DETERMINATION OF THE SPINS AND PARITIES OF $N_{1/2}^*(1688)$ AND $N_{3/2}^*(1920)$

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This paper is a preliminary report on the results of two counter experiments recently completed at Nimrod.

(i) Differential cross sections for π^+p and π^-p elastic scattering have been measured at a number of momenta in the region of the isobar $N_{1/2}^*(1688)$ and $N_{3/2}^*(1920)$, which occur at pion laboratory momenta of 1030 and 1505 MeV/c, respectively.

(ii) Polarization effects have been measured in the same region using a polarized proton target of the type[1] recently described by Chamberlain et al.,[2] and Schultz,[3]

The differential cross section $d\sigma/d\Omega$ may be written

$$d\sigma/d\Omega = \sum_n C_n^{\pm} P_n(\cos\theta), \qquad (1)$$

where θ is the center-of-mass scattering angle. The expansion coefficients C_n^+ and C_n^-

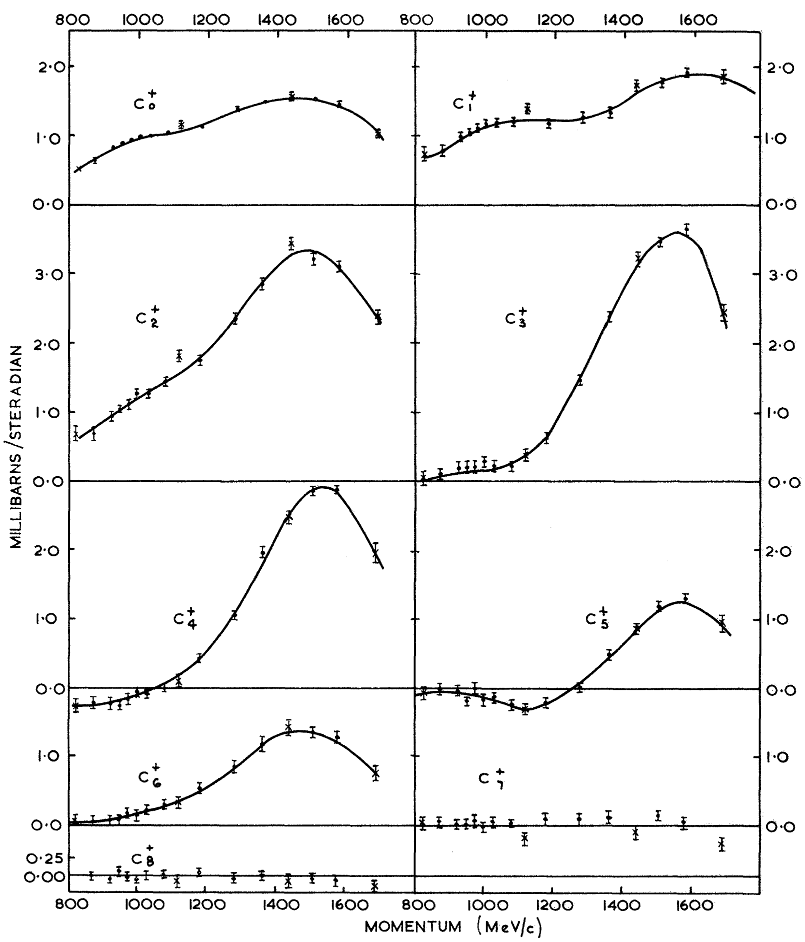

FIG. 1. The coefficients C_n^+ defined in Eq. (1) for the elastic scattering of π^+ mesons by protons. The points at 1120, 1440, and 1690 MeV/c are from the work of Helland et al.[4]; the rest are from this experiment.

