Rev. 91, 1257 (1953); C. M. Sommerfield, Phys. Rev. 107, 328 (1957); A. Petermann, Helv. Phys. Acta 30, 407 (1957); M. Baranger, F. J. Dyson, and E. E. Salpeter, Phys. Rev. 88, 680 (1952).

[9]Some of the bounds on individual integrals have been found to be violated, and there is some question as to whether or not the total contribution is within the given bounds (private communication from Max Soto).

[10]E. E. Salpeter, Phys. Rev. 87, 328 (1952); T. Fulton and P. C. Martin, Phys. Rev. 95, 811 (1954).

[11]D. F. Jackson, Proc. Phys. Soc. (London) 76, 949 (1960).

[12]E. E. Salpeter, Phys. Rev. 89, 92 (1953).

[13]J. F. Streib, Phys. Rev. 100, 1797 (1955).

[14]We exclude the new measurements of R. T. Robiscoe, Phys. Rev. 138, A22 (1965); and private communication, which are about 0.3 Mc/sec higher than Lamb's for both hydrogen and deuterium.

SIGNAL VELOCITY IN A REGION OF RESONANT STIMULATED EMISSION*

N. S. Shiren
IBM Watson Research Center, Yorktown Heights, New York
(Received 19 July 1965)

In this Letter we report both experimental and preliminary theoretical results on the signal (or pulse) velocity for wave propagation in a narrow bandwidth amplifying medium. The measurements were made on 16.45-Gc/sec longitudinal ultrasonic waves propagating through a phonon maser amplifier. Previously reported measurements[1] and theory[2,3] showed that the signal velocity decreases in the neighborhood of a resonance absorption line. The present results are quite different, however. For all levels of amplification, up to a maximum attainable 12.5 db/cm, the velocity was found to be the same, within ~±1%, as in the absence of the maser interaction. This result is also predicted by our theory which is valid for any one-dimensional wave propagation in a medium for which the resonant interaction obeys a Lorentz dispersion relation.

The phonon maser utilized the Ni^{2+} $\Delta M=2$ spin-resonance transition in MgO at 1.6°K. 0.5-μsec ultrasonic pulses were generated in a quartz transducer bonded to the 1.5-cm MgO crystal. Population inversion was accomplished by adiabatic rapid passage through the $\Delta M=1$ transitions at x band. Descriptions of the experimental apparatus and measurement techniques have been previously reported.[4,1]

The experimental results are illustrated in Fig. 1. Echo patterns are shown for 15-db amplification, for 15-db absorption, and for zero interaction (i.e., off resonance). For the amplification condition the input was 15 db smaller than for the other two conditions in order to avoid saturation of the resonance. If the power is high and saturation is allowed to occur, the amplified signal exhibits the well-known pulse-sharpening effect. An example is shown in Fig. 2. In Fig. 1 the velocity of the amplified pulse is seen to be unchanged from the off-resonance value. The attenuated pulse, however, exhibits the characteristic velocity decrease (in this case ~15%),

(a)
(b)
(c)

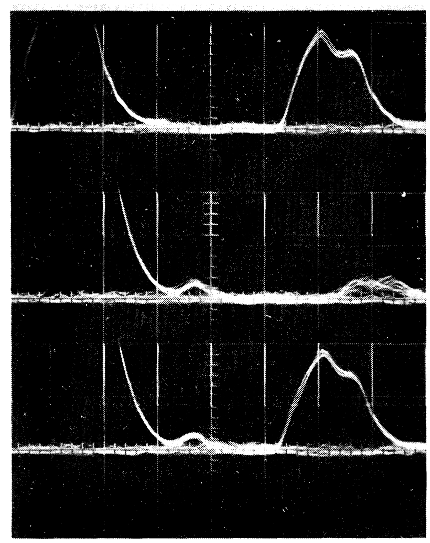

FIG. 1. Oscilloscope traces showing effect of Ni^{2+} resonance interaction on ultrasonic signal velocity. Time runs from left to right. The first pulse is a bond echo and has traveled through the quartz transducer only. The second pulse has made one round trip through the MgO. The time between pulses is 2.27 μsec. (a) 15-db amplification, ultrasonic frequency on peak of Ni^{2+} $\Delta M=2$ resonance. (b) 15-db attenuation, on resonance, input power 15 db higher than in (a). (c) Off resonance, input power same as (b).

