ASYMMETRY IN THE CRITICAL SURFACE CURRENT OF TYPE-2 SUPERCONDUCTORS

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In a recent Letter[1] Fink has described how the order parameter ψ near the plane surface of a type-2 superconductor varies with position when a magnetic field H_0 is applied parallel to the surface. He obtains his result by integrating the Ginzburg-Landau equations[2] numerically, and has pointed out that surface solutions for which ψ tends to zero deep in the body of the material exist not only when H_0 lies between H_{c2} and H_{c3} (the maximum nucleation field),[3] but also below H_{c2}, the upper critical field of the mixed state. A calculation of ψ treating it as though it were zero in the body of the material (as Fink does) may be expected to produce a good approximation to the exact solution near the surface while taking into account the existence of the mixed state, as long as the maximum value of ψ in the mixed state ψ_B (estimated from Abrikosov's solution for an infinite medium[4]) is small compared with the value of $|\psi|$ at the surface. Since at the surface $|\psi| \sim \psi_0$, where ψ_0 is the value of ψ in zero field ($|\psi/\psi_0| \simeq 0.7$ near H_c), this condition is satisfied if $\psi_B \ll \psi_0$, as long, that is, as $(1-H_0/H_{c2}) \ll 1$. We have extended Fink's calculation in order to see in what way his surface solutions are altered by a current, and in particular to calculate the critical current and how it varies with magnetic field.

Whatever the total current in the surface layer may be, there is, as we point out below, a current at any point, depth x below the surface, which flows normal to the field and whose density is given by Eq. (4). We shall consider here only the situation in which the total surface current J is directed normal to the field. J is taken as positive when it is diamagnetic and flows in the same direction as the current density at $x=0$, negative when it is paramagnetic and flows in the opposite direction. (In all the situations we have had to consider $d>0$.) Now the form of the solutions for ψ depends on the direction of J, whether it is positive or negative. It is natural to ask whether the critical current J_c might not depend upon the direction of the current also. We find that it does. Thus the critical surface is asymmetric and must vary with the angle between the current and the magnetic-field directions (both lying in the plane of the surface). The difference between the critical currents in the two directions perpendicular to the field increases as the field decreases. Calling h_2 the reduced field, H_0/H_{c2}, the ratio, increases from 1.02 when $h_2=1.61$, to 1.73 when $h_2=0.67$. The critical current is lower when it is diamagnetic than when it is paramagnetic.

In our calculation we have assumed that the parameter κ of the Ginzburg-Landau theory is large ($\kappa \gg 1$): The penetration depth λ_0 is then much greater than the thickness of the surface layer ($\sim\lambda_0/\kappa$), and it will be reasonable to assume a uniform field H_0 throughout the layer.

Assuming the surface solutions to take the form

$$\psi/\psi_0 = e^{iky} f(x),$$

where k is independent of x, the Ginzburg-Landau equations reduce to

$$\frac{h_2}{b}\frac{d^2f}{dx^2} - bh_2(d-x)^2 f + f - f^3 = 0, \tag{1}$$

where $b = h_2\kappa^2/\lambda_0^2$ and $d = k/b$. At the surface,

$$df/dx = 0 \text{ at } x = 0, \tag{2}$$

and we seek solutions for which, deep in the metal,

$$f(x) \to 0, \text{ and } df/dx \to 0 \text{ as } x \to \infty. \tag{3}$$

The current density j (in emu) is given by

$$j = (H_0/4\pi\lambda_0^2)(d-x)f^2. \tag{4}$$

By integrating (4) by parts and using (1), (2), and (3), the total current J can be related to d and $f(0)$. If we define a quantity Z, independent of H_c and κ by

$$Z = [f^2(0)/2h_2][bh_2d^2 + \tfrac{1}{2}f^2(0) - 1], \tag{5}$$

then in terms of Z,

$$J = Z(5H_c/\pi\kappa\sqrt{2}) \text{ A/cm}.$$

In the calculation, values of Z, b, and h_2 are chosen, and $f(0)$ is varied until numerical