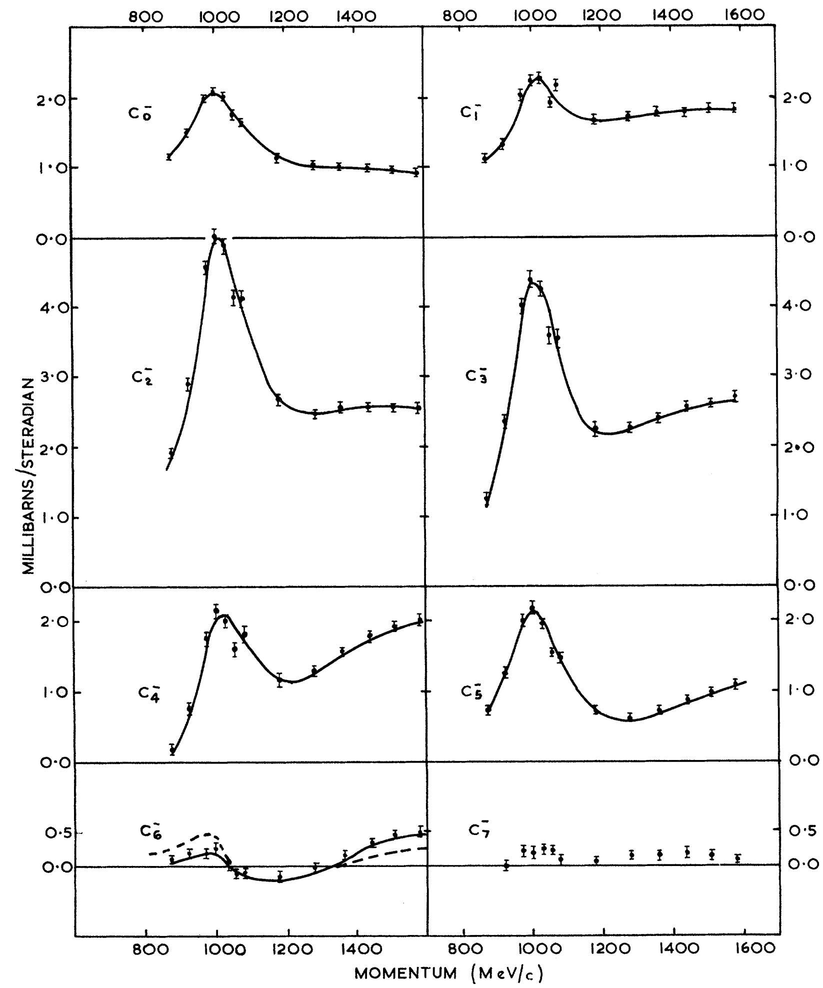

FIG. 2. The coefficients C_n^- defined in Eq. (1) for the elastic scattering of π^- mesons by protons (data from this experiment only). The dotted curve is the prediction for C_6^- described in the text.

for scattering of π^+ and π^- were obtained from the differential cross sections by the method of least-squares fitting and are plotted as functions of beam momentum in Figs. 1 and 2. Also shown are points from the work of Helland et al.[4,5]

Similarly, one can write

$$\frac{1}{\sin\theta}\frac{A^\pm}{P}\frac{d\sigma}{d\Omega}=\sum_n D_n^{\pm} P_n(\cos\theta), \qquad (2)$$

where $A^\pm$ is the asymmetry in scattering from a proton target with polarization P. The coefficients D_n^- for π^- scattering obtained from our experimental data are shown in Fig. 3.

Let the scattering matrix M be defined by the equation

$$M=f(\theta)+ig(\theta)\vec{\sigma}\cdot\vec{n},$$

where $\vec{\sigma}$ is the Pauli spin operator, $\vec{n}=(\vec{k}_i\times\vec{k}_f)/(|\vec{k}_i\times\vec{k}_f|)$, $\vec{k}_i, \vec{k}_f$ being the initial and final momenta. Then

$$d\sigma/d\Omega=|f(\theta)|^2+|g(\theta)|^2$$

and

$$\frac{A}{P}\frac{d\sigma}{d\Omega}=-2\,\mathrm{Im}[f^*(\theta)g(\theta)],$$

where

$$f(\theta)=\sum[(l+1)A_{l+}+lA_{l-}]P_l(\cos\theta)$$

and

$$g(\theta)=\sum[A_{l+}-A_{l-}]\sin\theta P_l'(\cos\theta).$$

$A_{l\pm}$ are the scattering amplitudes for states of total angular momentum $J=l\pm\frac{1}{2}$, and $P_l'(\cos\theta)=[d/d(\cos\theta)]P_l(\cos\theta)$.

