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[7]We have understood a λ line in the Pippard sense, namely, a line separating two phases, so that $C_p \to \infty$ at least from one side.

VELOCITY OF SOUND IN LIQUID HELIUM AT ITS LAMBDA POINT*

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As a consequence of the Pippard relations[1,2] for a λ transition,

$$C_p = \alpha V T \beta + C_0, \tag{1}$$

$$\beta = \alpha K_T + \beta_0, \tag{2}$$

where the symbols are as follows: p, pressure; T, temperature; V, specific volume; C_p, isobaric specific heat; β, isobaric thermal expansion coefficient; K_T, isothermal compressibility; α, C_0, and β_0 are quantities measured along the λ phase-transformation line $[\alpha = dp_\lambda/dT_\lambda, C_0 = T_\lambda dS_\lambda/dT_\lambda$, where S is the specific entropy, $\beta_0 = (1/V_\lambda)(dV_\lambda/dT_\lambda)]$. Chase[3] has shown that the velocity of sound, u, obeys the following relation:

$$\frac{1}{u_\lambda^{\,2}} - \frac{1}{u^2} = \frac{2(u-u_\lambda)}{u_\lambda^{\,3}} = \frac{C_0^{\,2}}{\alpha^2 V^2 T C_p}, \tag{3}$$

where u_λ, the sound velocity at λ point is given by

$$u_\lambda^{\,2} = \frac{\alpha^2 V^2 T}{C_0 - \alpha\beta_0 V T} \tag{4}$$

and $(u-u_\lambda) \ll u_\lambda$. These relations [Eqs. (1), (2), and (3)] are expected to hold on both sides of the λ point, but only in its immediate vicinity. The experimental situation as regards their verification (attention will be restricted to the ${}_2\mathrm{He}^4$ λ transition) is as follows.[4] The specific heat at saturated vapor pressure, C_S, has been measured[2] with considerable accuracy as close to T_λ as $(T-T_\lambda) = \pm 10^{-6}\,{}^\circ\mathrm{K}$ and is found to be given by $C_S = [4.55 - 3.00 \log_{10}|T - T_\lambda| - 5.20\Delta]$ J/g deg; $\Delta = 0, T < T_\lambda$; $\Delta = 1, T > T_\lambda$. The quantity β has been measured to within about $T-T_\lambda = \pm 10^{-4}\,{}^\circ\mathrm{K}$ at vapor pressure.[5,6] For $T-T_\lambda < 10^{-2}\,{}^\circ\mathrm{K}$, β and C_p obey the first Pippard relation, although there is not complete agreement as to the appropriate values of α and C_0 or, in fact, to whether different values of C_0, namely $C_{0\mathrm{I}}$ and $C_{0\mathrm{II}}$, are required for He I and He II.[5] The isothermal compressibility has not been directly measured close enough to the λ point to permit a check of the second Pippard relation. In this connection it should be mentioned that in the range of $T-T_\lambda$ mentioned above, the fractional changes in K_T will be far less than those in β, and consequently a successful experiment would require high resolution of K_T values. Sound-velocity measurements have been made in the vicinity of the λ point.[7] These measurements and phase-transformation theory[8] indicate that there is a characteristic relaxation time $\tau \sim (T_\lambda - T)^{-1}$. At the frequency used, 1 Mc/sec, $\omega\tau$ is not negligible near the λ point, and as Chase points out,[3] the resulting attenuation and, more particularly, dispersion, interferes with a test of Eq. (3). The purpose of this Letter is to report the results of measurements at 9.75 kc/sec where this is not the case.

The experimental method consisted of the determination of a plane-wave-mode resonance of a cylindrical chamber using as exciting and sending transducers two exactly similar electrostatic units whose active elements are gold-coated Mylar diaphragms.[9]

