INVESTIGATION OF MICROWAVE RADIATION EMITTED BY JOSEPHSON JUNCTIONS*†

D. N. Langenberg, D. J. Scalapino, B. N. Taylor, and R. E. Eck

Department of Physics and Laboratory for Research on the Structure of Matter,
University of Pennsylvania, Philadelphia, Pennsylvania
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Several years ago, Josephson[1] predicted that an ac supercurrent of frequency $\nu = 2eV/h$ would exist in a superconductor-insulator-superconductor tunnel junction biased at a voltage V if the insulating layer were sufficiently thin.[2] The first support for this prediction came from experiments which showed that if electromagnetic fields were induced in the insulating layer by an externally applied microwave field, structure appeared in the dc current-voltage (I-V) characteristic of the junction.[3] Subsequently, such structure was observed in the absence of any externally applied microwave field, thus demonstrating the presence of electromagnetic radiation in the junction generated by the Josephson ac supercurrent itself.[4-6] Using two superimposed junctions with contiguous oxide layers, Giaever[7] has shown that the electromagnetic fields generated in one of the junctions can be detected by observing the structure it produces in the I-V characteristic of the junction. Recently, Yanson, Svistunov, and Dmitrenko reported the first direct observation of radiation emitted by a Josephson junction.[8] In this Letter, we report the results of an investigation of this radiation including (1) a discussion of the junction–wave-guide–magnetic-field configuration and a theoretical estimate of the magnitude of the radiated power, (2) experimental observation of 10^{-12} W of emitted radiation at 9.2 Gc/sec with a spectral purity greater than 1 part in 10^4, and (3) observation and explanation of harmonic and fractional subharmonic radiation.

In Fig. 1, a cutaway view of the wave-guide–junction–magnetic-field configuration is shown. The axis of the wave guide defines the z axis. The externally applied dc magnetic field H_0 was oriented in the plane of the junction perpendicular to this axis producing a spatial modulation of the Josephson ac supercurrent along the length L of the junction (Fig. 1). This supercurrent excites TM waves[5,9] which propagate in the insulating layer parallel to the z axis with a phase velocity $\bar{c}$. Thus, a junction of length L has characteristic frequencies[5] given by $\omega_n = n\pi\bar{c}/L (n = 1, 2, 3, \cdots)$. The excitation of these modes gives rise to steps in the I-V characteristics at bias voltages $V_n = \hbar\omega_n/2e$ (Fig. 2). For the junctions used in these experiments, $L = 0.16$ cm and $\bar{c} = 0.05c$ (c is the velocity of light), so

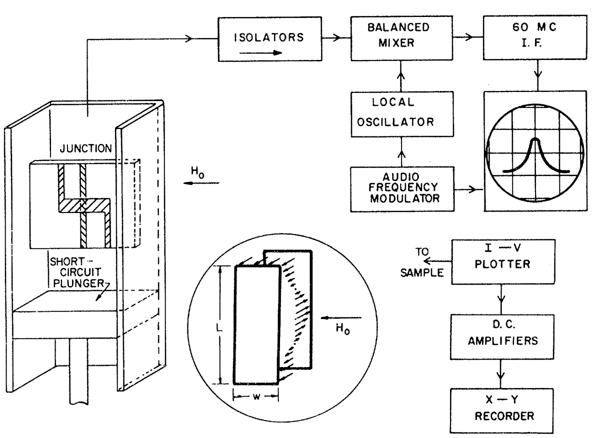

FIG. 1. Cutaway view of the sample holder and short-circuit plunger, and a block diagram of the superheterodyne detector. The inset shows the electric-field distribution in the junction when it is biased on the second mode.

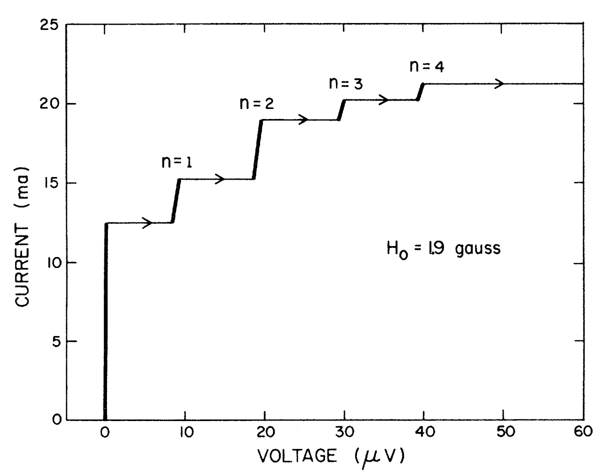

FIG. 2. A typical I-V curve for the Sn–Sn-oxide–Sn junctions used in these experiments. The voltage separation of the modes (labeled by n) corresponds to a frequency separation of approximately 4.6 Gc/sec. By adjusting the magnitude of H_0, a greater portion of each mode could be observed as well as several other higher modes.