Expressions for those expansion coefficients

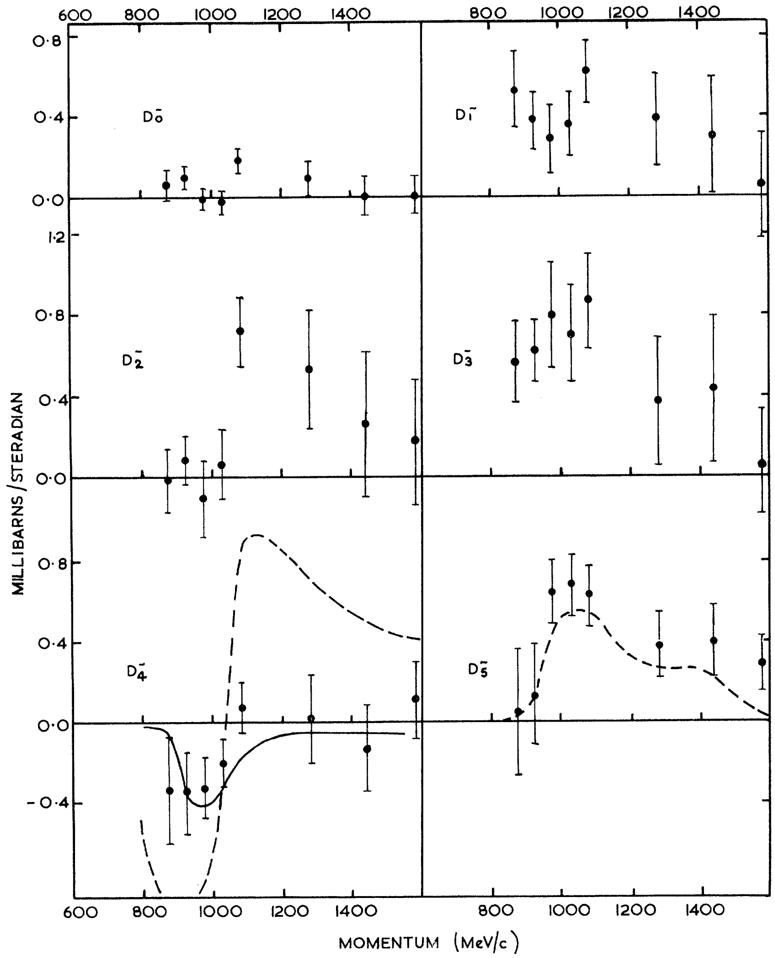

FIG. 3. The coefficients D_n^- defined in Eq. (2) for the elastic scattering of π^- mesons by polarized protons. The curves for D_4^- and D_5^- are the predictions described in the text.

relevant to the analysis of our data are given below. Factors of $\frac{1}{2}$ have been omitted; e.g. $D5$ means $D_{5/2}$.

$$C_4 = 10.29\,\mathrm{Re}(D5^*D3) + 10.29\,\mathrm{Re}(F5^*P3) + 2.57|D5|^2 + 2.57|F5|^2 + 8\,\mathrm{Re}(F7^*P1) + 5.71\,\mathrm{Re}(F7^*P3)$$
$$+ 4.68\,\mathrm{Re}(F7^*F5) + 4.21|F7|^2 + 5.71\,\mathrm{Re}(G7^*D3) + 4.68\,\mathrm{Re}(G7^*D5) + 4.21|G7|^2 + 8\,\mathrm{Re}(G7^*S)$$
$$+\text{terms with } G9 \text{ or higher waves;}$$

