HYPERSONIC BODY MOTION THROUGH RAREFIED PLASMA

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The exact solution of the problem of rarefied plasma flow near a body is of great interest.[1] We shall consider the case when the body size R_0 is much less than the electron mean free path, so one can consider the plasma to be collisionless. Moreover, the flow velocity V_0 is supposed to be much greater than the ion thermal velocity $V_0 \gg v_T = (2T/M)^{1/2}$, and the Debye radius $D_0 = (T/4\pi e^2 N_0)^{1/2}$ much less than a typical body size. In this case, near the sharp edge of a body at the distance $x-R_0$ of order $D_0 < x-R_0 < R_0$, we have plasma motion with similitude, like fluid flow near a wedge in hydrodynamics.[2] In other words, the ion distribution function, particle concentration, electrical-field potential, and other parameters depend only on $(x-R_0)/Z$, where x is the coordinate along the normal to the edge, and Z is the coordinate along the undisturbed plasma flow. The self-similar kinetic equation for the ion distribution function has the form[3]

$$(u-\tau)\frac{\partial f}{\partial \tau}-\frac{1}{2}\frac{\partial f}{\partial u}\frac{d}{d\tau}\{\ln\int_{-\infty}^{\infty} f du\}=0, \qquad (1)$$

where $u = v/v_T$, $\tau = [(R_0-x)/Z](V_0/v_T)$.

The results of numerical solution of Eq. (1) are shown on Fig. 1. On Fig. 1(a) the distribution function is shown for various values of τ. One can see that this function for $\tau \gg 1$ is very different from the initial Maxwellian ($\tau = -3$); it has the form of a needle. Consequently, for $\tau \gg 1$ one can use the hydrodynamic approximation and obtain the following results for the concentration $N(\tau)$ and mean velocity $\bar{u}(\tau)$:

$$N(\tau) = 0.70 N_0 \exp(-\sqrt{2}\tau), \quad \bar{u}(\tau) = \tau + 1/\sqrt{2}. \qquad (2)$$

(The factor of 0.70 is determined by matching with the numerical solution.) The ion effective temperature, proportional to the velocity dispersion, decreases sharply with τ:

$$T_{\text{eff}} \approx 2.5 T \exp(-2\sqrt{2}\tau). \qquad (3)$$

The solid line on Fig. 1(b) corresponds to the ion and electron concentration $N(\tau)/N_0$, where N_0 is the concentration in the undisturbed flow.

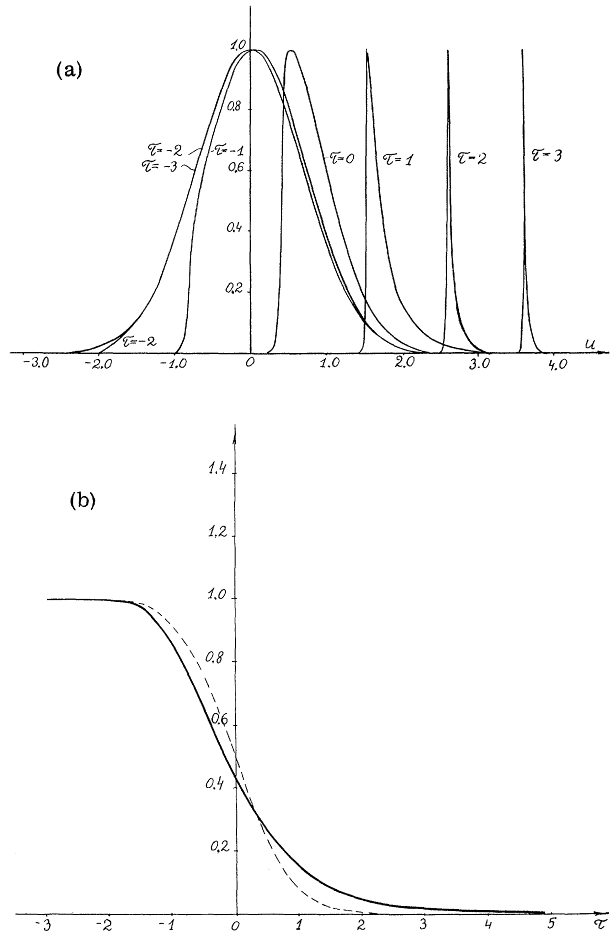

FIG. 1. (a) The ion distribution function, for various values of τ, numerically obtained from Eq. (1). (b) The ratio of the ion to electron concentration (solid line) compared with the function $\pi^{-1/2}\int_\tau^\infty e^{-u^2} du$ (dashed line).