that the $n=2$ mode occurred at 9.2 Gc/sec. The insert in Fig. 1 shows the spatial variation of the electric field for this mode which was chosen so that the electric fields at the ends of the junction were in phase and would contribute coherently.

A simple estimate of the magnitude of the power radiated by the junction into a wave guide which is terminated by its characteristic guide impedance Z_g can be obtained by treating the junction as a strip line of characteristic impedance Z_J. The effective reactance associated with the local field modes at the edge of the junction is small so that the transmission coefficient from the junction to the wave guide is simply[10] $T=4Z_gZ_J/(Z_g+Z_J)^2$. If U_2 is the average value of the electromagnetic energy stored in the $n=2$ mode, then the power incident on the ends of the junction is $\bar{c}U_2/L$, and multiplying by the transmission coefficient we obtain an estimate of the power radiated:

$$P_{\mathrm{rad}}=(\bar{c}/L)U_2T. \tag{1}$$

The characteristic guide impedance for the $TE_{1,0}$ mode operating at 9.2 Gc/sec is approximately $Z_0=377\Omega$.[10] Treating the junction as a strip line, we find that its characteristic impedance is[11]

$$Z_J=Z_0\frac{c}{\bar{c}}\frac{l}{w\epsilon}, \tag{2}$$

where w is the width of the junction and l and ϵ are the thickness and relative dielectric constant of the oxide layer, respectively. For the junction used $w=0.025$ cm, $l\approx 10$ Å, and $\epsilon\approx 4$, so that $Z_g\approx 2\times 10^{-5}Z_0$ and $T\approx 10^{-4}$.

The magnitude of U_2 can be estimated from the dc I-V characteristic. On the second step, the $n=2$ mode (Fig. 2), dc current is 16 mA and the dc voltage is 19 μV, so that the dc power P fed into the junction is 3×10^{-7} W. As discussed later, we expect that this power is divided into about five junction modes, each with its own characteristic frequency ω_n; thus $U_2=PQ/5\omega_2$. Here Q represents the loaded Q of the $n=2$ mode which we estimated from the slope of the I-V characteristic at the second step to be about 5. Using these estimates $P_{\mathrm{rad}}\approx 5\times 10^{-12}$ W.

In order to investigate this radiation, samples were mounted in a wave guide containing a short-circuit plunger (Fig. 1) which could be adjusted from outside the cryostat. The radiation emitted by the sample passed through a series of isolators to an X-band superheterodyne receiver consisting of a balanced mixer, a preamplifier, and a 60-Mc/sec i.f. amplifier with a bandwidth of 4 Mc/sec. The larger signals were displayed directly on an oscilloscope by using a 100 cps sinuosoidal voltage to modulate the local oscillator (LO) frequency and also to drive the X axis of the oscilloscope. The video output signal of the i.f. amplifier was applied to the Y axis of the oscilloscope.[12]

The initial observations of radiated power were made with a junction having the I-V characteristic shown in Fig. 2 and biased on the $n=2$ mode. Figure 3 shows a signal observed for this case corresponding to a power level of 10^{-12} W, the largest power observed in these experiments. The magnitude of the signal depended on the distance between the junction and the short and was maximum when this distance was approximately $\lambda_g/2$ where λ_g is the guide wavelength.

