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THEORETICAL ESTIMATE OF DOUBLE QUANTUM PHOTODETACHMENT OF IODINE IONS

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The experimental value recently obtained by Hall, Robinson, and Branscomb[1] for the two-photon ionization probability of the iodine ion I^- is in disagreement with a theoretical estimate by Geltman.[2] It has been suggested[1] that this may be due to the inadequacy of the plane-wave approximation for the virtual states made by Geltman in evaluating the second-order contribution from the part of the electromagnetic interaction which is linear in the electron charge, $H^{(1)}=(e/mc)\vec{A}\cdot\vec{p}$.

The above discrepancy can rather be interpreted as arising from the interaction term $H^{(2)}=(e^2/2mc^2)\vec{A}^2$ having previously been disregarded.[2] This term, which has nonvanishing matrix elements between two-photon states and the photon vacuum, is in fact responsible for the greater part of the two-photon ionization probability in this case.

This has been ascertained by the following calculations:

(i) Using for the bound electron in I^- the approximate eigenfunction $\psi(r)=(\alpha/2\pi)^{1/2}e^{-\alpha r}/r$ [$\alpha=(2mE/\hbar^2)^{1/2}$, $E=3.076$ eV being the electron affinity], plane waves for the positive-energy electron eigenfunctions and the incident electromagnetic wave

$$\vec{A}=\left(\frac{2}{\pi}\right)^{1/2}A\vec{\epsilon}\cos\left[\frac{2\pi}{\lambda}(\vec{n}\cdot\vec{\chi}-ct)\right] \tag{1}$$

($\vec{\epsilon}$ and $\vec{n}$ denoting unit vectors in the directions of polarization and propagation, respectively; $\lambda=6943$ Å, $h\nu=1.785$ eV), the transition amplitude due to $H^{(2)}$ is found to be

$$M_{fi}^{(2)}=\frac{e^2\kappa^3}{mc^2}(2\pi\alpha)^{1/2}\left[\hbar^2\alpha^2+\left(\vec{p}-\frac{2\pi\hbar\vec{n}}{\lambda}\right)^2\right]^{-1} \times A^2\delta(E_f-E_i). \tag{2}$$

Here $\vec{p}$ is the momentum of the ejected electron, $E_i=-E+2h\nu$ the initial energy, and $E_f=\vec{p}^2/2m$ the final energy.

Disregarding the contribution from $H^{(1)}$, the square of $M_{fi}^{(2)}$, integrated over the momentum of the ejected electron and multiplied by the appropriate numerical factors, yields the ionization probability per unit time

$$W=\left(\frac{e^2}{4\pi\hbar c}\right)^2\frac{[E(2h\nu-E)]^{1/2}}{(h\nu)^4}\frac{\pi^2\hbar^5c^4}{(mc^2)^2}F^2 \approx 24.2\times10^{-51}F^2, \tag{3}$$

where $F=2\nu A^2/\hbar c$ is the photon flux in the units used in references 1 and 2.

A similar calculation employing the transition amplitude $M_{fi}^{(1,1)}+M_{fi}^{(2)}$, where

$$M_{fi}^{(1,1)}=-(2c^2/h\nu)(\vec{p}\cdot\vec{\epsilon})^2 \times[4mc^2+h\nu-2c\vec{n}\cdot\vec{p}]^{-1}M_{fi}^{(2)} \tag{4}$$

is the second-order amplitude arising from $H^{(1)}$, yields

$$W\approx19.8\times10^{-51}F^2.$$

This 20% reduction is due to the factor $[1-\frac{2}{3}\times(2h\nu-E/h\nu)]$ by which Eq. (3) is multiplied when $M_{fi}^{(1,1)}$ is also taken into account. Note that an estimate of a lower bound for the ratio of $M_{fi}^{(2)}$ and $M_{fi}^{(1,1)}$ gives

$$\frac{M_{fi}^{(2)}}{M_{fi}^{(1,1)}} \gtrsim \frac{h\nu}{2h\nu-E}.$$

(ii) The probability as arising from $H^{(2)}$ alone has been expressed in terms of the cross section σ for one-photon ionization due to photons of energy $2h\nu = 3.57$ eV. Taking[3] $\sigma = 2.1\times10^{-17}$ cm^2,

$$W \approx 1.95\times10^{-32}\sigma F^2 = 410\times10^{-51}F^2$$

has been obtained. Considering the reduction due to the contribution from $H^{(1)}$ and the fact that, as stated in reference 3, the value of σ may be in error by 50%, the above value does not seem incompatible with the experimental result.