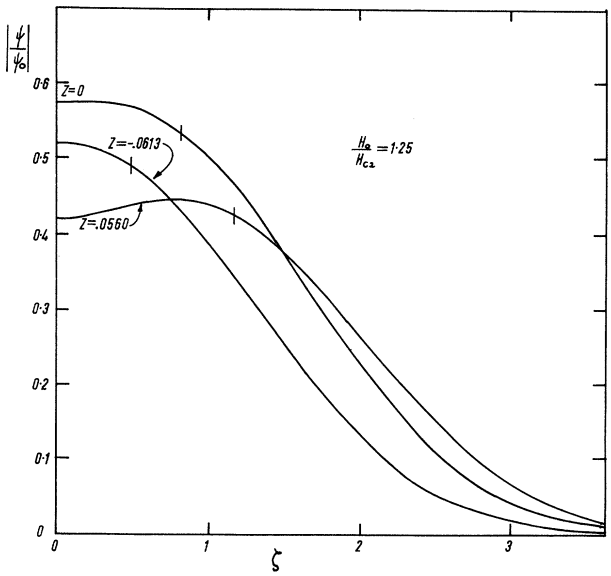

FIG. 1. Variation of the order parameter ψ with depth below the surface when the total surface current J is zero ($Z=0$), and when the current is very nearly critical: either diamagnetic, flowing in the same direction as the current at $x=0$ ($Z>0$), or paramagnetic, flowing in the opposite direction ($Z<0$). The depth is here in reduced units: $\zeta=x(\kappa/\lambda_0)(H_0/H_{c2})^{1/2}$. ψ_0 is the value of the order parameter in zero field and $J=Z(5H_c/\pi\kappa/2)$ A/cm. The vertical lines crossing the curves mark the points of zero current density, $x=d$.

integration of (1) starting from condition (2) satisfies condition (3). Examples of "allowed" solutions obtained in this way for $h_2=1.25$ are shown in Fig. 1.

If Z has one sign or the other and Z is increased beyond a certain value, "allowed" solutions can no longer be found. We have assumed that this value (Z_c) corresponds to the critical current. The solutions of Fig. 1 correspond to currents close to the critical values. Results of the calculation are given in Table I: The critical current $J_c=IH_c/\kappa$ A/cm, i.e., $I=1.125Z_c$. H_c is the thermodynamic critical field in gauss.

These results are plotted in Fig. 2 together with the results of a calculation of Abrikosov[5]: He calculated ψ, in the presence of a current parallel as well as perpendicular to the field, by a variational method, using for his purpose a Gaussian error function centered on the surface. Near H_{c3} he finds the critical current to be proportional to $(H_{c3}-H_0)^{3/2}$. Over the range of our calculation his results for critical current (which he finds to be isotropic in the plane of the surface) follow very closely

Table I. The dependence of critical surface current on reduced field H_0/H_{c2} when the current is either in the same direction as the current at the surface $x=0$ (diamagnetic) or the the opposite direction (paramagnetic). The critical current is IH_c/κ A/cm (H_c in gauss). The error limits given describe the uncertainty in locating the limiting values of current J for which "allowed solutions" can be found, at the present stage of calculation.

H_0/H_{c2}	I(diamagnetic)	I(paramagnetic)
1.613	0.0047 ± 0.0002	0.0048 ± 0.0002
1.538	0.0125 ± 0.0003	0.0130 ± 0.0004
1.428	0.0285 ± 0.0005	0.0298 ± 0.0003
1.250	0.0631 ± 0.0002	0.0692 ± 0.0002
1.000	0.126 ± 0.001	0.153 ± 0.002
0.667	0.201 ± 0.001	0.348 ± 0.003

the same dependence on reduced field h_2 as I(paramagnetic) does—they are larger by a factor between 1.64 and 1.71. I(diamagnetic), on the other hand, depends on h_2 in a different way, not surprisingly perhaps because although a Gaussian error curve may be a good approximation to the solution for ψ when J and Z are negative, it is a poor one when they are positive (see Fig. 1).

