optically flat, a more reasonable dependence of the absorption intensity on θ was observed, as compared to the almost complete isotropy previously observed. If the transducer electrode was not flat, this would be expected to contribute to acoustic-mode conversion. Further evidence is the fact that the observed angular dependence of the absorption intensity for both longitudinal- and transverse-wave excitation can be understood if one assumes a mixture of modes present. One way to diminish the mode mixing is to use larger diameter samples and operate at higher acoustic frequencies. In experiments done on I^{127} in KI by us and others,[12] using more favorable sample geometry, the deviation from the expected angular dependence can be explained in terms of an anisotropic $g(\nu)$. Experiments using larger diameter tantalum samples, as well as experiments on other metals, temperature dependence of the resonance, effects of sample purity, etc., are in progress. A more detailed experimental and theoretical description of this work will be published later.

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EFFECT OF OPEN ORBITS ON HELICON AND ALFVÉN-WAVE PROPAGATION IN SOLID-STATE PLASMAS

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Since the original suggestion of Konstantinov and Perel[1] and Aigrain[2] that certain low-frequency electromagnetic (helicon) waves can be propagated with little attenuation in noncompensated solid-state plasmas, and of Buchsbaum and Galt[3] that magnetohydrodynamic waves can propagate in compensated solids, there has been great activity in this area by both experimentalists and theorists.[4] The work to date has dealt with materials under conditions such that only closed cyclotron orbits were present. There is then a close analogy between the characteristics of helicon and Alfvén waves in solids, and the corresponding waves in gaseous plasmas. In this Letter we discuss a phenomenon which is peculiar to solids, namely, the effect of open orbits on helicon and Alfvén-wave propagation. We will show that the presence of a single band of open orbits causes strong damping of the helicon wave in an uncompensated material and, in extreme circumstances, can convert the helicon wave into a damped Alfvén wave. This effect has been seen by Grimes.[5] The additional damping is easily observable since it is a strong function of magnetic-field orientation. It falls rapidly to zero as the field is turned away from directions which give rise to open orbits. In uncompensated metals, the study of this effect may provide information about Fermi-surface topology similar to that obtainable from magnetoresistance measurements.

In compensated materials, we will show that one of the Alfvén waves is strongly affected by carriers in open orbits, while the other remains unperturbed.

The properties of helicon and Alfvén waves in solids are closely related to those of the galvanomagnetic coefficients (the Hall coefficient and magnetoresistance) at high magnetic field. This connection is a useful one since the effect of open orbits on the Hall and magnetoresistance coefficients is well understood as a result of the work of Lifshitz, Azbel, and Kaganov.[6] They show that in an uncompensated metal with no open orbits, the transverse magnetoresistance saturates with increasing magnetic field and the Hall coefficient, R, depends only on the net charge density, irrespective of the details of the band structure. Currents in such a material flow nearly perpendicular to the electric field when the applied magnetic field is large. As a result, the medium will support a relatively lossless, low-frequency electromagnetic wave—which is the helicon wave. Its dispersion relation (ignoring losses) is

$$k^2=\left(\frac{\omega^2}{c^2}\right)\frac{{\omega_P}^2}{\omega\omega_c\cos\theta}=\frac{4\pi\omega}{c^2}\left(\frac{R}{B}\right)\frac{1}{\cos\theta}, \qquad (1)$$

where k is the wave vector of the helicon, ω is its frequency, ω_c is the cyclotron frequency, $\omega_P=(4\pi Ne^2/m)^{1/2}$ the plasma frequency, and θ is the angle between k and the static magnetic field, B. The imaginary part of k is determined by the small component of the current flowing parallel to the electric field and is of order $(\omega_c\tau)^{-1}$—where τ is the collision time—compared to the real part in the high-field limit.

Now let us imagine that a single band of open orbits exists at right angles to B, in real space, say along the x axis, with B along the z axis of a Cartesian coordinate system. Then, in an uncompensated solid, the Hall coefficient is no longer a good measure of the charge, and the magnetoresistance contains a term whose coefficient is proportional to the number of carriers in open orbits and which is a quadratic function of B (provided the conduction current has a component along the y axis).[6] This term causes current flow in the direction of the electric field of the helicon wave and thereby increases its damping. In the extreme high-field limit, the carriers in open orbits dominate the dispersion relation and change the character of the electromagnetic wave from that of a helicon to a damped Alfvén wave.

