Table I. Energy-gap and critical-field data.

Superconductor	$\dfrac{\Delta(0)}{kT_c}$	$\dfrac{T_c}{H_0}\left(-\dfrac{dH_c}{dT}\right)_{T_c}$	$\dfrac{\Delta(0)/kT_c}{(T_c/H_0)(-dH_c/dT)_{T_c}}$
weak-coupling limit of BCS theory	1.75[a]	1.75[a]	1.00
aluminum	1.68 - 1.75[b]	1.715[c]	0.98-1.02
tin	1.73,[d] 1.76[e]	1.75[f]	0.99, 1.01
thallium	1.78[g]	1.79[h]	0.99
tantalum	1.80[e]	1.80[f]	1.00
indium	1.82[d]	1.80[i]	1.01
niobium	1.92[e]	1.99[j]	0.96
lead	2.16[d]	2.13[k]	1.01
mercury	2.25[l]	2.03[m]	1.11

[a]These values were obtained from University of Illinois Department of Physics Technical Report No. 13, 25 May 1959 (unpublished), which contains a reprint of reference 1 with several of the numerical results amended. From B. Muhlschlegel, Z. Physik 155, 313 (1959), the weak-coupling limit of the BCS theory gives $\Delta(0)/kT_c = 1.76$ and $(T_c/H_0)(-dH_c/dT)_{T_c} = 1.74$, in good agreement with the values in Table I.

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values predicted by the weak-coupling theory, their ratio does not. The relationship is also a useful one, for critical-field measurements can be carried out much more easily and accurately than tunneling measurements, and Relation (1) provides a means for calculating energy gaps from critical-field measurements.

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EVIDENCE FOR PARITY-SU(3) MIXING IN THE MESON NONETS*

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(Received 23 July 1965)

The search for higher symmetry groups which mix space-time and the internal-symmetry group SU(3) has proceeded along two lines: (1) the enlargement of SU(3) into a higher symmetry group which does not commute with the parity—to this category belong groups like $\overline{W}(3)$,[1,2] $S\overline{W}(3)$,[3,4] and R(6)[5]; (2) the enlargement of SU(3) into a higher symmetry group which does not commute with the proper orthochronous Lorentz group—to this category belong SW(6),[6] W(6),[7] and $\tilde{U}(12)$.[8] During the past year, groups of category (2) have received a great deal of attention, and some promising results have been obtained. However, these attempts have encountered formidable obstacles, such as breakdown of unitarity,[9] the need to work with an infinite-dimensional Lie algebra,[10] etc. None of these conceptual difficulties arises for groups of category (1). Moreover, it is quite possible that a parity-SU(3) mixing type of group is a physically interesting subgroup of a category (2) group and may provide insight into the proper choice of this more all-encompassing group.