The dashed line corresponds to the function

$$\frac{1}{\sqrt{\pi}}\int_\tau^\infty e^{-u^2} du,$$

which gives the free neutral molecule flow concentration near the edge. Such a solution was used in previous papers[4,1] to approximate the ion distribution. One can see that for negative and small positive values of τ the difference between the two curves is small, so in this region one can neglect the influence of the electrical field on the ion motion. On the contrary, for large positive values of τ, e.g. in the plasma region near the body, the concentration increases: $N \sim \exp(-\sqrt{2}\tau)$, instead of $N \sim \exp(-\tau^2)$ as for a neutral gas.

The second peculiarity of the solution obtained is the sharp decrease of ion effective temperature T_{eff} with τ [Eq. (3) and Fig. 1]. This leads to the important physical effect of plasma instability. In fact, in the after-body region not far from the body ($\tau > 1$), there is a secondary plasma flow with a needle-form distribution function. Because of the decrease of the ion effective temperature, the ion acoustical waves decay more slowly, and this makes the flow less stable. Considering a finite body, we have to take into account that the secondary flow from one edge of a body goes through the rarefied after-body region and enters the plasma region near the other edge. As a result of this collision between the flow and the plasma an instability may take place.

To analyze the effects near a body of finite dimensions, we considered the flow near a body, the cross section of which was an infinite plate in the x, y plane with a width equal to $2R_0$. The flow is normal to this plate. For example, an infinite cylinder with radius R_0 presents such a cross section. The analytical solution was found (by means of a self-similar solution) in the region

$$\frac{1}{\sqrt{2}\ln(R_0/D_0)} < \frac{Zv_T}{R_0V_0} \lesssim 1.$$

The left limitation is connected with the growth of the Debye radius in a rarefied zone in the neighborhood of the body. Far from the body, $Zv_T/R_0V_0 \gtrsim 1$, the problem was solved numerically. At the distance $Z \gtrsim R_0V_0/v_T$, the ion distribution appeared to be close to the neutral particle distribution, which was used in references 4 and 1 to approximate the ion distribution. Near the body the ion concentration is higher than the approximation used in references 4 and 1. In particular, if $V_0/v_T < \sqrt{2} \times \ln(R_0/D_0)$, the ion current j on a spherical trap (probe), situated at the distance R_0 along the Z axis, may be written in a form

$$j = 1.40 N_0 S_0 V_0 \left(1 + \frac{v_T}{\sqrt{2}V_0}\right) \exp\left\{-\frac{\sqrt{2}V_0}{v_T}\right\}. \quad (4)$$

We supposed here that the ion current is due to all ions which came on the surface of the trap.

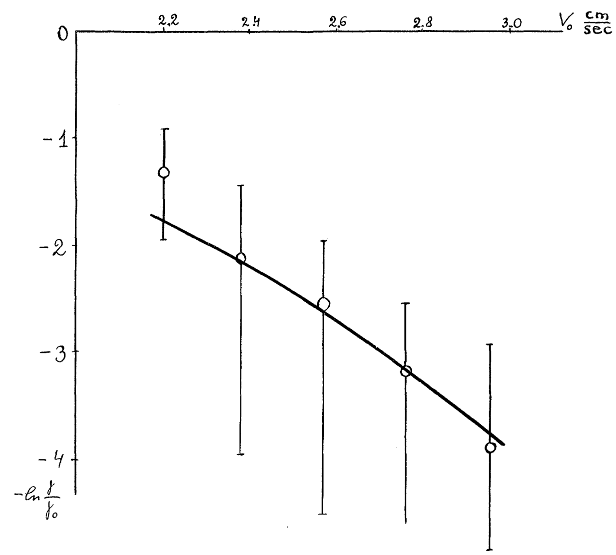

FIG. 2. The ion current on a spherical probe from Eq. (4). The points are the experimental values of Norman and Willmore.[5]

The trap cross area is S_0. The ion current to the trap situated on the Z axis ahead of the body is equal to $j_0 = N_0 S_0 V_0 (1 + v_T^2/2V_0^2)$. The ratio j/j_0 depends on V_0/v_T only, e.g. on the body velocity V_0, on the ion temperature T, and on the mass M. The ratio j/j_0 was measured experimentally in the ionosphere by Norman and Willmore.[5] The experimental results are shown in Fig. 2. The solid line is the result of calculations according to Eq. (4). In this calculation we took $M = 16$, T varying from 900° at altitude 270 km to 1500° at altitude 480 km according to present ionosphere models. One can see good agreement between the theory and experiment.