$$C_5 = 14.29\,\mathrm{Re}(F5^*D5) + 13.33\,\mathrm{Re}(F7^*D3) + 5.71\,\mathrm{Re}(F7^*D5) + 13.33\,\mathrm{Re}(G7^*P3) + 5.71\,\mathrm{Re}(G7^*F5)$$
$$+ 6.59\,\mathrm{Re}(G7^*F7) + \cdots;$$

$$C_6 = 18.18\,\mathrm{Re}(F7^*F5) + 3.03|F7|^2 + 18.18\,\mathrm{Re}(G7^*D5) + 3.03|G7|^2 + \cdots;$$

$$C_7 = 22.84\,\mathrm{Re}(G7^*F7) + \cdots;$$

$$C_8 = 27.41\,\mathrm{Re}(G9^*G7) + 3.43|G9|^2 + \cdots;$$

$$D_4 = -12.86\,\mathrm{Im}(F5^*D5) + 12\,\mathrm{Im}(F7^*D3) + 0.86\,\mathrm{Im}(F7^*D5) - 12\,\mathrm{Im}(G7^*P3) - 0.86\,\mathrm{Im}(G7^*F5)$$
$$-22.60\,\mathrm{Im}(G7^*F7);$$

$$D5 = 16.67\,\mathrm{Im}(F7^*F5) - 16.67\,\mathrm{Im}(G7^*D5).$$

(i) Spin of $N_{3/2}^*(1920)$.—In the region of this resonance (around 1500 MeV/c) all the π^+p coefficients C_n^+ with n up to 6 have maxima, while C_7^+ and C_8^+ are small. Assuming that the peak is due to a single resonant state, the absence of a peak in C_8^+ shows that $J \leq \frac{7}{2}$ for the resonant state. The smallness of C_8^+ shows that nonresonant states with $J > \frac{7}{2}$ are only weakly excited, and the smallness of C_7^+ shows that a $J = \frac{7}{2}$ state of only one parity can be excited ($F_{7/2}$ and $G_{7/2}$ would interfere to give a nonzero C_7^+). Thus the peak in C_6^+ must be due to the term $18.1\,\mathrm{Re}(F_{7/2}{}^*F_{5/2})$, or $18.1\,\mathrm{Re}(G_{7/2}{}^*D_{5/2})$ in the case of a $J = \frac{5}{2}$ resonance (together with a nonresonant $J = \frac{7}{2}$ amplitude of the same parity), the term $3.03|G_{7/2}|^2$ for a $J = \frac{7}{2}$ resonance with no $J = \frac{5}{2}$ state, or a combination of the two terms if there is a $J = \frac{7}{2}$ resonance as well as a nonresonant but appreciable $J = \frac{5}{2}$ amplitude of the same parity. However, if $J = \frac{5}{2}$ for the resonance, then we must have a substantial $J = \frac{7}{2}$ amplitude to explain the peak in C_6^+, a substantial $J = \frac{5}{2}$ amplitude of the opposite parity to explain the peak in C_5^+, and contributions from lower J states to explain the magnitudes of the peaks in the lower coefficients. The values of the imaginary parts of these nonresonant amplitudes can be estimated from the values of C_n^+ at the resonance. Now $C_0 = \sum(J + \frac{1}{2})|A_J|^2$, where the sum is over all states and A_J is the amplitude for the state J; if $J = \frac{5}{2}$ is assumed for the resonance, then the estimates we obtain for the imaginary parts of the A_J lead to a value of C_0^+ much larger than our measured value. Therefore, $J = \frac{7}{2}$ for $N_{3/2}^*(1920)$.