In an early paper[1] on parity-SU(3) mixing groups, it was shown how a chiral decomposition of massless Dirac fields via a four-fermion interaction leads to such mixed groups, the nature of the mixed group depending on the choice of the interaction. The choice of a vector plus an axial-vector interaction among three Dirac (quark) fields yields the most plausible[11] of these parity-SU(3) mixing groups, namely $\overline{\mathrm{W}}(3) = \mathrm{U}(3)^{(+)} \otimes \mathrm{U}(3)^{(-)}$, where the two U(3) groups refer, respectively, to the positive and negative chiral projections of the triplet of quark fields. Subsequently, Gell-Mann[3] came upon the $\mathrm{S}\overline{\mathrm{W}}(3) = \mathrm{SU}(3)^{(+)} \otimes \mathrm{SU}(3)^{(-)}$ group by looking for the group generated, under equal time commutation, by the space integrals of the time components of the vector and axial-vector current octets. In later papers,[4,2] the consequences of the $\mathrm{S}\overline{\mathrm{W}}(3)$ and $\overline{\mathrm{W}}(3)$ groups[12] for mesons and baryons were spelled out in some detail. In reference 2, 16 possibilities were considered for mesons, with two possibilities for each of the following: underlying group [$\overline{\mathrm{W}}(3)$ or $\mathrm{S}\overline{\mathrm{W}}(3)$], tensor structure (TS) of the medium strong (MS) symmetry-breaking interaction,[13] $[(3,3^*)+(3^*,3)]$ or $[(8,1)+(1,8)]$, the particle representations (PR) to which the mesons belong, $[(3,3^*),(3^*,3)]$ or $[(8,1),(1,8)]$, and mixing or no mixing between the unitary octets and singlets. In all 16 cases treated, the very strong (VS) symmetry-breaking interaction —which reduced the symmetry from $\overline{\mathrm{W}}(3)$ to U(3) or from $\mathrm{S}\overline{\mathrm{W}}(3)$ to SU(3)—was assumed to have the tensor structure[14] $[(3,3^*)+(3^*,3)]$. Most of the mass relations (see below) were derived to lowest order in the VS and MS symmetry-breaking interactions, despite the expectation that the VS would be substantially larger than the MS interaction. This approximation in the theory,[15] plus the paucity of experimental information at the time, left the idea of parity-SU(3) mixing in an inconclusive state.

Recent experimental evidence for a $J=0^+$ meson at about 700 MeV,[16] together with increasing evidence for the existence of positive-parity mesons of higher spin[17] ($J=1^+, 2^+$), have led us to sharpen up the predictions of the $\overline{\mathrm{W}}(3)$ and $\mathrm{S}\overline{\mathrm{W}}(3)$ theories for mesons. The essential improvement over our previous calculation consists in taking account of the VS symmetry-breaking term to all orders. In reference 2, the first-order VS splitting had the effect of removing the mass degeneracy—in the $\overline{\mathrm{W}}(3)$ or $\mathrm{S}\overline{\mathrm{W}}(3)$ limit—between, say, the scalar nonet (denoted by S_9) and the pseudoscalar nonet (denoted by P_9) and producing m^2 spacings of the S_1, P_8, S_8, and P_1 multiplets[18] in the ratio 1:2:1. Since the VS interaction does not destroy the purity of the irreducible SU(3) representations, one may take account of its higher order effects by simply allowing for arbitrary spacings between S_1, P_8, S_8, and P_1. If we then switch on the (weaker) MS interaction to first order, we should obtain predictions with comparable accuracy to those which follow from the usual SU(3) theory.

Following this method, we have recalculated all 16 cases of reference 2. As illustrations, we give the new results for case 4 and case 10 (or 12)[19] of reference 2; if we express the m^2 (denoted in each instance by the particle symbol) of the four pseudoscalar mesons and of the four scalar mesons (denoted by additional primes) in terms of the five constants c, the common mass of P_8; d, the mass of P_1; e, the common mass of S_8; f, the mass of S_1; and b, the mass splitting induced by the MS symmetry-breaking term, we find for case 4

$$\begin{gathered} K=c,\ K'=e; \\ \pi=c+b,\ \pi'=e-b; \\ \eta_8=c-\tfrac{1}{3}b,\ \eta_8'=e+\tfrac{1}{3}b; \\ X_1=d-\tfrac{2}{3}b,\ X_1'=f+\tfrac{2}{3}b; \\ (\eta_8X_1)=(X_1\eta_8)=-\tfrac{1}{3}\sqrt{2}b,\ (\eta_8'X_1')=(X_1'\eta_8')=\tfrac{1}{3}\sqrt{2}b. \quad (1)\end{gathered}$$