Experimental evidence for anisotropy in the critical surface current is scanty. Effects of the anisotropy will not normally appear in measurements of the critical current of rectangular strips in a field transverse to the current and parallel to one of the faces[6,7] even when it is clear that most of the current is carried on the surface, for the two signs of current are present at once, one on each side of the strip. They will appear, however, if one face of the strip has properties at the surface different from the other, or if the cross section is not rectangular, as in the experiments of Swartz and Hart[6] on a Pb 5% Tl rod of triangular cross section. They observe on reversing the current a change in magnitude of critical current by a factor of between 2 and 3 when the field is parallel to one of the faces (at $H_0/H_{c2}=0.68$). Anisotropy in critical current, rather than the curvature of the faces used by the authors in interpreting their results, might well provide an explanation for the effect, and should in any case make an important contribution to it. We do not know what the ratio of diamagnetic to paramagnetic critical current should be for their alloy: κ is 1.5, not large enough for our results to be applicable.

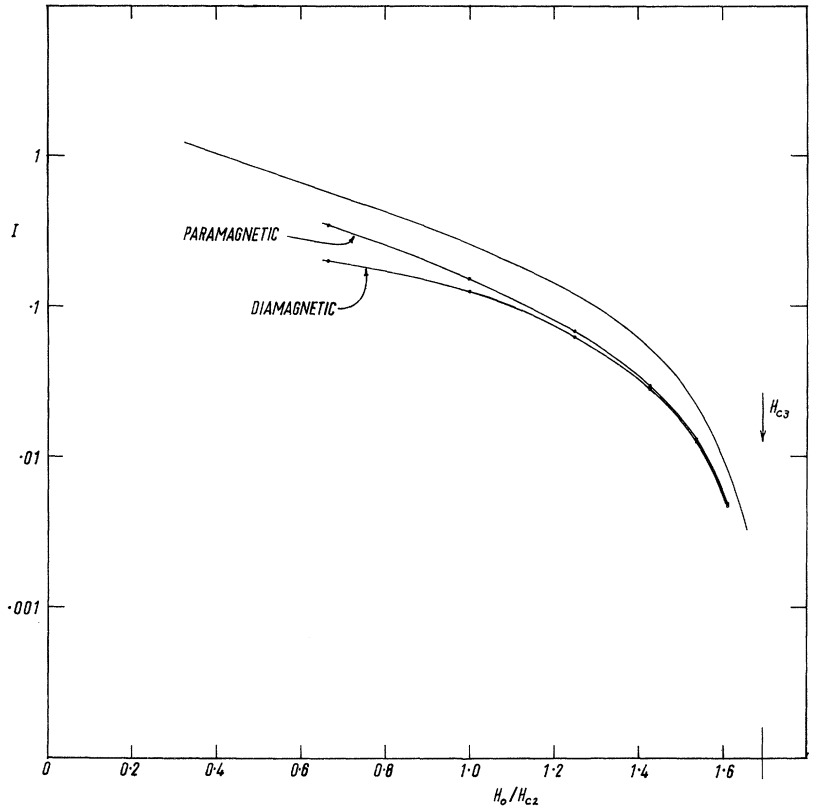

FIG. 2. Variation of critical surface current with reduced field H_0/H_{c2} as given by Abrikosov's calculation[5] upper curve) and by the calculation described here (lower curve and calculated points) for a superconductor with $K \gg 1$. The critical surface current $J_c = IH_c/\kappa$ A/cm.

It is possible that currents in the surface layer have significant effects on the magnetization curve of a type-2 superconductor (or its reversibility) in situations where a large proportion of the surface makes small angles with the field. Effects of this kind have already been noted outside the mixed state, below[8,9] H_{c3}: They may also be important in the presence of the mixed state. To give an idea how large the effects might be compared with the mixed-state magnetization, we remark only that the critical current in the surface layer is larger than the surface current required by the mixed state above a reduced field h_2 of 0.7. This is easily shown. If we call the minimum field of the mixed state B_m, then according to Abrikosov,[3] near enough to H_{c2}

$$(H_0 - B_m) = 0.7(1-h_2)H_c/\kappa.$$

About 70% of this difference in field is produced by a current flowing around the boundaries of the mixed state (the rest is produced by diamagnetic currents in the body of the material). If B_S is the field inside the superconductor produced by the critical diamagnetic current in the surface layer,

$$(H_0 - B_S) \simeq [0.15 + 0.03(1-h_2)]H_c/\kappa. \qquad (6)$$

Thus $(H_0 - B_S) > 0.7(H_0 - B_m)$, when $h_2 > 0.7$.