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[1] J. L. Hall, E. J. Robinson, and L. M. Branscomb, Phys. Rev. Letters 14, 1013 (1965).

[2] S. Geltman, Phys. Letters 4, 168 (1963).

[3] B. Steiner, M. L. Seman, and L. M. Branscomb, J. Chem. Phys. 37, 1200 (1962).

[4] The idea of expressing two-photon in terms of one-photon ionization amplitudes is due to Geltman (see reference 2).

CORRECTION TO THE (He^3, $2\,^3S_1$) TO (He^{3+}, $2\,^2S_{1/2}$) HYPERFINE-STRUCTURE RATIO*

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Tests of the theory of hfs in simple atoms which do not depend upon nuclear structure are of special interest because of the (45 ± 17) parts per million (ppm) difference[1] between the theoretical and experimental hfs of atomic hydrogen. Various ratios of hfs measurements have this property and therefore afford unambiguous tests of the quantum-electrodynamic corrections. The theoretical ratio of the $2S$ state hfs to the $1S$ state hfs agrees[2,3] with the measured value to a precision of several parts in 10^8 for the H atom, and a precision of several parts in 10^7 for D and He^{3+}. There has been, however, a discrepancy in the ratio of the $2\,^3S_1$ state hfs of the He^3 atom[4] to the $2\,^2S_{1/2}$ state hfs of the He^{3+} ion,[5] the calculated[6,7] value being too large by 10 ppm. The purpose of this note is to point out the omission of a correction of -4 ppm in this theoretical ratio.

The second-order radiative corrections to hfs have been calculated in powers of the Coulomb field, i.e., in powers of $Z\alpha$. They give a correction to the hfs of the form

$$1+\alpha[a+b(Z\alpha)+c(Z\alpha)^2+\cdots].$$

Here[8] $a=\frac{1}{2}\pi$ and $b=\frac{5}{2}-\log 2$; c depends upon the atomic state and contains $\log(Z\alpha)$ and $\log^2(Z\alpha)$. Thus the atom/ion ratio should have a previously omitted factor

$$F=\frac{1+\alpha[a+b(Z\alpha)]+[\lambda c_1{}^a+(1-\lambda)c_2{}^a]\alpha(Z\alpha)^2}{1+\alpha[a+b(Z\alpha)+c_2{}^i(Z\alpha)^2]}$$

$$=1+[c_1{}^a-c_2{}^i+(1-\lambda)(c_2{}^a-c_1{}^a)]\alpha(Z\alpha)^2+\cdots,$$

where λ is the fraction of the atomic hfs due to the interaction of the nucleus and the $1s$ electron, and $(1-\lambda)$ is the fraction due to the $2s$ electron; exchange effects are neglected.

The atomic hfs can be obtained[9] to within 3.6% by assuming that all the hfs arises from a single unscreened $1s$ electron, or by using the simplest two-parameter variational wave function.[10] In the first case $\lambda=1$, and in the second $\lambda=0.95$; we will use $\lambda=1$ for simplicity. Since the radiative corrections arise at distances from the origin comparable to the electron Compton wavelength, screening is negligible, and $c_1{}^a\approx c_1{}^i$. Thus,

$$F\approx1+(c_1{}^i-c_2{}^i)\alpha(Z\alpha)^2=1-(4.2\pm0.4)\times10^{-6},$$

using Zwanziger's result,[11] $c_1{}^i-c_2{}^i=-2.7$ for $Z=2$, and allowing for a 10% error due to our approximations.

The remaining discrepancy of 6 ppm may be due to inaccuracies in the relativistic corrections,[6] which have an estimated uncertainty