In Eqs. (1), η_8 and X_1 are the bare particles, and η and X are the physical particles (similarly for η_8', X_1' and η', X'); (η_8X_1) and $(\eta_8'X_1')$ are the mixing coefficients. From Eqs. (1), we deduce one mass relation[20] (2) for the pseudoscalar nonet, an identical relation (3) for the scalar nonet, and a third relation (4) connecting the pseudoscalar and scalar octets:

$$\eta=\frac{2K(2X+2\pi-3K)-\pi(\pi+X)}{(3X+\pi-4K)}, \quad (2)$$

$$\eta'=\frac{2K'(2X'+2\pi'-3K')-\pi'(\pi'+X')}{(3X'+\pi'-4K')}, \quad (3)$$

$$K'-\pi'=\pi-K. \quad (4)$$

Relation (4) was known before,[21] but (2) and (3) are new. Case 10 (or 12) of reference 2 leads to the relations[22]

$$\eta = \frac{4K(X+2\pi-2K)-\pi(X+3\pi)}{(3X+\pi-4K)}, \tag{2'}$$

$$\eta' = \frac{4K'(X'+2\pi'-2K')-\pi'(X'+3\pi')}{(3X'+\pi'-4K')}, \tag{3'}$$

$$K'-\pi' = K-\pi. \tag{4'}$$

Again, Relation (4′) is old,[21] but (2′) and (3′) are new.[23]

We now examine the empirical evidence, starting with the $J=0$ mesons. Since the existence of a ninth pseudoscalar meson (X at 960 MeV) seems to be established,[17] and the usual pseudoscalar octet does not satisfy the Gell-Mann–Okubo formula too well,[24] we presume at first (see below) that we are dealing with a pseudoscalar nonet. If we insert the values for the masses of the neutral members (K^0, π^0) in (2), we predict $m(\eta)=552$ MeV (compared to the experimental value of 548.8 MeV); the mixing angle between η_8 and X_1 is found to be $-10.4°$. On the other hand, Eq. (2′) yields $m(\eta)=495$ MeV, a very poor result. The other cases of reference 2 lead to results as poor as case 10 (or 12), or to no predictions at all (since too many arbitrary parameters occur). If we tentatively accept case 4 for the pseudoscalar nonet {underlying group SW(3), PR[(3, 3*), (3*, 3)], TS[(3, 3*) + (3*, 3)]}, we can use Eqs. (3) and (4) to predict two scalar masses if we know two others.[25] Taking the recent evidence for an $I=0$ (η') scalar particle at 700 MeV and the more elusive evidence[26] for a K' particle at 725 MeV, we predict $m(\pi')=870$ MeV and $m(X')=400$ MeV. The mass of X' is in the range desired by theorists,[27] but otherwise has no direct experimental foundation; experimental evidence is lacking also for π! Thus, while the pseudoscalar nonet is well explained by case 4, the situation for the scalar nonet is still obscure and we can not as yet exclude other PR and TS assignments to the $J=0$ mesons (see below).

The situation is much more promising for the $J=1$ and $J=2$ mesons, apart from the fact that the very existence of both $J=1^-$ and 2^+ particles is favorable to the idea of parity-SU(3) mixing. The same considerations which were applied to the $J=0$ mesons now lead to a choice between cases 4 and 10 (or 12), i.e., Eqs. (2)-(4) or Eqs. (2′)-(4′), for the $J=1$ and $J=2$ mesons. Since at least three masses are known for the $J=1^-$ and[28] $J=2^+$ nonets, we may use Eqs. (2) and (2′) to predict the fourth (η-type[29]) mass. Moreover, since the G parity[30] of the $B(1215)$ meson is even[15] (the same as the ρ meson), it is assigned to the $J=1^+$ nonet; on the other hand, since the G parity[30] of the $A_1(1090)$ meson[17] is odd (the same as the π meson), it is assigned to the $J=2^-$ nonet. Since we then know the masses of the π'-type mesons in the $J=1^+$ and $J=2^-$ nonets, we may use Eqs. (4) and (4′) to predict the masses of the K'-type mesons in the same nonets. There is insufficient experimental information to make predictions concerning the η'-type mesons in the $J=1^+$ and $J=2^-$ nonets on the basis of Eqs. (3) and (3′). Our predictions for the η-type and K'-type mesons are given in Table I with the experimental values shown in parentheses (and the "name" of the meson attached to the experimental value).