The junction voltage at which signals appeared were found to obey (to within the ~0.1-μV experimental error of the voltage measurements) the equation

$$V=\frac{h}{2e}(\nu_{\mathrm{LO}}\pm\nu_{\mathrm{i.f.}}), \tag{3}$$

where ν_{LO} is the LO frequency and $\nu_{\mathrm{i.f.}}$ is 60

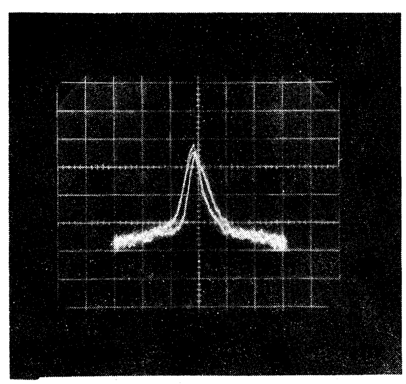

FIG. 3. Signal observed when the sample in Fig. 2 was radiating 10^{-12} W at 9.2 Gc/sec. The X axis is approximately linear in the LO frequency with a total variation of about 25 Mc/sec. The signal on the Y axis is the output of the detector and the peak results from sweeping the difference frequency between the LO and the radiation from the junction through the 4-Mc/sec pass band of the i.f. amplifier. (The dual trace is caused by a phase shift between the X and Y signals).

Mc/sec. For example, when the LO frequency was 9.125 Gc/sec, signals were observed when the junction was biased at 18.8 and at 19.0 μV. The best experimental value for the difference between these two voltages was found to be 0.25 ± 0.05 μV, which corresponds to a frequency difference of 121 ± 24 Mc/sec, in agreement with the expected difference of 120 Mc/sec. Whenever the LO frequency was changed the bias voltage had to be reset according to Eq. (3), and the two signals were always separated by a voltage corresponding to a frequency difference of about 120 Mc/sec. These facts leave little doubt that the junction was actually radiating.[13]

A signal like that in Fig. 3 could be observed for at least several minutes if the bias voltage was occasionally readjusted. Although the 4-Mc/sec pass band of the i.f. amplifier corresponds to a junction voltage range of only 8 nV, the slopes of the current steps in the I-V curves are so steep (typically 0.2 μV/mA) that a current source with noise and long-term drift of as much as 10 μA can hold the junction bias voltage to within 2 nV.

An upper limit on the spectral purity of the radiation emitted can be obtained from the swept-frequency display in Fig. 3. All of the signals observed by this method had a frequency width equal to the 4-Mc/sec bandwidth of the i.f. amplifier so that an upper bound of about 1 part in 10^4 can be placed on the fractional intrinsic spectral width of the power emitted by the junction at constant voltage.

In addition to the expected signals observed with the junction biased on the $n=2$ mode, signals were observed with the junction biased on modes $n=1$, 3, 4, 5, and 6, even though the LO was set to detect a frequency corresponding to the second mode. The magnitude of these signals ranged from 10^{-14} to 10^{-12} W, and for each mode signals were observed at two different voltages (as for the $n=2$ mode).

In order to show that these signals were not due to higher harmonic mixing in the mixer, a high-pass wave-guide filter (cutoff frequency = 15 Gc/sec, attenuation of 100 db at 9.2 Gc/sec) was inserted between the mixer and junction. All signals were observed to disappear, thus showing that radiation was indeed being emitted by the junction at the second mode frequency even though it was biased on a mode corresponding to a higher or lower frequency.[14] (A signal at a frequency corresponding to the $n=1$ mode would be completely attenuated by the X-band guide itself.)

An explanation of this harmonic and fractional harmonic radiation follows from a self-consistent solution of the junction equations discussed in reference 5. There the equations were linearized assuming that the induced ac voltage was small compared to the dc bias voltage. Here we briefly discuss the nonlinear effects which arise when the ac voltage is larger that the dc bias and derive a criterion for multimode excitation.

If the junction is biased to a voltage $V=\hbar\omega_n/2e$, the nth mode will be excited with an ac voltage amplitude v_n. If we assume that the $(n-m)$th mode is simultaneously excited[15] with an amplitude v_{n-m}, the Josephson ac current will contain frequency components at both ω_n and ω_{n-m}. Ohm's law relating v_{n-m} to the $(n-m)$th current harmonic provides a self-consistency condition which must be satisfied in order for such multimode excitation to occur.