$$\Delta(1/v)\simeq\beta(\omega)/\delta. \tag{1}$$

$\beta(\omega)$ is the absorption per cm at the input fre-

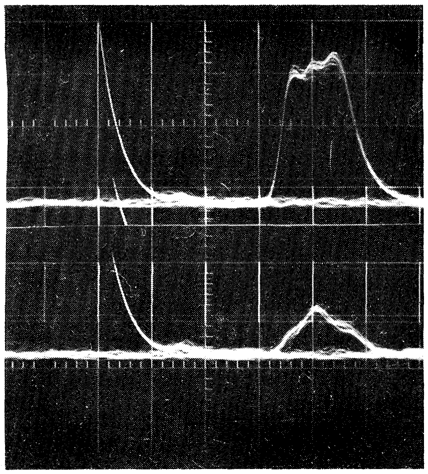

FIG. 2. Oscilloscope traces showing pulse sharpening due to amplification and saturation at high input power. (a) Ultrasonic frequency on resonance. (b) Off resonance.

quency, ω, due to the resonance transition centered at ω_0. δ is the half-width at half-amplitude due to either homogeneous broadening $(1/T_2)$ or small- (spatial-) scale inhomogeneous broadening.[1]

For an input signal of the form

$$F(t,0)=0, \quad t\leqslant 0,$$

$$F(t,0)=\sin \omega t, \quad t>0,$$

the solution to the one-dimensional wave equation for a plane wave propagating in the x direction is

$$F(t,x) = \lim_{\epsilon\to 0} -\frac{1}{2\pi}\mathrm{Re}\int_{-\infty}^{\infty}\frac{\exp\{-i[\omega'\tau-\kappa(\omega')x]\}}{\omega'-\omega+i\epsilon}d\omega'. \tag{2}$$

$\tau=(t-x/c)$, and for the dispersion relation we assume the Lorentz form for a homogeneously broadened spin-resonance transition, at resonant frequency ω_0.

$$k(\omega)\simeq \omega/c+\kappa(\omega),$$

$$\kappa(\omega)=\pm\frac{\beta_0\delta}{\omega_0-\omega-i\delta}=[\alpha(\omega)+i\beta(\omega)]. \tag{3}$$

c is the off-resonance velocity, $\beta_0\equiv|\beta(\omega_0)|$ (always positive), and the upper or lower sign is taken according to whether absorption or amplification, respectively, is being considered. Thus $\beta(\omega)$ is positive for absorption and negative for emission. Equation (3) is a valid approximation under the condition $\delta/\omega_0\ll 1$, $\beta_0\ll\omega/c$.

The results of the earlier theories are not directly applicable to the amplification case, although the method of integration used could be applied to obtain a result for negative β. However, we have instead resorted to a different method due to Lynch, Holland, and Hamermesh[5] and evaluated Eq. (2) for both positive and negative β. As in the earlier theories, the signal velocity may be expressed in closed form only in the asymptotic limit $\beta_0 x\to\infty$. For the absorption case our result, in this limit, is the same as that given by Eq. (1), and for the amplification case it is $\Delta(1/v)=0$, independent of ω. These asymptotic results are upper limits, and therefore the result for the amplification case is in agreement with experiment even though the largest $\beta_0 x$, for which measurements were made, was ~3 (26-db power gain). Comparison with experiment for the absorption case is considered in reference 1.