The internal consistency of our predictions in Table I is quite striking. The same theory (case 10 or 12) which predicts the best values for the η-type mesons (with $J=1^-, 2^+$) also predicts the best values for the masses of the opposite parity K'-type mesons with the same spins ($J=1^+, 2^-$, respectively). Thus, case 10 or 12 is distinctly favored over case 4, and hence we must assign the PR [(3, 3*), (3*, 3)] and the TS [(8, 1) + (1, 8)] to the $J=1$[31] and $J=2$ nonets. We have the option as to whether the underlying group is $\overline{\mathrm{W}}(3)$ (case 10) or $\mathrm{S}\overline{\mathrm{W}}(3)$ (case 12).

The last option is lost if we combine our results for the $J=1$ and $J=2$ mesons with our earlier tentative results for the $J=0$ mesons.

Table I. Mass predictions for the $J=1$ and $J=2$ mesons.

Particle	Nonet	Mass predictions (MeV)	Eq. used
η type	$J=1^-$	951	(2)
		1011	(2′)
		φ(1020)	
	$J=2^+$	1480	(2)
		1510	(2′)
		f'(1525)	
K' type	$J=1^+$	1150	(4)
		1310	(4′)
		C'(1330)	
	$J=2^-$	940	(4)
		1220	(4′)
		C(1215)	

We must then conclude that the underlying group is $S\overline{W}(3)$ and that, moreover, the PR is $[(3,3^*),(3^*,3)]$ and the TS changes from predominantly $[(3,3^*)+(3^*,3)]$ for the $J=0$ to predominantly $[(8,1)+(1,8)]$ for the $J=1$ and $J=2$ meson nonets. Apart from the novel requirement of a "spin dependence" of the MS tensor structure, the same PR assignment $[(3,3^*),(3^*,3)]$ to the $J=0^-$ and $J=1^-$ mesons yields too large a decay rate for $\varphi\to\rho+\pi$.[32]

If we insist on the same TS$[(8,1)+(1,8)]$ for all mesons, we would be forced into the PR$[(8,1),(1,8)]$ for the $J=0$ nonet [since case 10 (or 12) is ruled out by the $J=0^-$ nonet—see above], and at the same time explain naturally the low decay rate of $\varphi\to\rho+\pi$. No mass predictions would follow for the ninth $J=0^-$ (or $J=0^+$) meson, independently of whether the underlying group is $\overline{W}(3)$ or $S\overline{W}(3)$. Under these circumstances, we may choose the parity-SU(3) mixing group as $\overline{W}(3)$ for all mesons, and we may then construct a completely consistent quark model (with four-fermion interactions[33]) with explicit expressions for the VS and MS symmetry-breaking terms[34] as well as for the particle representations of all the mesons. This will be done elsewhere; here we only remark that the PR's for the $J=0$ and $J=1$ mesons would now differ in the triality quantum number[35] at the $\overline{W}(3)$ level, and that this difference would complicate the search[36] for a higher Lorentz-SU(3) mixing group of which $\overline{W}(3)$ is a subgroup. We could then only combine the $J=0$ and $J=1$ mesons into the same supermultiplet of the higher group if this supermultiplet contained irreducible representations of $\overline{W}(3)$ with different trialities.[37]

While further experimental information is urgently needed, we believe that our results for the $J=1$ and $J=2$ meson nonets provide the first serious evidence for a higher symmetry group which mixes parity and SU(3). If the relevance of parity-SU(3) mixing is confirmed as well for the $J=0$ mesons and definite PR and TS assignments can be made, it will be possible to decide between the two interesting alternatives discussed above and to carry on, perhaps more profitably, the search for higher symmetry groups.

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*Work supported in part by the U. S. Atomic Energy Commission.