To make these ideas more concrete, we assume that the open orbits are due to a cylindrical piece of Fermi surface oriented along the y axis in momentum space, containing n_e electrons/cc. We also assume a spherical piece which contains N_e electrons/cc. This model is a rather schematic one in that it assumes a very simple-shape Fermi surface and an isotropic relaxation time. We believe, however, that it contains the essential physics of the problem and that our results are qualitatively correct (particularly as regards magnetic-field dependence) for real metals. To discuss the details of wave propagation in this medium, it is necessary to solve the dispersion relation obtained by setting equal to zero the determinant of the coefficients of the wave equation

$$\vec{k}\times(\vec{k}\times\vec{E})+(\omega^2/c^2)\epsilon\cdot\vec{E}=0, \qquad (2)$$

in which $\vec{E}$ is the electric field of the wave and ϵ the dielectric tensor. For simplicity, we assume that ω is sufficiently small, that $\omega\tau \ll 1$ and $kv_F\tau\ll 1$, where v_F is a typical Fermi velocity. The latter requirement insures the locality of the relation between current and field and, in metals, can be satisfied over a considerable range of experimentally interesting conditions. With these assumptions ϵ takes the form

$$\epsilon=\begin{pmatrix}\epsilon_\perp+\epsilon_0 & \epsilon_x & 0\\ -\epsilon_x & \epsilon_\perp & 0\\ 0 & 0 & \epsilon_{\parallel}+\epsilon_0\end{pmatrix}, \qquad (3)$$

where

$$\epsilon_\perp=\epsilon_{\text{lattice}}+\frac{i{\omega_P}^2\tau}{\omega[1+(\omega_c\tau)^2]},$$

$$\epsilon_x=\frac{i{\omega_P}^2\omega_c\tau^2}{\omega[1+(\omega_c\tau)^2]},$$

$$\epsilon_{\parallel}=\epsilon_{\text{lattice}}+i{\omega_P}^2\tau/\omega,$$

$$\epsilon_0=i{\omega_{P0}}^2\tau/\omega. \qquad (4)$$

$\omega_P = (4\pi N_e e^2/m^*)^{1/2}$ is the plasma frequency of electrons in the spherical portion of the Fermi surface, and $\omega_{P0} = (4\pi n_e e^2/m_0^*)^{1/2}$ that of the electrons in the cylinder. ϵ_0 is the contribution of these open-orbit electrons to the dielectric tensor. Throughout Eqs. (4) we have neglected ω compared to $1/\tau$, since the locality condition is usually violated (in the ϵ_0 term) before the equality $\omega = 1/\tau$ is reached.

In an uncompensated material in high fields ($\omega_c\tau \gg 1$), we have $|\epsilon_x| \gg |\epsilon_\perp|$. If there are no open orbits present ($\epsilon_0 = 0$), the off-diagonal element, ϵ_x, is dominant, and the dispersion relation for the propagating helicon wave is that given by Eq. (1). The presence of open orbits changes matters considerably. We first note that $|\epsilon_0| > |\epsilon_\perp|$ if $f \equiv (n_e/N_e) > [m_0^*/m^*(\omega_c\tau)^2]$. Thus, in pure materials in high fields, a relatively small number of open orbits is sufficient to make itself felt. To see what the effect of ϵ_0 is, we first consider the case of propagation along the static magnetic field ($k = k_z$). Inserting Eqs. (4) into the dispersion relation, we find

$$\frac{c^2k^2}{\omega^2} = (\epsilon_\perp + \tfrac{1}{2}\epsilon_0) \pm (\tfrac{1}{4}\epsilon_0^{\,2} - \epsilon_x^{\,2})^{1/2}. \tag{5}$$

Two interesting cases now arise. The first, and most easily attained experimentally, is that in which $\epsilon_\perp < \epsilon_0 < \epsilon_x$. Under these circumstances, Eq. (5) takes the approximate form

$$c^2k^2/\omega^2 \simeq \pm(-\epsilon_x^{\,2})^{1/2} + (\epsilon_\perp + \tfrac{1}{2}\epsilon_0). \tag{6}$$

The solution with positive sign represents a propagating wave which is damped by the imaginary term $(\epsilon_\perp + \epsilon_0/2)$. Since $\epsilon_0 > \epsilon_\perp$, it is clear that a relatively small number of electrons in open orbits can markedly change the absorption of the helicon wave.[7] Moreover, it is easy to show that the quantity ϵ_0 falls quite rapidly to a small value (of order $f\epsilon_\perp$) as the magnetic field is turned away from the open-orbit direction. It follows from these results that the helicon-wave absorption should be a rapidly varying function of magnetic-field direction in an uncompensated solid containing open orbits. It has sharp maxima for field directions which give rise to the open orbits.

The more interesting limit of Eq. (5) is that in which $\epsilon_0 \gg \epsilon_x$, or $f \gg [m_0^*/m^*\omega_c\tau]$. It can be reached[5] experimentally with large fields in pure materials. In this limit the dispersion relation has the approximate solutions.

$$c^2k^2/\omega^2 = \epsilon_0 \text{ and } \epsilon_x^{\,2}/\epsilon_0. \tag{7}$$

Both solutions represent heavily damped waves ($\mathrm{Im}k = \mathrm{Re}k$). The second of these waves is the easier one to observe and is, therefore, of particular interest. Its index varies linearly with B, and it is linearly polarized. These properties are more nearly those of an Alfvén wave than a helicon. Thus, under extreme conditions, the open-orbit electrons completely change the character of the electromagnetic modes in the plasma.