Figure 1(a) shows the velocity of first sound as a function of $T-T_\lambda$. In Fig. 1(b) the sound velocity is plotted against $1/C_p$ (determined by its known temperature dependence). In He I these two quantities are linearly related [as required by Eq. (3)] over a range of two decades in $T-T_\lambda$ by

$$u_{\rm I}=u_{{\rm I}\lambda}+A(C_p)^{-1}, \tag{5}$$

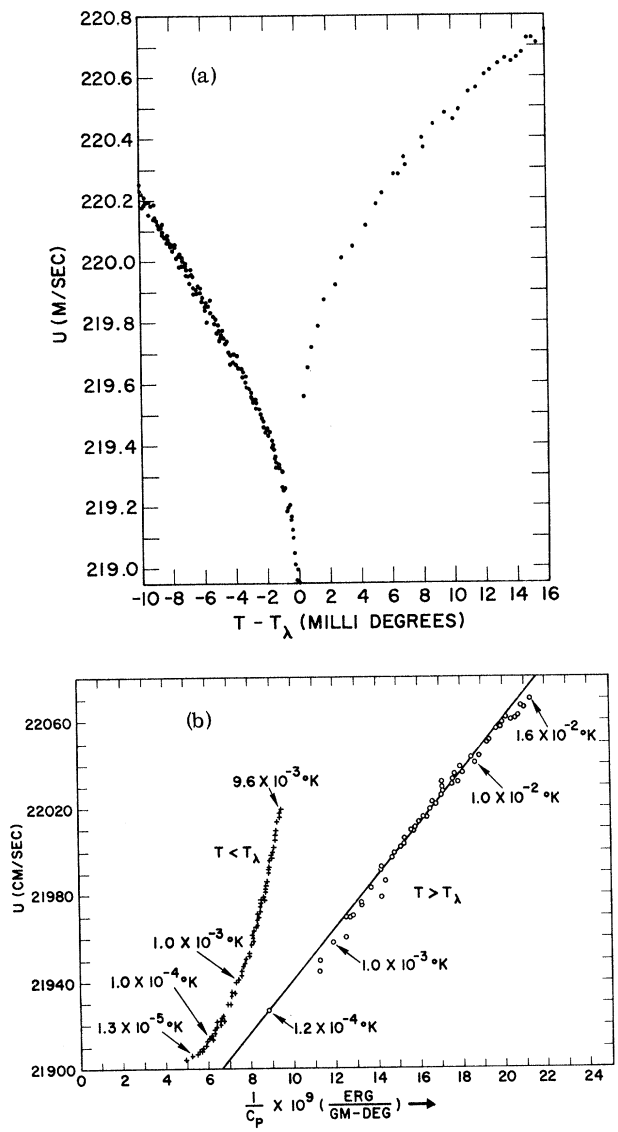

FIG. 1. (a) The measured velocity of sound in liquid helium near the lambda point. The points were obtained by measuring the frequency at 9.75 kc/sec of a plane-wave-mode resonance of a cylindrical chamber as a function of temperature. (b) The measured velocity of sound in He I and He II plotted against $(C_p)^{-1}$ as determined in reference 2. The data in He I fit Eq. (3), and the slope of the line drawn gives $C_0/\alpha=-4.88\times10^{-2}$ J g^{-1} atm^{-1}. The data for He II do not fit Eq. (3) and are replotted in Fig. 2.

with $u_{{\rm I}\lambda}=218.2$ msec^{-1} and $A=12\times10^9$ (c.g.s., °K). The quantity $C_0\alpha^{-1}=T_\lambda dS_\lambda/dP_\lambda$ can be determined from A and is found to be -4.88×10^{-2} J g^{-1} atm^{-1}. If one uses the value[6] $C_0=6.3$ J g^{-1} (°K)$^{-1}$, then α is found to be -129 atm deg^{-1}. This can be compared with the following values: -130 atm deg^{-1} (reference 2), -118.4 atm deg^{-1} (reference 5), -89 atm deg^{-1} (reference 6), -98 atm deg^{-1} (see Lounasmaa and Kaunisto[10]). Using Eq. (4) and the value $u_{{\rm I}\lambda}$, β_0 can be calculated and is found to 1.85(°K)$^{-1}$. This is in good agreement with the value of 1.90(°K)$^{-1}$ in reference 2, but not with the values 1.378(°K)$^{-1}$ in reference 6, or 1.223(°K)$^{-1}$ in reference 10. The dominant term in the denominator of Eq. (4) is the second one, and the principle source of differences in the β_0 values reported here and in references 2 and 6 are due to the differences in the α values used in the calculation.