(ii) Spin of $N_{1/2}^*(1688)$.—The amplitude for π^-p elastic scattering is $A(\pi^-p) = (\frac{2}{3})A(I = \frac{1}{2}) + (\frac{1}{3})A(I = \frac{3}{2})$, where the two terms on the right are isotopic-spin eigenamplitudes. We attribute the peaks in C_0^- to C_5^- to a resonant state with $I = \frac{1}{2}$. For a $J = \frac{7}{2}$ resonance it follows from the size of the bump in C_0^- that C_6^- should have a peak of at least 2 mb/sr at the resonant energy; the absence of any such peak shows that $J = \frac{5}{2}$ for $N_{1/2}^*(1688)$; the peak in C_5^- must be due to interference with a $J = \frac{5}{2}$ amplitude of the opposite parity. (C_5^0, the corresponding coefficient in the charge-exchange process $\pi^- + p \to \pi^0 + n$, also has a positive peak.[6] In terms of the isotopic-spin eigenamplitudes, D_{15}, D_{35}, F_{15}, and F_{35}, for the $J = \frac{5}{2}$ states with $I = \frac{1}{2}$ and $\frac{3}{2}$, the expressions for C_5^0 and C_5^- are

$$C_5^0 = 14.29\,\mathrm{Re}[2/9F_{15}{}^*D_{15} + 2/9F_{35}{}^*D_{35} - 2/9F_{15}{}^*D_{35} - 2/9F_{35}{}^*D_{15}],$$

$$C_5^- = 14.29\,\mathrm{Re}[4/9F_{15}{}^*D_{15} + \tfrac{1}{9}F_{35}{}^*D_{35} + 2/9F_{15}{}^*D_{35} + 2/9F_{35}{}^*D_{15}].$$

Since the magnitude of the peak observed in C_5^0 is approximately half that in C_5^-, and since both peaks are positive, it follows that both $J = \frac{5}{2}$ states must be dominated by the $I = \frac{1}{2}$ amplitudes.)

(iii) Parity of $N_{1/2}^*(1688)$.—The energy dependence of C_6^- in the neighborhood of $N^*(1688)$ has the form of an interference between a resonance state and a predominantly real nonresonant state whose real part is positive—viz., the coefficient is positive below resonance, negative above resonance, and passes through zero at the resonant energy. In order to have this behavior in C_6^-, the $J = \frac{5}{2}$ resonant amplitude must be interfering with a $J = \frac{7}{2}$ amplitude of the same parity. Since higher J values are not appreciable, the behavior of the coefficient D_5^- in the asymmetry distribution can be predicted. We have $D_5^- = 16.67\,\mathrm{Im}(E_{7/2}{}^*F_{5/2}) - 16.67 \times \mathrm{Im}(G_{7/2}{}^*D_{5/2}) \approx 16.67\,\mathrm{Re}(F_{7/2})\,\mathrm{Im}(F_{5/2}) - 16.67 \times \mathrm{Re}(G_{7/2})\,\mathrm{Im}(D_{5/2})$, using the knowledge from C_6^- that the $J = \frac{7}{2}$ amplitude is predominantly real. Since C_7^- is small, only one of $F_{7/2}$ or $G_{7/2}$ is appreciable, so only one of the terms contributes. The imaginary part of a partial wave amplitude for elastic scattering must be positive, so we expect D_5^- to be positive if $N^*(1688)$ is $F_{5/2}$ and negative if it is $D_{5/2}$ (or zero if the interpretation of C_6^- is incorrect). The experimental result is that D_5^- is positive in this energy region, so $N^*(1688)$ must be an $F_{5/2}$ state.

(iv) Parity of $N_{1/2}^*(1920)$.—This resonance has $I = \frac{3}{2}$, so it occurs in the π^--p elastic-scattering amplitude with a coefficient of $\frac{1}{3}$. It must therefore contribute at least part of the $J = \frac{7}{2}$ amplitude necessary to explain C_6^- and D_5^-. $N^*(1688)$ is more than one full width away from $N^*(1920)$, so the $J = \frac{7}{2}$ amplitude would be expected to be predominantly real and positive if it is contributed mostly by the $I = \frac{3}{2}$ state. Preliminary data on charge-exchange scattering,[6] $\pi^- + p \to \pi^0 + n$, show that the corresponding coefficient C_6^0 is positive in the energy region where C_6^- is negative. This shows that the main contribution to C_6^- is interference between an $I = \frac{1}{2}$ amplitude and an $I = \frac{3}{2}$ amplitude. It has already been established in (iii) that the $J = \frac{7}{2}$ state and the resonant $J = \frac{5}{2}$ state in π^--p elastic scattering have the same parity, so the $N^*(1920)$ resonance must be given the assignment $F_{7/2}$.

