm{Max}(m_1, m_2), \quad M_2 > \mathrm{Max}(m_1, m_2)$$

(these conditions are those commonly made in strong interactions, e.g. for the proton and neutron fields). Suppose there exists a conserved current (e.g. the isotopic spin current) $j^\mu(x)$ such that $\int [j^0(\vec{x}, t), \varphi_1(y)] d^3x = \varphi_2(y)$. Then there are states in the theory with quantum numbers of $\varphi_1{}^*\varphi_2$ and arbitrarily small mass.

Define $j_1(x) = (\Box + m_1^2)\varphi_1(x)$. Consider the integral

$$\int d^3x \langle p_2 | [j^0(\vec{x}, t), j_1(y)] | 0\rangle = (\Box + m_1^2)\langle p_2 | \varphi_2(y) | 0\rangle.$$

We now take the Fourier transform of both sides with respect to time t, and the four vector y, to obtain

$$\langle p_2 | [j^0(\vec{q}, q^0), j_1(p_1)] | 0\rangle |_{\vec{q}=0}$$

$$= c_1{}^1\delta^4(p_1 + p_2)\delta(q^0)(m_2{}^2 - m_1{}^2).$$

The right-hand side is nonzero. The left-hand side has two contributions. The first,

$$\langle p_2 | j^0(0, q^0) j_1(p_1) | 0\rangle,$$

is zero unless $j_1(p_1)|0\rangle$ is a physical state, i.e., $p_1^2 \geq M_1^2 > m_2^2$, and so this term cannot contribute to the right-hand side. The second term,

$$\langle p_2|j_1(p_1)j^0(0,q^0)|0\rangle,$$

must therefore contribute to $q^0=0$; thus there exist some states with the quantum numbers of $\varphi_1{}^*\varphi_2$ and arbitrarily small energy. Q.E.D.

We can immediately apply the theorem to the proton-neutron mass difference. Thus $M_n > M_p$, and considering simply $\langle p|[Q,\psi_n]|0\rangle$ we see that there must exist charge-one particles with arbitrarily small mass, which have never been seen. Similarly, one may not explain the breaking of the SU(3) mass multiplets by spontaneous breakdown.

---

*The research reported in this document has been sponsored in part by the Air Force Office of Scientific Research through the European Office of Aerospace Research, U. S. Air Force.

[1]J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[2]G. S. Guralnik, C. R. Hagen, and T. W. Kibble, Phys. Rev. Letters 13, 585 (1964).

[3]R. F. Streater, to be published.

[4]S. Glashow, Phys. Rev. 130, 2132 (1963).

[5]For a good review of the situation to date, see G. Guralnik, Lectures at the Conference on Unified Field Theory, Munich, July 1965 (to be published).

PSEUDOSPIN MODEL OF LIQUID He^4

P. R. Zilsel

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey,*
and Department of Physics, Stanford University, Stanford, California†

(Received 16 August 1965)

A simple pseudospin model for a gas of hard-core bosons with attractive interactions was introduced by Matsubara and Matsuda,[1] and its zero-temperature properties were discussed by Witlock and Zilsel.[2]

The purpose of this Letter is to point out that, with a slight modification, the model is applicable to liquid He^4 and appears to account, at least semiquantitatively, for the liquid-vapor equilibrium as well as for the λ transition, the off-diagonal long-range order[3] and superfluidity of the low-temperature liquid phase, and the nature of the elementary excitation spectrum.

The model is a quantum generalization of the classical lattice-gas (Ising) model.[4] Alternatively, it may be viewed as a cell approximation to Siegert's[5] formalism for hard-core boson fields: The volume Ω of the system is divided into M cubical cells of size d^3 where d is of the order of, but somewhat larger than, the hard-core radius a. (We shall take $d=2.85$ Å, which makes $a/d \simeq \frac{5}{6}$.) Field amplitudes ψ_j, ψ_j^+ $(j=1,\cdots,M)$, are associated with each cell (rather than each point in space), and the hard-core constraint is represented approximately by $\psi_j^2=0$, or $n_j^2=n_j$, where $n_j=\psi_j^+\psi_j$ is the particle number operator for the jth cell. This means that the states for any one cell are those of a Fermi oscillator. However, the amplitudes for <u>different</u> cells <u>commute</u>, the underlying field being Bose, so that the commutation relations are

$$[\psi_i,\psi_j]_-=0,\ \ [\psi_i,\psi_j{}^+]_-=(1-2n_j)\delta_{ij}. \tag{1}$$

The algebra defined by (1) is that of the generators of an SU(2), the symmetry being between particles and vacancies in each cell. Using the notation of reference 2, we define pseudospin components

$$\sigma_j{}^{(1)}=\psi_j+\psi_j{}^+,\quad \sigma_j{}^{(2)}=i(\psi_j{}^+-\psi_j),\quad \sigma_j{}^{(3)}=1-2n_j, \tag{2}$$

which have all the properties of Pauli operators.

In the <u>original version of the model</u>,[1,2] the continuum kinetic-energy operator $(\hbar^2/2m)\times\int\nabla\psi^+\cdot\nabla\psi d^3r$ is replaced by its finite difference approximation

$$\begin{aligned} K &= (\hbar^2/2md^2)\sum_{\langle ij\rangle}(\psi_i{}^+-\psi_j{}^+)(\psi_i-\psi_j) \\ &= (\hbar^2/md^2)\sum_{\langle ij\rangle}[\tfrac{1}{4}(1-\vec{\sigma}_i\cdot\vec{\sigma}_j)+n_in_j], \end{aligned} \tag{3}$$

and the attractive part of the potential energy by a "square well" attraction

$$V=-v\sum_{\langle ij\rangle}n_in_j, \tag{4}$$

where $\langle ij\rangle$ stands for nearest-neighbor pairs in the cubic lattice space. (The hard-core repulsion is, of course, represented in the com-