The results in the He II are unexpected. The temperature dependence is best indicated by the plot of Fig. 2, where it is seen that over a range of three decades in $T_\lambda-T$,

$$u_{\rm II}=u_{{\rm II}\lambda}+B(T_\lambda-T)^{1/2}, \tag{6}$$

with $u_{{\rm II}\lambda}=219.01$ m/sec and $B=1.2\times10^3$ cm sec^{-1} (°K)$^{-1/2}$. Also, this temperature dependence can be shown to extend out still another octave (to $T_\lambda-T=2\times10^{-2}$). Furthermore, the same temperature dependence yields values for the velocity of sound which never differ from the experimental values by more than

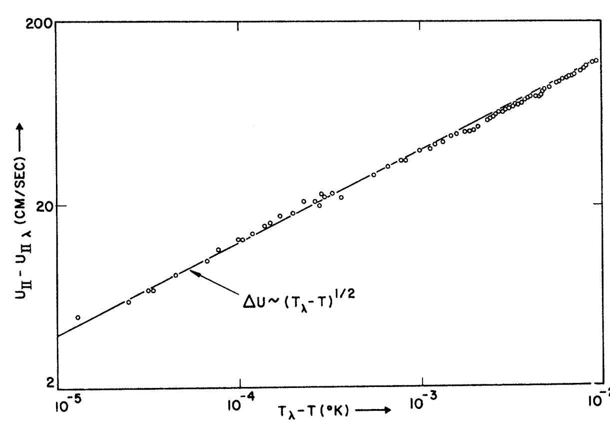

FIG. 2. The measured sound velocity in He II, $u_{\rm II}$, plotted against $(T_\lambda-T)$. $u_{{\rm II}\lambda}=219.01$ cm sec^{-1} is an experimental quantity. Equation (6), which is the experimental result, is seen to hold over three decades in $(T_\lambda-T)$.

3 % in the temperature range from the lambda point to 0°K. Equation (3) applies to the isentropic sound velocity given by $(dp/d\rho)_S^{1/2}$, whereas the measurements determine the velocity of first sound. These two velocities differ by an amount which depends on the constant, $(1-\gamma^{-1})$ (where $\gamma = C_p/C_V$), which couples temperature and pressure waves[11]; γ is known to have a singularity at the λ point. However, calculations of this difference indicate it is negligible, amounting to less than 1 cm $\sec^{-1}$ in the pertinent temperature range. Other sources of error which can be shown to be immaterial are yielding of the resonator walls, and viscous and heat-conduction effects at these walls.

It would appear at first glance that the observed temperature dependence of the sound velocity in He II implies a violation of the Pippard relations. Such a conclusion is, however, unwarranted, as the following considerations show. If it is assumed that the first Pippard relation is experimentally verified in He II for $T_\lambda - T < 10^{-2}$, one can determine K_T by substituting Eq. (1) into Eq. (3) and using $u^2 = \gamma V/K_T$. Thus,

$$K_T = \frac{(C_p - C_0)^2}{\alpha^2 VTC_p} - \frac{2VB(T_\lambda - T)^{1/2}}{u_{\mathrm{II}\lambda}^3} + \frac{V}{u_{\mathrm{II}\lambda}^2}. \tag{7}$$

When these values are compared with those calculated assuming Eq. (2) holds rigorously, they are found to differ by less than 0.2 % for $T_\lambda - T < 10^{-3}$ °K (this difference increases to 0.4 % for $T_\lambda - T = 10^{-2}$ °K). Such deviations are well within experimental accuracy. This calculation emphasizes that only small systematic deviations of K_T (or for that matter C_p or β) from the expected logarithmic temperature dependence suffice to produce the kind of temperature dependence shown in Eq. (6) rather than that of Eq. (5). This is perhaps less surprising if it is kept in mind that both γ and K_T approach infinity logarithmically at the lambda point while u changes very little – in fact the total range of velocity changes in this experiment is less than 1 %.