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[9] R. Blankenbecler, M. Goldgerger, K. Johnson, and S. Treiman, Phys. Rev. Letters 14, 518 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters 14, 509 (1965); J. M. Cornwall, P. G. O. Freund, and K. T. Mahanthappa, Phys. Rev. Letters 14, 515 (1965).

[10] Cf. S. Okubo and R. E. Marshak, reference 7; also, R. F. Dashen and M. Gell-Mann, to be published; L. O'Raifeartaigh, Phys. Rev. Letters 14, 575 (1965).

[11] As was pointed out in references 2 and 5, only $\overline{W}(3)$ [of the three parity-SU(3) mixing groups considered: R(6), $\overline{W}(3)$, and U(6)] does not have the undesirable property of mixing representations of SU(3) with zero and nonzero triality in its own irreducible representations.

[12] Strictly speaking, the quark model (with a four-fermion interaction) leads to the $\overline{W}(3)$ (rather than the $S\overline{W}(3)$ group because of the extra gauge degree of freedom.[1] The approach through the "algebra of currents"[3] can yield $S\overline{W}(3)$ because the generators can always be made traceless.

[13] (R_1,R_2) denotes an irreducible representation of $\overline{W}(3)$ or $[S\overline{W}(3)]$, with R_1, R_2 referring to irreducible representations of $U(3)^{(+)}$[or $SU(3)^{(+)}$], $U(3)^{(-)}$[or $SU(3)^{(-)}$], respectively; R^* denotes the conjugate representation of R.

[14] The tensor structure $[(3,3^*)+(3^*,3)]$ of the VS symmetry-breaking interaction was based on the quark model. Actually, any representation of the form $[(R_1,R_2)+(R_2,R_1)]$ can be used provided the reduction of (R_1,R_2) to the SU(3) level contains a singlet [so that there is no mixing within any irreducible representation of SU(3)]. Note that this rules out the simple assignment of $[(8,1)+(1,8)]$ for the VS symmetry-breaking term.

[15] M. Gell-Mann (cf. reference 3) based all of his arguments with regard to the PR's for the mesons, Goldberger-Treiman relations, etc., on the same (poor) approximation for VS.

[16] M. Feldman et al., Phys. Rev. Letters 14, 869 (1965); V. Hagopian et al., Phys. Rev. Letters 14, 1077 (1965); L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters 14, 329 (1965).

[17] For references to experimental results, see A. H. Rosenfeld et al., Rev. Mod. Phys. 36, 977 (1964); and University of California Radiation Laboratory Report No. UCRL-8030, 1965 (unpublished).

[18] S_1 (P_1) and S_8 (P_8) are the scalar (pseudoscalar) SU(3) singlet and octet, respectively; since mesons of differ-

ent spin are not related in the present theory, $S_{1,8}$ ($P_{1,8}$) could equally well refer to positive- (negative-) parity SU(3) multiplets of arbitrary spin. Unless stated otherwise, we shall use the $J=0$ notation.

[19]The only difference between case 10 and 12 is that the underlying group is $\overline{\mathrm{W}}(3)$ for case 10 and $\mathrm{S}\overline{\mathrm{W}}(3)$ for case 12; these two cases lead to precisely the same mass relations when the VS interaction is taken into account to all orders (this is not true when the VS symmetry breaking is calculated to first order—cf. reference 2).

[20]One can write Eq. (2) in the more symmetrical form $(X-\eta_8)(\eta-\eta_8)=-(2/9(K-\pi)^2$, with $\eta_8=\frac{1}{3}(4K-\pi)$. From this, it is evident why it is preferable to solve for η rather than X when η is close to η_8.

[21]Cf. Eq. (30) of reference 2; we get the same result because it was proved in reference 2 that this relation was correct to any order in the VS interaction. Equation (4′) is Eq. (33) of reference 2.