Integration of Eq. (2) results in the following expressions for $F(x,\tau)$, $\tau\geqslant 0$ $[F(x,\tau)=0$ for $\tau<0]$.

$$F(x,\tau) = \mathrm{Re}\, i\exp[-i(\omega_0-i\delta)\tau]\sum_{n=0}^{\infty}[-i\kappa(\omega)x]^{-n} \times(\pm\beta_0 x\delta t)^{n/2}J_n[2(\pm\beta_0 x\delta t)], \tag{4}$$

or alternatively,

$$F(x,\tau)=F_1(x,\tau)+F_2(x,\tau);$$

where

$$F_1(x,\tau)=\mathrm{Re}\, i\exp\{-i[\omega\tau-\kappa(\omega)x]\} \tag{5}$$

is the stationary part of the signal, and

$$F_2(x,\tau) = -\mathrm{Re}\, i\exp[-i(\omega_0-i\delta)\tau]\sum_{n=1}^{\infty}[i\kappa(\omega)x]^{n} \times(\pm\beta_0 x\delta t)^{-n/2}J_n[2(\pm\beta_0 x\delta t)] \tag{6}$$

is the transient part. The upper or lower sign is taken as in Eq. (3). The definition of signal velocity is somewhat arbitrary. There is always a transient or precursor which travels with the velocity c. However, it is usual to define the signal velocity in terms of the time, τ_S, at which the stationary part, $F_1(x,\tau)$, is first observed. The problem is thus one of detection of $F_1(x,\tau)$ in the presence of the transient, $F_2(x,\tau)$. It might thus appear that τ_S will depend on such factors as detector bandwidth and detection sensitivity since they will determine the extent to which the transient must have decayed before the stationary part can be detected. However, the stationary amplitude varies as $e^{\beta x}$, whereas the amplitude of $F_2(x,\tau)$ varies as $e^{-\delta\tau}f(x,\tau)$, where the variation of $f(x,\tau)$ is slower than exponential in βx or $\delta\tau$. Therefore, in the asymptotic limit, the exponential factors dominate and the actual numerical value of the ratio F_2/F_1, which is chosen as a detection criterion, will be unimportant. For the same reason the results are insensitive to the shape of the input pulse as long as the pulse duration is large compared with $1/\delta$ and $1/\omega$. We may then take τ_S as the time at which the envelopes of $F_1(x,\tau)$ and $F_2(x,\tau)$ become equal. Thus for $\tau \geqslant \tau_S$ we require

$$\langle[F_1(x,\tau)]^2\rangle = e^{2\beta(\omega)x} \geqslant \langle[F_2(x,\tau)]^2\rangle, \qquad (7)$$

where the angular brackets indicate time averages over many cycles of ω and ω_0 but over times small compared with $1/\delta$.

Equation (7) may be conveniently tested in the two regions $\delta\tau \gg (\beta_0 x)^{-1}$ and $\delta\tau \ll (\beta_0 x)^{-1}$. In the asymptotic limit, $\beta_0 x \to \infty$, the gap between the two regions becomes negligible, and in the following discussion this limit is assumed.

<u>(1) $\delta\tau \gg (\beta_0 x)^{-1}$</u>.–$(\beta_0 x\delta\tau) \to \infty$ and we take the asymptotic limits of the Bessel functions. We further divide the region according as $\tau \gtrless |\beta(\omega)x/\delta|$.

<u>(a) $\tau > |\beta(\omega)x/\delta|$</u>.–The series of Eq. (6) converges and the inequality, Eq. (7), is satisfied for all τ in this region for both amplification and absorption.

<u>(b) $\tau < |\beta(\omega)x/\delta|$</u>.–The series of Eq. (4) converges. We write $F_2(x,\tau)$ as $[F(x,\tau)-F_1(x,\tau)]$. Equation (7) is not satisfied for absorption in this region of τ. However, for amplification we find

$$\lim_{\beta_0 x \to \infty} \langle[F(x,\tau)-F_1(x,\tau)]^2\rangle = \langle[F_1(x,\tau)]^2\rangle$$

throughout the region.