One might hope to make ϵ_0 real, and thereby convert this wave into a propagating (rather than damped) Alfvén wave by working at frequencies $\omega \gg (1/\tau)$. However, under practical experimental conditions, this is not possible because the locality condition is violated in the ϵ_0 part of the dielectric tensor when $\omega \gg 1/\tau$. A more complete calculation shows that in the nonlocal case ϵ_0 remains imaginary, even when $\omega \gg 1/\tau$, so the damping of the wave persists.

The calculations outlined above apply to the case $\vec{k} \parallel \vec{B}$. Results for helicon propagation in other directions are similar. In particular, the conclusion that helicon damping is enhanced when open orbits are present remains true for arbitrary propagation directions.

Now we consider the effects of open orbits in compensated materials. The electromagnetic modes of a compensated plasma are the fast and slow Alfvén waves which, in contrast to the helicon wave, propagate with low loss only if $\omega \gg (1/\tau)$.[3] Unfortunately, for frequencies in this range and for metallic electron densities, the relationship between the current and the electric field is usually nonlocal. On the other hand, for $\omega \ll 1/\tau$, the relation is local and the modes are damped Alfvén waves ($\mathrm{Im}k = \mathrm{Re}k$). Such waves have been observed experimentally[5,8] and may turn out to be of considerable interest in compensated metals. To see how these waves are affected by carriers in open orbits, we again consider the simple case of propagation along the magnetic field. Equation (5) still determines the wave vector, but now, because of the presence of holes, the coefficients $\epsilon_\perp$ and ϵ_x must be suitably redefined. As a result of the near compensation, the coefficient ϵ_x is small ($\epsilon_x/\epsilon_0 \sim 1/\omega_c\tau$), and the solutions of Eq. (5) are approximately

$$c^2k^2/\omega^2 \simeq (\epsilon_0+\epsilon_\perp) \text{ and } \epsilon_\perp, \quad (8)$$

where $\epsilon_\perp = ic^2/\omega\tau V_A{}^2$ and V_A is the Alfvén speed.[3,4] The first wave, which is linearly polarized in the x direction, is strongly affected by the carriers in open orbits. They dominate its behavior when $f(\omega_c\tau)^2 > 1$. In this limit, the refractive index of the wave is independent of magnetic field and varies as $\omega^{-1/2}$ with frequency. This behavior is the same as that of the zero-field classical skin effect. The second wave, which is polarized in the y direction, is unaffected by the open orbits and retains its damped Alfvén-wave character. These results have been derived for the special case of propagation parallel to the field, but more detailed calculations show that the behavior is similar for arbitrary propagation directions.

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OBSERVATION OF THE EFFECT OF OPEN ORBITS ON HELICON-WAVE PROPAGATION

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We report the first observations of the effects of open orbits on helicon-wave propagation in metals. Buchsbaum and Wolff[1] have shown theoretically that open orbits in an uncompensated medium lead to damping of a helicon wave and, in a limiting case, completely change the character of the medium and allow it to support a damped Alfvén wave. We have observed such effects in helicon propagation in a single crystal of silver. A second consequence of open orbits, which we have studied in crystals of Sn and Pb, arises from the existence of singular magnetic-field directions along which helicon waves can propagate with little damping in an otherwise compensated medium. This observation greatly extends the range of materials in which helicon-wave propagation can be studied.

The experimental technique which we have employed to transmit helicon waves through the thin, disk-shaped specimen is similar to that of Grimes and Buchsbaum.[2] In this technique, the helicon wave is excited by driving an audiofrequency current through a transmitter coil placed near one face of the specimen. The transmitted energy is picked up by a second coil near the opposite face of the specimen and fed to a phase-sensitive detector which derives its phase reference from the transmitter. The detected signal is displayed, versus either the magnitude or the direction of the applied magnetic field, while the frequency is held constant.

In an uncompensated medium (a medium having unequal densities of electrons and holes) with only closed orbits, helicon waves propagate according to the simple dispersion relation[1,3]

$$k^2 = \frac{\omega_p{}^2\omega(1+i/\omega_c\tau)}{c^2\omega_c\cos\Theta}, \quad (1)$$

provided $\omega \ll \omega_c$, $\omega_c\tau \gg 1$, and $kv_F\tau \ll 1$. Here $\vec{k}$ is the wave vector; ω is the angular frequency of the helicon wave; $\omega_p{}^2 = 4\pi Ne^2/m$ is the plasma frequency; $\omega_c = eH/mc$ is the cyclotron frequency; τ is the electron relaxation time; Θ is the angle between $\vec{k}$ and the magnetic field, $\vec{H}$. With $\vec{H}$ directed approximately parallel to $\vec{k}$, and in the absence of open orbits, helicon