Let us note that the agreement obtained exceeds the accuracy of both experiment and theory. For example, the ratio V_0/v_T is of order $2 + 3$, and it is not legitimate to neglect the influence of the details of the body shape. In the experiment a plane probe was used and the finite ratio R_0/D_0 could be important. So under the conditions of this experiment, only an exponential term in (4) is quite accurate. It is important that the experiment shows the linear relation between $\ln(j/j_0)$ and V_0/v_T predicted by the theory. One should note that for a neutral gas, e.g. in the approximation in which the influence of the electric field on ion motion was not taken into account, the value of $\ln(j/j_0)$ is of order $-(V_0/v_T)^2$. Numerical values of j/j_0

are in this approximation less than the experimental values.[6]

The stability of the solution obtained was investigated. It was shown that the longitudinal ion acoustical waves in the after-body region are unstable. The wave frequency $\omega \sim (0.1\text{-}1)\Omega_i$, where $\Omega_i = (4\pi e^2 N_0/M)^{1/2}$ is the Langmuir ion frequency. It is possible that the plasma oscillations in the ionosphere observed by Bowen, Boyd, Henderson, and Willmore[7] in the wake of a body are the result of this instability mechanism.

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NONANALYTIC FORM OF THE COEXISTENCE CURVE OF HELIUM AT THE CRITICAL POINT*

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During the years between 1956 and 1963 I measured the density of saturated He^4 liquid and vapor along the coexistence curve from 0.30 to $0.994T_c$, by refractive index measurements with a modified Jamin interferometer.[1-3] We analyzed our data on the coexistence curve between 0.98 and $0.994T_c$ by extending Landau and Lifshitz's theory using a fit to a power-series expansion in volume and temperature.[3] If, as is now believed, singularities occur at the critical point, this whole expansion procedure is questionable, since the coexistence curve cannot, in principle, be represented asymptotically by a Taylor series in the density about such a nonanalytic point.[4] A reanalysis of these same data will now be presented, without the use of a Taylor-series expansion. The result of this analysis confirms that the critical point is indeed a nonanalytic point.

The coexistence curve of saturated liquid and vapor densities ρ_l and ρ_g, as a function of temperature, is symmetrical, not about the critical density, but about a "rectilinear diameter." This line of mean densities of vapor and liquid passes through ρ_c, the critical density, and for He^4 would extrapolate linearly to $1.1\rho_c$ at $T=0$. The symmetry of the coexistence curve is obscured on a plot of saturated molar volume V_l and V_g of liquid and vapor since the mean volume curves sharply away from the temperature axis at lower temperatures. In examinations of the shape of coexistence curves, it is customary to consider the quantity $(\rho_l-\rho_g)$ or $(\rho_l-\rho_g)/2\rho_c$ as a function of (T_c-T). If one attempts to write

$$(\rho_l-\rho_g)/2\rho_c = A(T_c-T)^\beta, \qquad (1)$$

where A and β are constant, then classically (van der Waals), $\beta=\frac{1}{2}$, whereas a variety of experiments suggests that $\beta=\frac{1}{3}$ for a certain range of temperatures. Accurate values of T_c are needed for meaningful tests of such a relationship. Furthermore, the slope of the "rectilinear diameter," or line of mean densities, differs from substance to substance, so that the similarity of coexistence curves may be obscured by comparing experimental data in that manner. Buckingham[5] has suggested that the shape of the coexistence curve should be analyzed using the natural variable

$$X \equiv \frac{\rho_l-\rho_g}{\rho_l+\rho_g} \equiv \frac{V_g-V_l}{V_g+V_l} \neq \frac{\rho_l-\rho_g}{2\rho_c}. \qquad (2)$$

Advantages of this variable are (1) X ranges from 1 to 0 as T ranges from 0 to T_c for any substance; (2) there is equal symmetry using either density or molar volume, and the effect of the slope of the "rectilinear diameter" is entirely removed; and (3) if we plot X^n (where $n=1/\beta$) against T, we need not know T_c or ρ_c or V_c and, in fact, may determine T_c by such plots.