<u>(2) $\delta\tau \ll (\beta_0 x)^{-1}$</u>.–$(\beta_0 x\delta\tau) \ll 1$ and we take the Bessel functions in this limit. Then using Eq. (6),

$$\langle[F_2(x,\tau)]^2\rangle = e^{-2\delta\tau}[e^{\beta(\omega)x}-1]^2.$$

As in (1b), Eq. (7) cannot be satisfied in absorption for large $\beta_0 x$, but is satisfied in amplification throughout the stated region of τ.

In summary, we have the following. Amplification: The transient amplitude is never larger than the stationary amplitude, and so Eq. (7) is satisfied for all τ; therefore $\tau_S = 0$. Absorption: The transient amplitude is initially larger than the stationary amplitude but decays as $e^{-\delta\tau}$, becoming equal to the latter at $\tau = \tau_S = \beta(\omega)x/\delta$, in agreement with Eq. (1).

Finally, we note that similar signal-velocity effects were observed in the double-quantum phonon maser recently reported.[6]

I wish to acknowledge several discussions regarding the theory with Dr. G. J. Lasher and Dr. P. B. Miller. Mr. T. G. Kazyaka's assistance in the experimental work was invaluable.

---

*Work supported by U. S. Army Electronics Laboratory under Contract No. DA36-039MAC-02280(E).

[1]N. S. Shiren, Phys. Rev. 128, 2103 (1962).
[2]H. Baerwald, Ann. Physik 7, 731 (1930).
[3]L. Brillouin, Ann. Physik 44, 203 (1914).
[4]N. S. Shiren, in Physics of Quantum Electronics, edited by B. Lax (McGraw-Hill Book Company, Inc., New York, to be published).
[5]F. J. Lynch, R. E. Holland, and M. Hamermesh, Phys. Rev. 120, 513 (1960).
[6]N. S. Shiren, to be published.

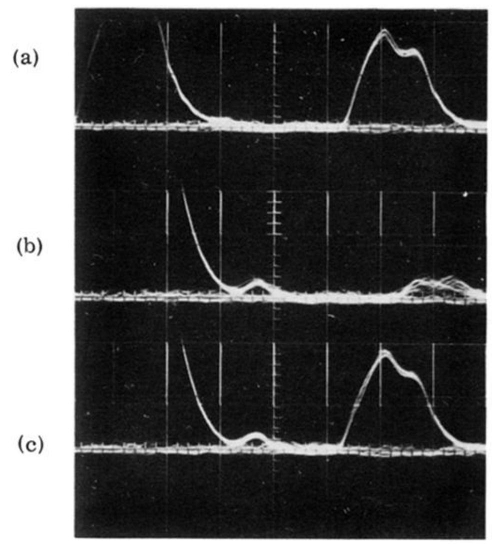

FIG. 1. Oscilloscope traces showing effect of Ni^{2+} resonance interaction on ultrasonic signal velocity. Time runs from left to right. The first pulse is a bond echo and has traveled through the quartz transducer only. The second pulse has made one round trip through the MgO. The time between pulses is 2.27 μsec. (a) 15-db amplification, ultrasonic frequency on peak of Ni^{2+} $\Delta M = 2$ resonance. (b) 15-db attenuation, on resonance, input power 15 db higher than in (a). (c) Off resonance, input power same as (b).

(a)

(b)

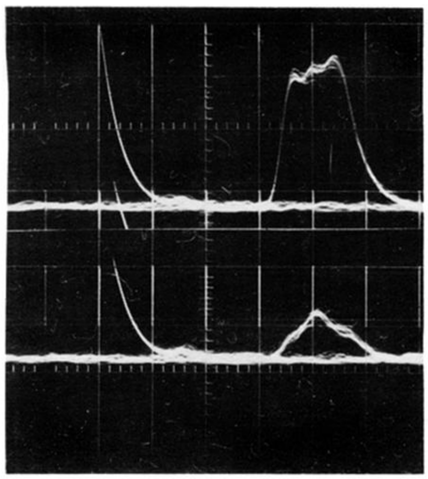

FIG. 2. Oscilloscope traces showing pulse sharpening due to amplification and saturation at high input power. (a) Ultrasonic frequency on resonance. (b) Off resonance.