Quantitative predictions of C_6^- and D_5^- have been made by using Breit-Wigner amplitudes to represent the two resonances

$$A_{\mathrm{BW}} = x\lambda/(\epsilon - i),$$

where

$$x = \frac{\text{elastic cross section}}{\text{total cross section}},$$

$$\epsilon = 2(E_\gamma^* - E^*)/\Gamma,$$

E^* = total energy in center-of-mass system,

Γ = width.

The parameters x and Γ can be estimated from the heights and widths of the bumps in the total cross-section curves, given the J values and a smooth background curve (which is rather arbitrary). Independent values can be obtained from $\sigma_{\mathrm{el}} = 4\pi C_0$. For $N^*(1688)$ we find $x = 0.80$, $\Gamma = 100$ MeV and for $N^*(1920)$, $x = 0.41$, $\Gamma = 170$ MeV. It is necessary to allow Γ for $N^*(1688)$ and x for $N^*(1920)$ to be energy dependent, becoming $\Gamma = 157$ MeV at $E^* = 1920$ MeV and $x = 0.69$ at 1688 MeV, respectively. With these parameters one obtains the curves for C_6^- and D_5^- shown in the figures. It can be seen that a reasonable representation of the energy dependence between the two resonances is obtained, justifying the assumption that the two resonant states are dominant in this region.

We may also attempt to calculate the coefficient D_4^-. As indicated above, the $F_{7/2}$ amplitude is significant but small, and the $G_{7/2}$ amplitude negligible near 1000 MeV/c. The coefficients C_5^- and D_4^- must therefore be dominated by $F_{5/2}D_{5/2}$ interference in this region. Assuming that the $F_{5/2}$ amplitude is resonant, the $D_{5/2}$ amplitude may be estimated from C_5^-. If it is taken to be purely imaginary and slowly varying, as seems reasonable from the approximate symmetry of the peak in C_5^- about the resonant energy, the dotted curve shown in Fig. 3 is obtained for D_4^-. This clearly does not agree with the values obtained experimentally. An improvement may be obtained by introducing a real part into the $D_{5/2}$ amplitude which must, however, be positive below the resonance and negative above it. It is also necessary to allow $\mathrm{Im}(D_{5/2})$ to vary in a similar way to $\mathrm{Im}(F_{5/2})$. This behavior of the $D_{5/2}$ amplitude is also required for consistency with the symmetric peak in C_5^- and suggests that the $D_{5/2}$ state may itself be resonant. If we assume that it can be described by a Breit-Wigner formula with $E_\gamma^* = 1674$ MeV, $\Gamma = 100$ MeV, $x = 0.42$, then the prediction for D_4^-, as shown by the solid curve in Fig. 3, is in very good agreement with experiment. A reasonable interpretation of our data is therefore that there is an $I = \frac{1}{2}$, $D_{5/2}$ resonance as well as the $F_{5/2}$ resonance near 1000 MeV/c.[7] A complete account of these experiments will be published.

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[1]In this experiment the asymmetry was measured in π^- scattering from a polarized target at 16 angles $(0.83 > \cos\theta^* > -0.95)$ at each of eight beam momenta.

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[6]F. Bulos et al., Phys. Rev. Letters 13, 558 (1964).

[7]Evidence for the existence of both a $D_{5/2}$ and an $F_{5/2}$ resonance in this energy region has also been found independently by A. Donnachie, A. T. Lea, and C. Lovelace, private communication; B. H. Bransden, P. J. O'Donnell, and R. G. Moorehouse, private communication; and P. Bareyre, C. Bricman, A. V. Stirling, and G. Villet, private communication; from phase-shift analyses of the present data.