[22]Relation (2′) can be written in the more symmetrical form $(X-\eta_8)(\eta-\eta_8)=-(8/9)(K-\pi)^2$, and we then see (cf. reference 18) that it differs from relation (2) in having the factor 8/9 instead of 2/9 on the right-hand side.

[23]J. Schwinger [Phys. Rev. 135, B816 (1964)] has derived relation (2′) for $J=1^-$ mesons. His derivation is based on a group which does not mix parity and is limited to vector mesons. In our theory, Eq. (2′) [as well as Eq. (2)] is independent of J.

[24]M. Umezawa [Phys. Rev. 138, B1536 (1965)] satisfactorily explains the X-η mixing; however, the quark model does not allow us to use two different hypercharge splittings for the two chiral projections of the quark fields. See also R. H. Dalitz and D. G. Sutherland, to be published.

[25]Actually, Eq. (4) enables us to predict the mass of π' or K' if we know the mass of the other; Eq. (3) then relates the masses of η' and X'.

[26]For references to experiments where the K' was seen, cf. reference 16; not all attempts to find K' seem to succeed, however.

[27]R. Bryan, C. Dismukes, and W. Ramsay, Nucl. Phys. 45, 353 (1963); S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964); L. Brown, in Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January 1965, edited by B. Kurşunoğlu, A. Perlmutter, and I. Sakmar (W. H. Freeman & Company, San Francisco, California, 1965); P. G. Thurnauer, Phys. Rev. Letters 14, 985 (1965).

[28]In selecting the mass values and G parities of the $J=1$ and $J=2$ mesons, we have used a combination of reference 16 and the recent preprint by S. L. Glashow and R. H. Socolow (1965); in particular, we have taken $K^*(1430)$, $A_2(1320)$, $f(1250)$, $f'(1525)$ for the $J=2^+$ nonet.

[29]In order to avoid confusion, we refer to the $J=0^-, 1^-, 2^+$ mesons with the appropriate quantum numbers as π, K, η, X type, and to the $J=0^+, 1^+, 2^-$ mesons as π', K', η', X' type.

[30]We assume that all mesons with the same J and opposite parity posses the same G parities, i.e., are normal (cf. reference 2). This hypothesis is consistent with our final PR assignment, i.e., $[(3,3^*),(3^*,3)]$ to the $J=1,2$ mesons but may not be true for the $J=0$ mesons (i.e., the scalar mesons may be abnormal—see below).

[31]This result disagrees with the PR assignment $[(8,1)+(1,8)]$ to the $J=1$ mesons by Gell-Mann (cf. reference 4) on the basis of his "current algebra" approach; presumably Gell-Mann was misled by first-order arguments with regard to the VS symmetry-breaking term (cf. reference 15).

[32]Unless we modify the estimate for the $\omega\to\rho+\pi$ rate in the calculation of the $\varphi\to\rho+\pi$ rate; this problem, as well as estimates of all other decay rates on the basis of the present theory, will be discussed in a separate paper.

[33]We would be compelled to work with at least six-fermion interactions in order to accomodate the $\mathrm{S}\overline{\mathrm{W}}(3)$ group within the quark model.

[34]E.g., an MS tensor structure $[(8,1)+(1,8)]$ may be represented by

$$m'(\overline{\psi}_3\gamma_\mu\psi_3)\sum_{i=1}^{3}(\overline{\psi}_i\gamma_\mu\psi_i)$$

(cf. reference 2 for notation).

[35]The definition of triality given for U(3) [cf. S. Okubo, C. Ryan, and R. E. Marshak, Nuovo Cimento 34, 759 (1964)] is obviously generalized for $\overline{\mathrm{W}}(3)$.

[36]Gell-Mann's PR assignment to the $J=0$ and $J=1$ mesons (cf. reference 4) are now reversed compared to ours, but the trialities are still different and the same statement applies.

[37]This is not true of most of the higher groups considered until now [e.g. SW(6) and $\widetilde{\mathrm{U}}(12)$].