If the current is paramagnetic, $(H_0 - B_S)$ then depends much less linearly on h_2, but the result (6), being insensitive to the variation of $(H_0 - B_S)$ with h_2, is hardly affected.

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ELECTRON PARAMAGNETIC RESONANCE OF PHOTOEXCITED DONOR-ACCEPTOR PAIRS IN ZINC-SULFIDE CRYSTALS*

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We wish to report the observation of photoexcited electron paramagnetic resonance (EPR) for hexagonal ZnS:Cu, Ga single crystals which we attribute to paramagnetic states of donor-acceptor pairs. The photoexcited EPR signals are observed at 77 and 4.2°K with crystals and powders containing 10^{-3} to 10^{-5} Cu, Ga per ZnS (equivalent concentrations of Cu and Ga). At the same temperatures we also observed the orange-red luminescent emission attributed to electron-hole recombination on approximately the fifth nearest neighbor Cu-Ga pairs.[1]

A Varian V-4500 X-band EPR spectrometer with 100-kc/sec modulation and with a cavity operating in the TE_{012} mode was used. The cavity has a slotted window for optical irradiation. Without irradiation no signal was observed. With irradiation by blue light several EPR lines were observed, their number and intensities depending upon Cu, Ga concentration. For crystals containing 10^{-3} Cu, Ga four strong lines appear at $g = 1.056$, 1.142, 1.635, and 3.916. The two high-field lines are approximately one gauss in width; the two low-field lines are somewhat wider. All four are very nearly isotropic in spectral positions and intensities. With 10^{-5} Cu, Ga the intensities of the EPR signals are reduced by a factor of approximately 1000 and additional lines are resolved in the regions of the lowest field and two high-field lines. The g values in the neighborhood of high-field lines are 1.149, 1.145, 1.142, and 1.062, 1.059, 1.156. The relative intensities of the four lines are approximately the same in over 30 crystals measured. When the irradiation is extinguished the four EPR signals of the more heavily doped crystals all decay with the same rate, and the initial decay constant is of the order of 0.5 sec and is followed by a slower decay with a time constant of several seconds.

The intensity of the EPR signal as a function of wavelength of optical excitation, which we denote EPR excitation spectrum, is shown in Fig. 1 for the more heavily doped crystals. The maxima in the EPR excitation spectra are at the same wavelength for all four g values. For crystals with 10^{-5} Cu, Ga the maximum shifts to 5000 Å, compared to maximum EPR signal at 4500 Å for crystals with 10^{-3} Cu, Ga. There is some evidence for differences in the EPR excitation spectra of the different resolved lines of the more lightly doped crystals. The photoconductivity spectrum was measured, and no photoconductivity was found in the wavelenght range 4500 to 5000 Å, the threshold being at 4200 Å.

The EPR excitation spectrum, the absence of photoconductivity, the occurrence of pair

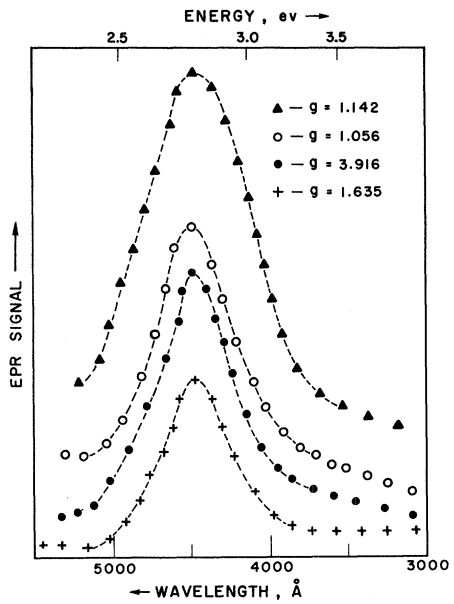

FIG. 1. Intensity of EPR signal versus wavelength of optical excitation.