With the above voltages present, the time dependence of the relative gap phase[16] φ is

$$\varphi=\omega_n t+\frac{v_n}{V}\sin(\omega_n t+\theta_n)+\frac{n}{n-m}\frac{v_{n-m}}{V}\sin(\omega_{n-m}t+\theta_{n-m}). \quad (4)$$

Near threshold where v_{n-m}/V is small, the Josephson current $j=j_1\sin\varphi$ becomes

$$j_1\sin\left[\omega_n t+\frac{v_n}{V}\sin(\omega_n t+\theta_n)\right]+j_1\frac{n}{n-m}\frac{v_{n-m}}{V}\sin(\omega_{n-m}t+\theta_{n-m})\times\cos\left[\omega_n t+\frac{v_n}{V}\sin(\omega_n t+\theta_n)\right]. \quad (5)$$

The amplitude of the $(n-m)$th harmonic can be obtained from (5). For the case of an odd fractional subharmonic, this amplitude is[17]

$$j_1\frac{n}{n-m}\frac{v_{n-m}}{V}J_1\left(\frac{v_n}{V}\right)\cos\theta_n, \quad (6)$$

where J_1 is the first-order Bessel function. This current drives the $(n-m)$th mode inducing a voltage of amplitude

$$v_{n-m}=j_1\frac{Q_{n-m}}{C\omega_{n-m}}\frac{n}{n-m}\frac{v_{n-m}}{V}\left|J_1\left(\frac{v_n}{V}\right)\cos\theta_n\right|, \quad (7)$$

where C is the junction capacitance and Q_{n-m} is the loaded Q of the $(n-m)$th mode. (This is just the Ohm's-law equation mentioned previously; $Q_{n-m}/C\omega_{n-m}$ is the junction resistance on resonance.) The nth current harmonic of (5) is $j_1 J_0(v_n/V)$, so that the voltage amplitude of the nth mode is determined by

$$v_n = j_1 \frac{Q_n}{C\omega_n}\left|J_0\left(\frac{v_n}{V}\right)\right|. \tag{8}$$

Dividing Eq. (7) by V and using Eq. (8), the threshold condition can be written as

$$\left|J_0\left(\frac{v_n}{V}\right)\right| = \frac{v_n}{V}\left|J_1\left(\frac{v_n}{V}\right)\right|\frac{n}{n-m}. \tag{9}$$

Here, we have taken $Q_{n-m}/\omega_{n-m} \approx Q_n/\omega_n$ and $\cos\theta_n \approx 1$. For finite excitation of the $(n-m)$th mode, the right-hand side of Eq. (9) must be larger than the left-hand side. For $n=5$ and $m=2$, this criterion is satisfied for values of v_5/V in certain intervals: The first is from 0.85 to 3.7, and the second from 3.95 to 6.95, etc. The higher gaps occur near the zeros of J_1 and are increasingly narrow. Using Eq. (7) with a value of 50 mA for j_1 and the experimental estimate of $Q_n/C\omega_n$, we find that the junctions used in these experiments were operating with v_5/V in the range 4 to 5.

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[2]$2e/h = 483.6$ Mc/sec/μV.

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[11]$Z_J = (L'/C')^{1/2}$, where L' and C' are the inductance and capacitance per unit length, respectively. See references 5 and 9.

[12]When greater sensitivity was desired, the LO frequency was not modulated. Instead, the radiation coming into the mixer was modulated at 100 cps using a microwave diode switch and the video output of the i.f. amplifier was fed into a lock-in amplifier followed by a recorder. This system could detect 5×10^{-16} W.

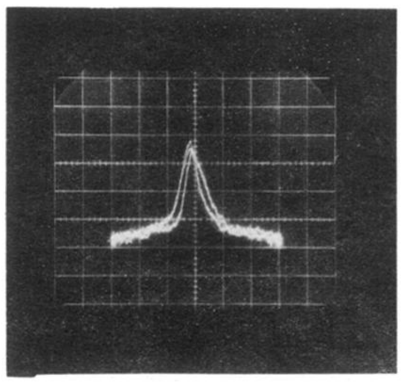
[13]Great care was taken to make sure that the signals were not somehow due to the 10^{-12} W of LO power incident on the junction from the mixer. Removal of isolators totaling 25 db of reverse attenuation had no effect, nor was there any change when the LO power was reduced by a factor of 5 (a change which did not affect the mixer gain).

[14]Indications of subharmonics were also reported by I. Giaever, reference 7.

[15]This corresponds, in general, to some fractional harmonic $(n-m)/n$ of the original frequency ω_n. Note that m can be either a positive or negative integer.

[16]Here the spatial dependence of the phase is suppressed since it does not affect the discussion and only complicates the notation.

[17]In the case of an even harmonic or fractional subharmonic, additional terms contribute, but the derivation of the threshold criterion is similar.



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