First- and second-order phase changes are characterized by the fact that the velocity of sound is discontinuous at the transition temperature. On the other hand, the sound velocity at lambda transformations is expected to be continuous. It is questionable whether Eqs. (5) and (6) hold for arbitrarily small values of $(T - T_\lambda)$, but it is clear that if they do then $u_{\mathrm{I}\lambda}$ and $u_{\mathrm{II}\lambda}$ represent the limiting sound-velocity values in the He I and He II, respectively, and they differ by 81 cm $\sec^{-1}$. If this possibility is accepted, it raises a serious question about the nature of the phase change – not at the vapor pressure where its lambda nature is well established, but at elevated pressures[12] where the experimental situation is less firm.

The origin of the square-root law, Eq. (6), is not understood at present. It seems not to be an inherent property of Landau second-order phase transitions,[13] nor does it follow from the thermodynamical functions derived for a dilute Bose system of hard spheres.[14]

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[12]If one accepts different values of C_0, namely $C_{0\mathrm{I}}$ and $C_{0\mathrm{II}}$, as found in reference 5, this implies latent heat at least in some regions along the λ line. It is, however, difficult to understand how $C_{0\mathrm{I}}$ can exceed $C_{0\mathrm{II}}$ as reported there.

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OBSERVATION OF STABLE SUPERFLUID CIRCULATION IN LIQUID-HELIUM II AT THE LEVEL OF ONE, TWO, AND THREE QUANTUM UNITS

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It has been proposed that the circulation of the superfluid component of liquid-helium II is quantized in integral multiples of h/m, where h is Planck's constant and m is the mass of the helium atom.[1,2] One of the most direct attempts to verify this proposal has been Vinen's ingenious experiment in which the circulation around a fine wire immersed in superfluid helium was measured by means of the influence that the circulation exerts on the transverse vibrations of the wire.[3] We wish here to report a repetition and extension of Vinen's experiment which we believe has given new evidence in support of the hypothesis of quantization of circulation. A related aspect of the hypothesis, the existence of quantized free vortices in the superfluid, has recently been given strong support by the experiments of Rayfield and Reif[4] and of Richards and Anderson.[5]

The principle of the measurement can be understood by considering the case of a cylindrically symmetric wire. In the absence of circulation such a wire can be regarded as having as its lowest modes of transverse vibration two degenerate circularly polarized modes. With circulation κ around the wire, the degeneracy of these modes is removed by the "lift" force,[6] resulting in a splitting, $\Delta\omega_\kappa = \rho_s\kappa/\mu$, between the angular frequencies of the two modes. Here ρ_s is the superfluid density, and μ is the mass per unit length of the wire plus that of the fluid displaced. If the two modes are excited simultaneously with equal amplitude, the result is vibration of the wire in a plane which precesses with angular frequency $\Delta\omega_\kappa/2$ in the same sense as the fluid is circulating.

The wire can be set into vibration by passing a current pulse through it in the presence of a steady transverse magnetic field. The free, slowly damped vibrations which follow can then be observed by means of the oscillatory emf induced along the wire. As the plane of vibration precesses, the induced emf sweeps out a decaying beat pattern with beat period $2\pi/\Delta\omega_\kappa = 2\pi\mu/\rho_s\kappa$. Thus, for a cylindrically symmetric wire, the circulation around the wire can be determined simply by measuring the beat period once μ and ρ_s are known. More accurately, the quantity κ measured in this way is a weighted average of the circulation around the wire taken along the wire's length, a quantity we shall call the apparent circulation.

In practice, however, the lowest vibrational normal modes of a wire in the absence of circulation are rarely degenerate, presumably because of some inherent asymmetry in the wire or its mounting. In our case, as in Vinen's, these modes always appear to be plane polarized, with mutually perpendicular planes of polarization. In such a circumstance it can be shown that the effect of the circulation will be to produce elliptically polarized modes whose total angular frequency difference, $\Delta\omega_t$, is given by $(\Delta\omega_t)^2 = (\Delta\omega_\kappa)^2 + (\Delta\omega_0)^2$, where $\Delta\omega_0$ is the angular frequency difference in the absence of circulation. Since the measured beat period is now $2\pi/\Delta\omega_t$, it is necessary in practice to know $\Delta\omega_0$ as well as μ and ρ_s in order to determine κ. It is helpful that at the beginning of a run $\Delta\omega_0$ can be adjusted to a convenient value by twisting the wire.

The basic elements of the apparatus, all of which are immersed in the liquid-helium bath,